TOPOLOGICAL 3D ELEVATION DATA INTERPOLATION OF ASTER GDEM BASED ON CONTINUOUS DEFORMATION

Dr. Ali Jamali, Universiti Teknologi Malaysia, Malaysia

Prof. Francesc Antón Castro, School of Mathematical and Computational Sciences, Yachay Tech University, Ecuador

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Introduction

The Digital Elevation Model (DEM) derived from the Shuttle Radar Topography Mission (SRTM) in February 2000 and its other versions are one of the most important free spatial datasets which contain void values (Reuter et al., 2007).

Introduction

Advanced Spaceborne Thermal Emission and Reflection Radiometer Global Digital Elevation Map (ASTER GDEM) is another free spatial dataset which was released in October 2011.

Problem Statement

Void values in SRTM and ASTER GDEM are required to be calculated with data interpolation algorithms.

Inverse Distance Weighting (IDW) (Lu et al., 2008) and Kriging (Stein, 2012; Tziachris et al., 2017) are two well-known interpolation algorithms.

Problem Statement

Topological surface reconstruction from elevation data has not been addressed properly.

Topological and geometrical surface reconstruction using continuous deformation (homotopy continuation) is introduced and discussed in this research.

Interpolation through homotopy

Cubic interpolation, shape-preserving piecewise cubic interpolation and linear interpolation are three continuous interpolation algorithms.

Cubic interpolation and shape-preserving piecewise cubic interpolation are in C^2 and require at least four points for interpolation.

Interpolation through homotopy

$$L_i(x) = a_i - b_i(x - x_i), i = 1,2,3, \ldots, n-1 \; and$$

$$L(x) = \begin{cases} L_0(x), & if \; x_0 < x < x_1 \\ \quad \vdots & \quad \vdots \\ L_{n-1}(x), & if \; x_{n-1} < x < x_n \end{cases} \quad (7)$$

Interpolation through homotopy

$C_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3, i = 1,2,3, \ldots., n - 1$ and

$$C(x) = \begin{cases} C_0(x), & if\ x_0 < x < x_1 \\ & \vdots \\ C_{n-1}(x), & if\ x_{n-1} < x < x_n \end{cases}$$

Homotopy Continuation

A homotopy between two continuous functions f_0 and f_1 from a topological space X to a topological space Y is defined as a continuous map $H: X \times [0, 1] \to Y$ from the Cartesian product of the topological space X with the unit interval [0, 1] to Y such that $H(x, 0) = f_0$, and $H(x, 1) = f_1$, where $x \in X$ (Allgower and Georg, 1990).

- The two functions f_0 and f_1 are called, respectively, the initial and terminal maps.

Homotopy Continuation

- The second parameter of H, λ, also called the homotopy parameter, allows for a continuous deformation of f_0 to f_1 (Allgower and Georg, 1990).

- Two continuous functions f_0 and f_1 are said to be homotopic, denoted by $f_0 \simeq f_1$, if and only if there is a homotopy H taking f_0 to f_1.

- Being homotopic is an equivalence relation on the set C (X, Y) of all continuous functions from X to Y.

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Homotopy Continuation

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Linear and non-linear homotopy are defined by

✓ $H(x,\lambda) = (1-\lambda) f_0(x) + \lambda f_1(x)$ where $\lambda \in [0,1]$ (linear)

✓ $H(x,\lambda,n) = (1-\lambda)^n f_0(x) + \lambda^n f_1(x)$ where $\lambda \in [0,1]$ (non-linear)

Interval valued Homotopy Continuation

What is new in this research?

We constructed an interval valued homotopy continuation and applied it to the problem of topologically guaranteed 3D topographic surface reconstruction.

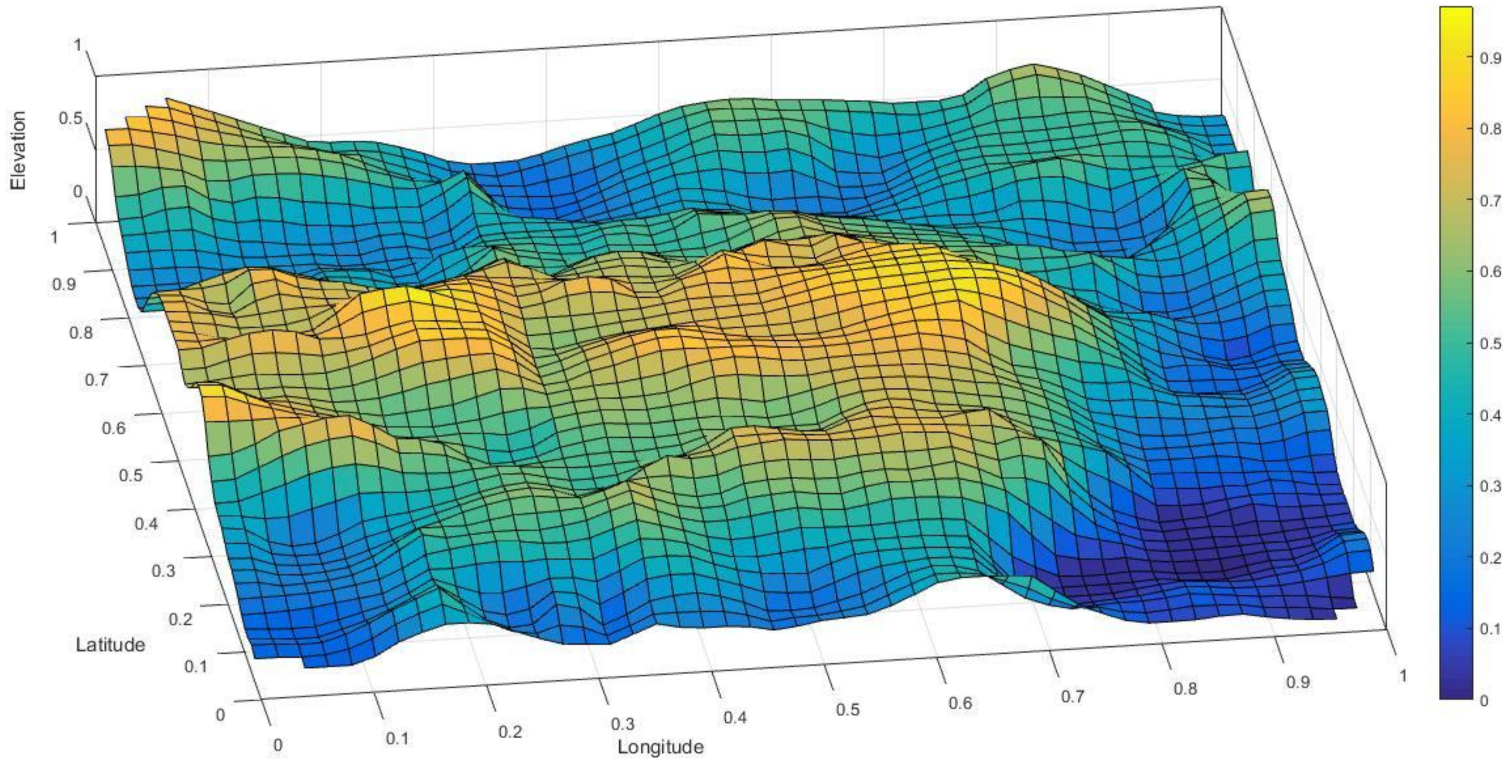
Interval valued Homotopy Continuation

$$H(x,\lambda)\begin{cases} H_0(x,\lambda), & if\ x_0 < x < x_1 \\ H_1(x,\lambda), & if\ x_1 < x < x_2 \\ & \vdots \\ H_i(x,\lambda), & if\ x_i < x < x_{i+1} \\ & \vdots \\ H_{n-1}(x,\lambda), & if\ x_{n-1} < x < x_n \end{cases}$$

Interval Valued Homotopy Continuation

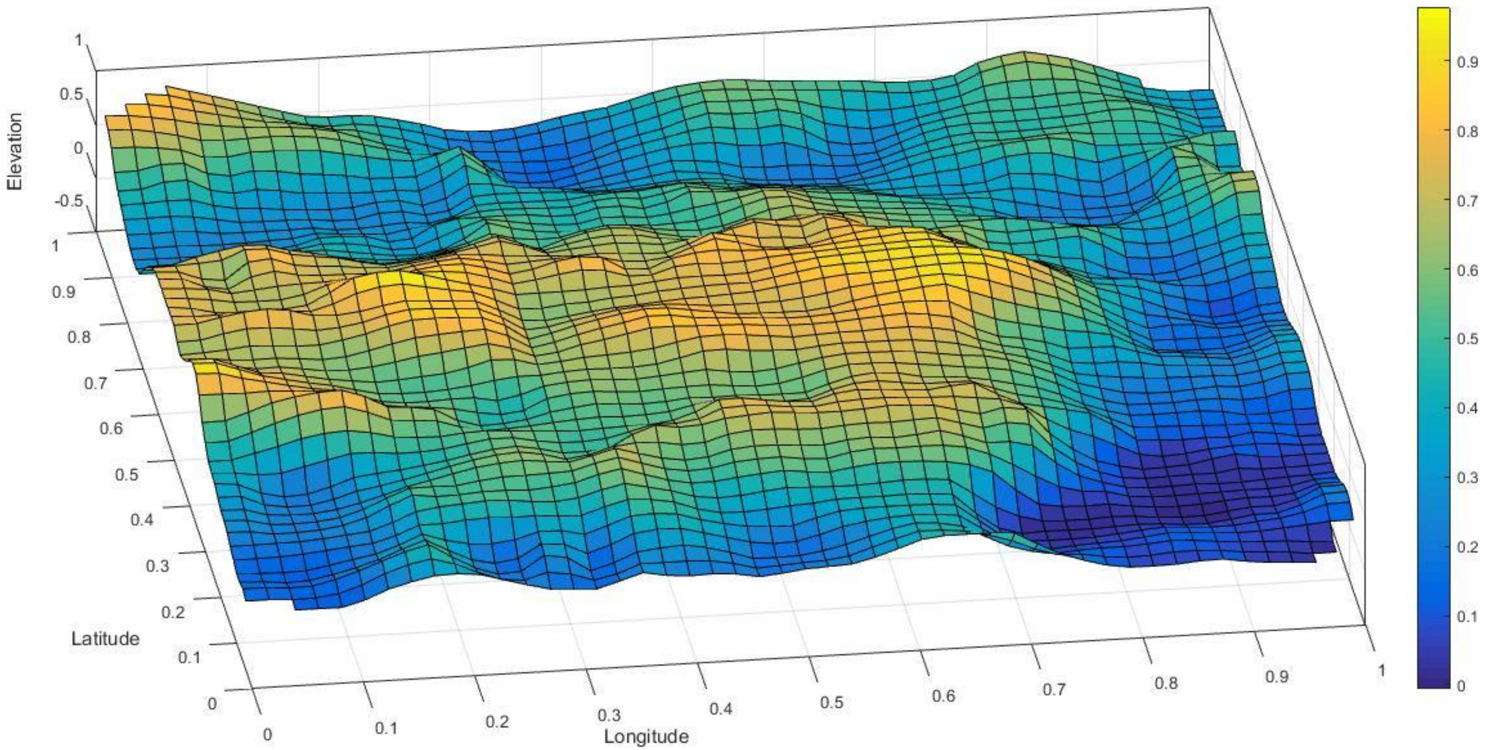
Where the H_i (x,λ) can have different degrees or all have the same degree. Considering that H_i (x,λ) can be any function as far as it is continuous.

Interval Valued Homotopy Continuation



3D elevation modelling using linear interval valued homotopy continuation.

Interval Valued Homotopy Continuation



3D elevation modelling using Cubic interval valued homotopy continuation.

Interval Valued Homotopy Continuation

Accuracy of interpolation methods in term of RMSE, Hausdorff distance and L-infinity (in meter).

Interpolation methods	Linear homotopy	Cubic homotopy	Cubic-linear homotopy	IDW	TIN
RMSE	0.2114	0.1984	0.1785	1.5439	7.5641
L-infinity	1.49	1.16	0.8741	4.97	18.06
Hausdorff distance	0.5	0.498	0.5	0.5833	14.1343

Conclusions

Two methods of interval valued homotopy (including linear and cubic) for spatial data interpolation were developed and evaluated against IDW and TIN methods.

Results of the linear and non-linear interval valued homotopy have better results compared to IDW and TIN in term of RMSE, L-infinity and Hausdorff distance.

Thank you