Defining simple *n*D operations based on prismatic *n*D objects

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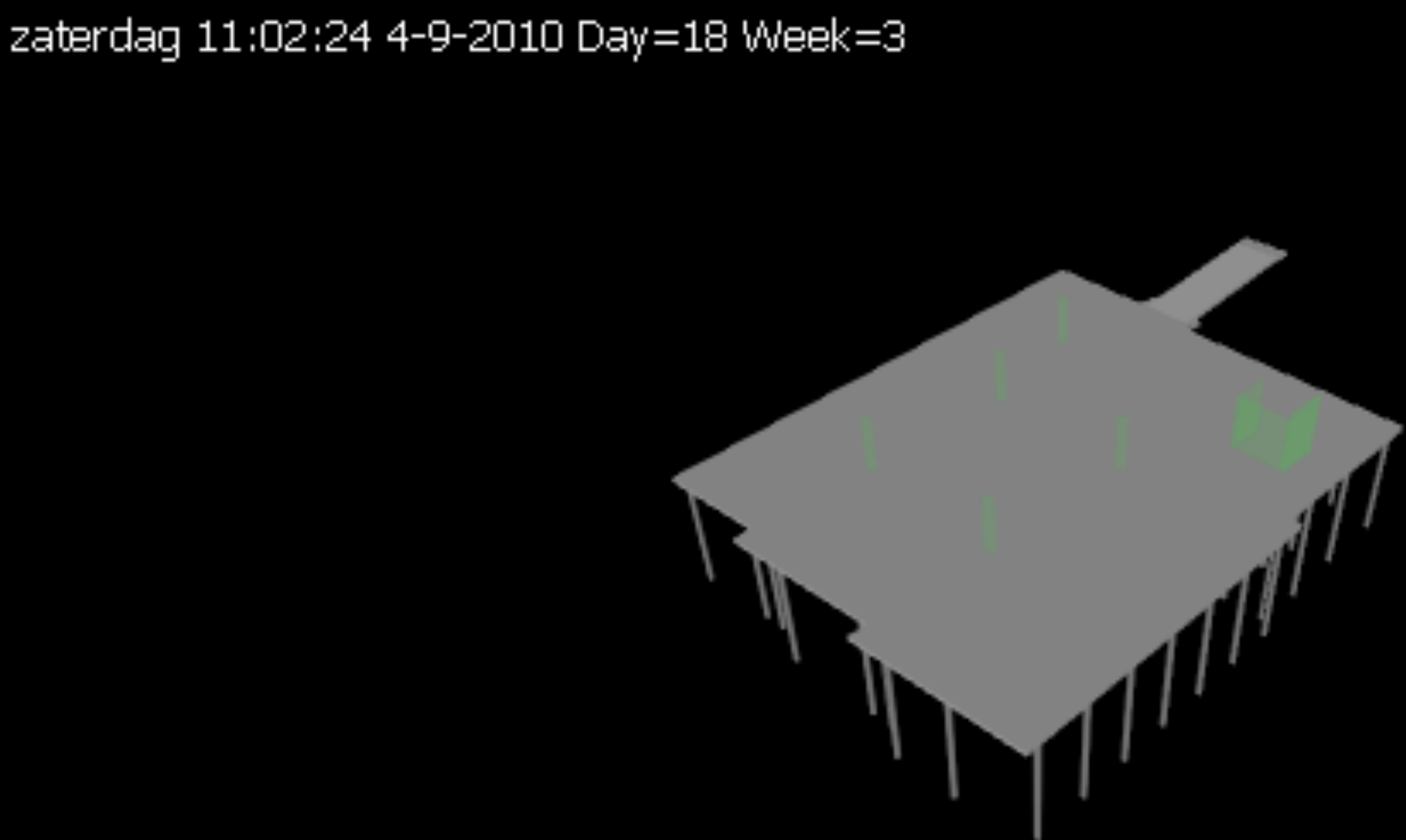
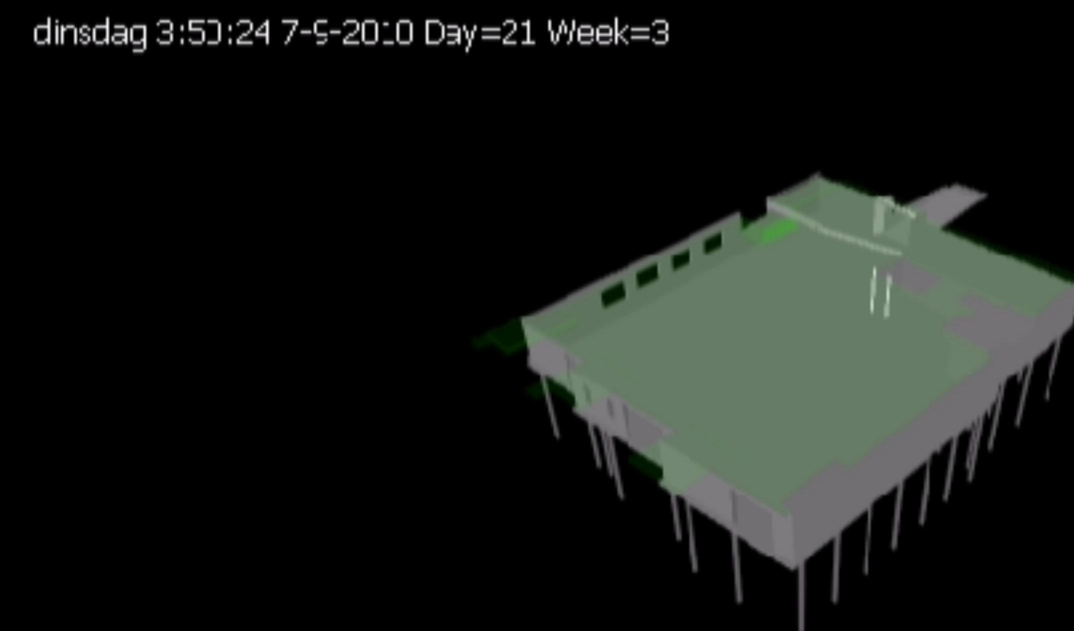
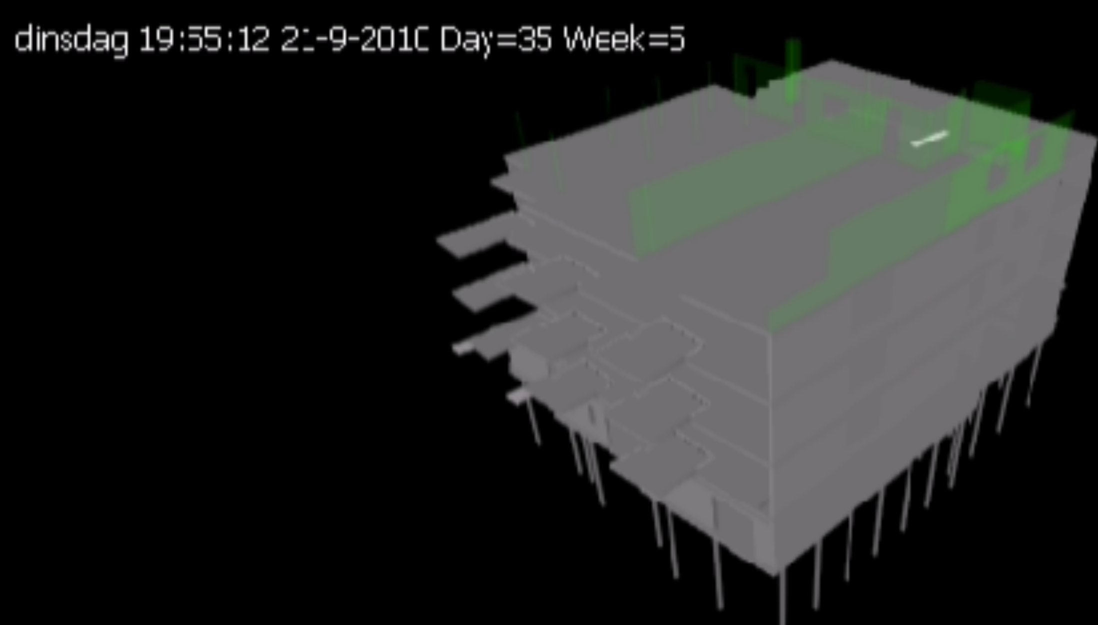
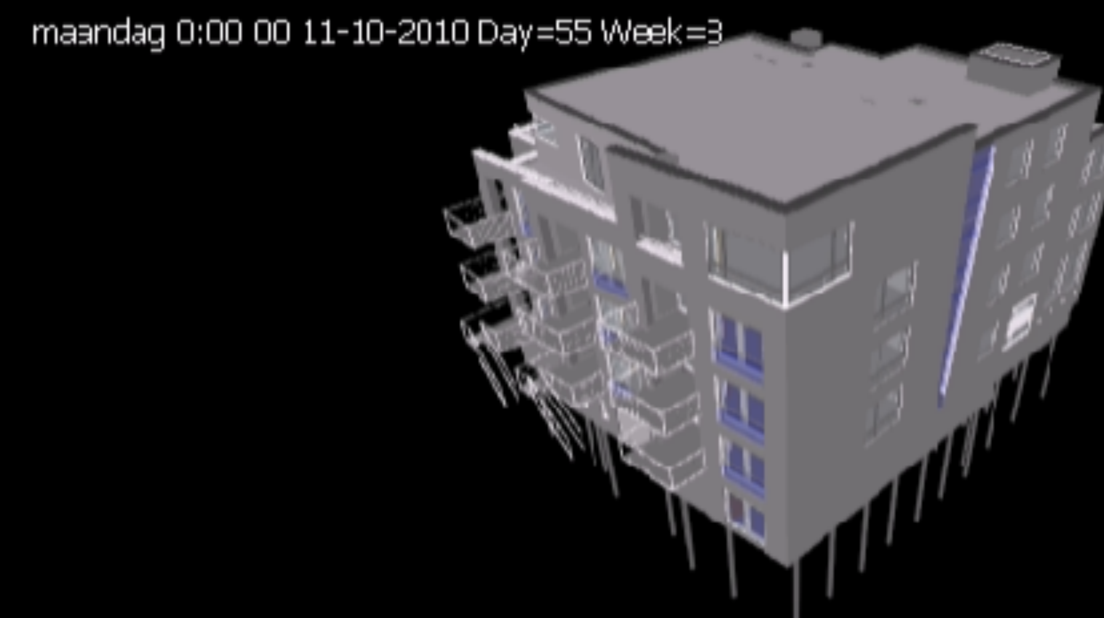
Higher-dimensional modelling



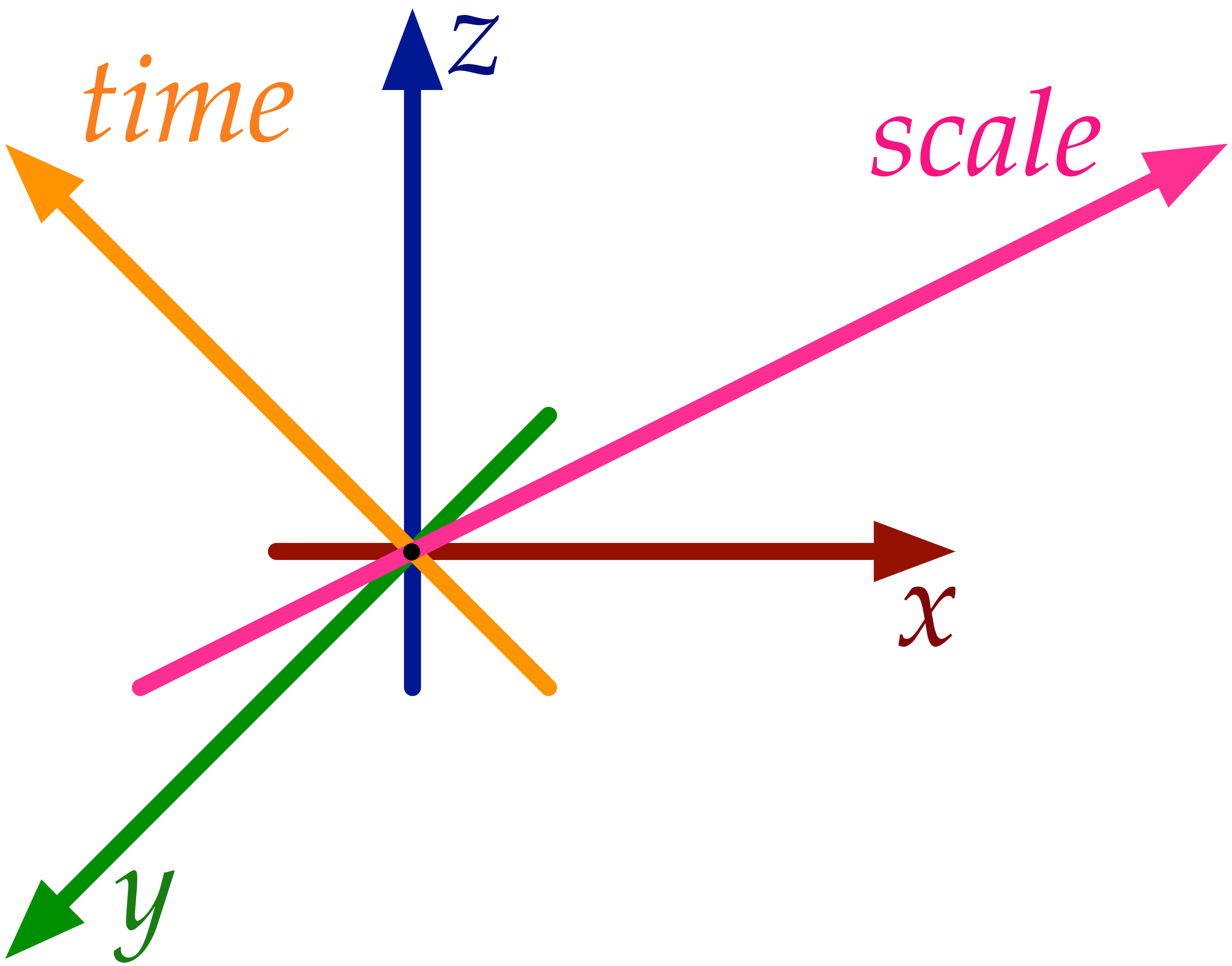
Higher-dimensional modelling



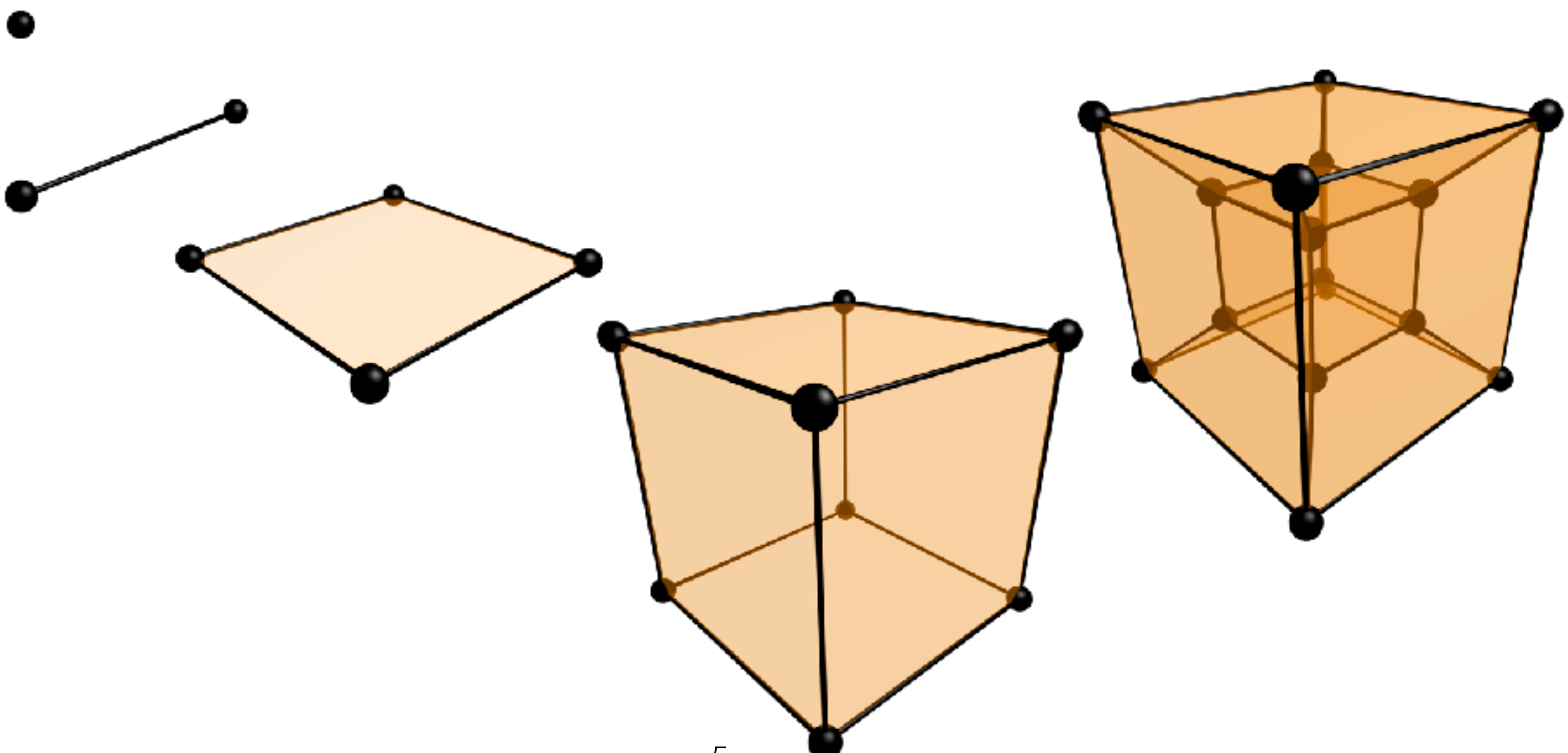




Higher-dimensional modelling



Higher-dimensional modelling



Higher-dimensional models

- Space-intensive but powerful
- Simple and consistent way to store the geometry, attributes and topological relationships between any objects of any dimension
- For instance, the complete history of a set of 3D objects in time and all of their possible representations at various scales can be modelled as a single 5D model.

Higher-dimensional models

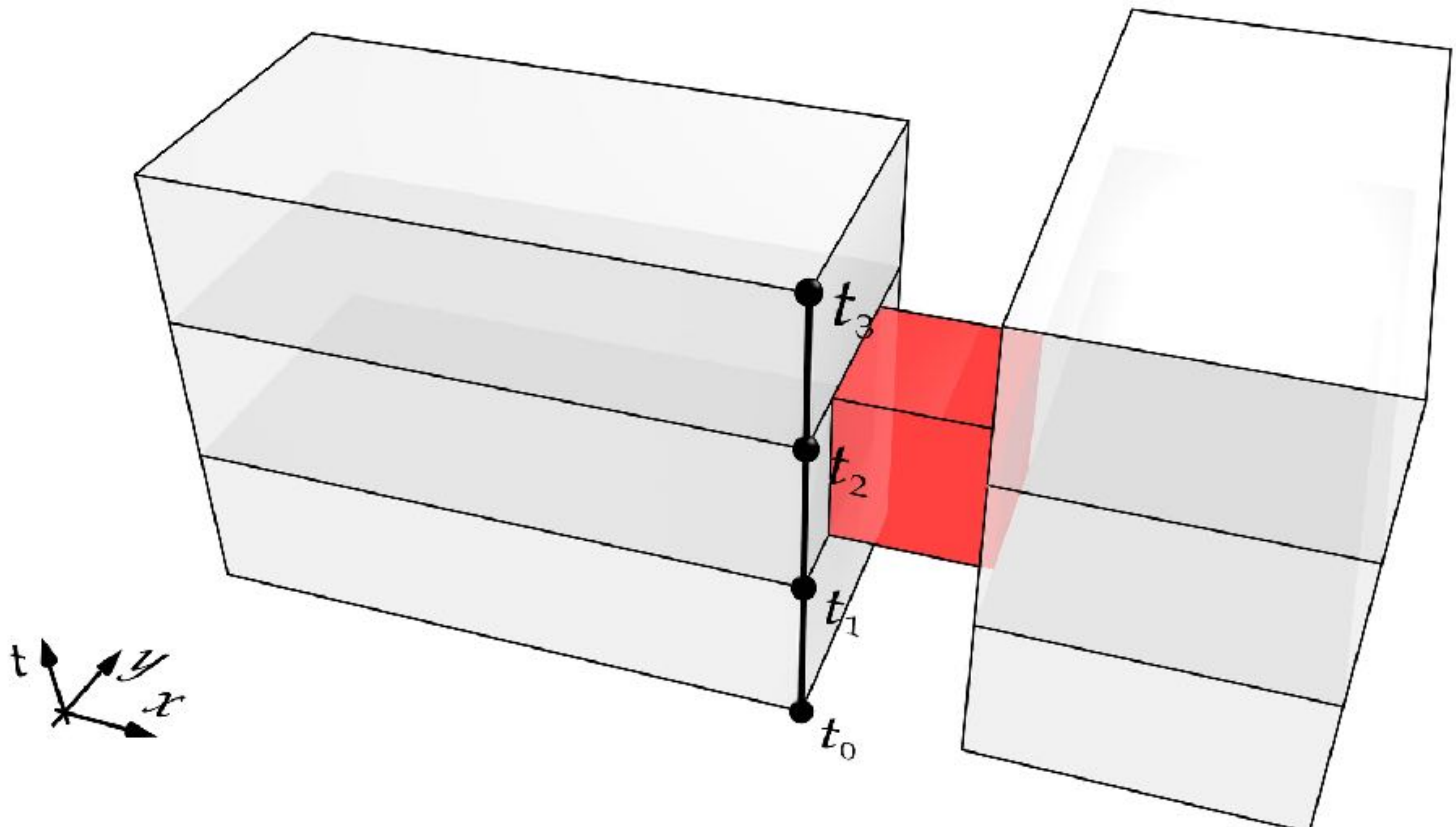
- However… building higher-dimensional models is not intuitive.

- While we are used to solving problems in 2D and 3D, and we thus have an intuitive understanding of 2D and 3D space, and of operations on 2D and 3D objects (e.g. the Euler operators typically used to create polyhedra), we do not have similar intuitive notions and experiences for *n*D objects.

Extrusion: our proposed solution

Extrusion (2D to 3D)

Extrusion (2D+time)



Extrusion: our proposed solution

- Use *n*D prismatic polytopes—analogous to prisms in 3D—as a base for the creation and manipulation of *n*D models.

- Prismatic polytopes essentially represent objects that are unchanged along a single dimension, such as objects that do not move or change in shape.

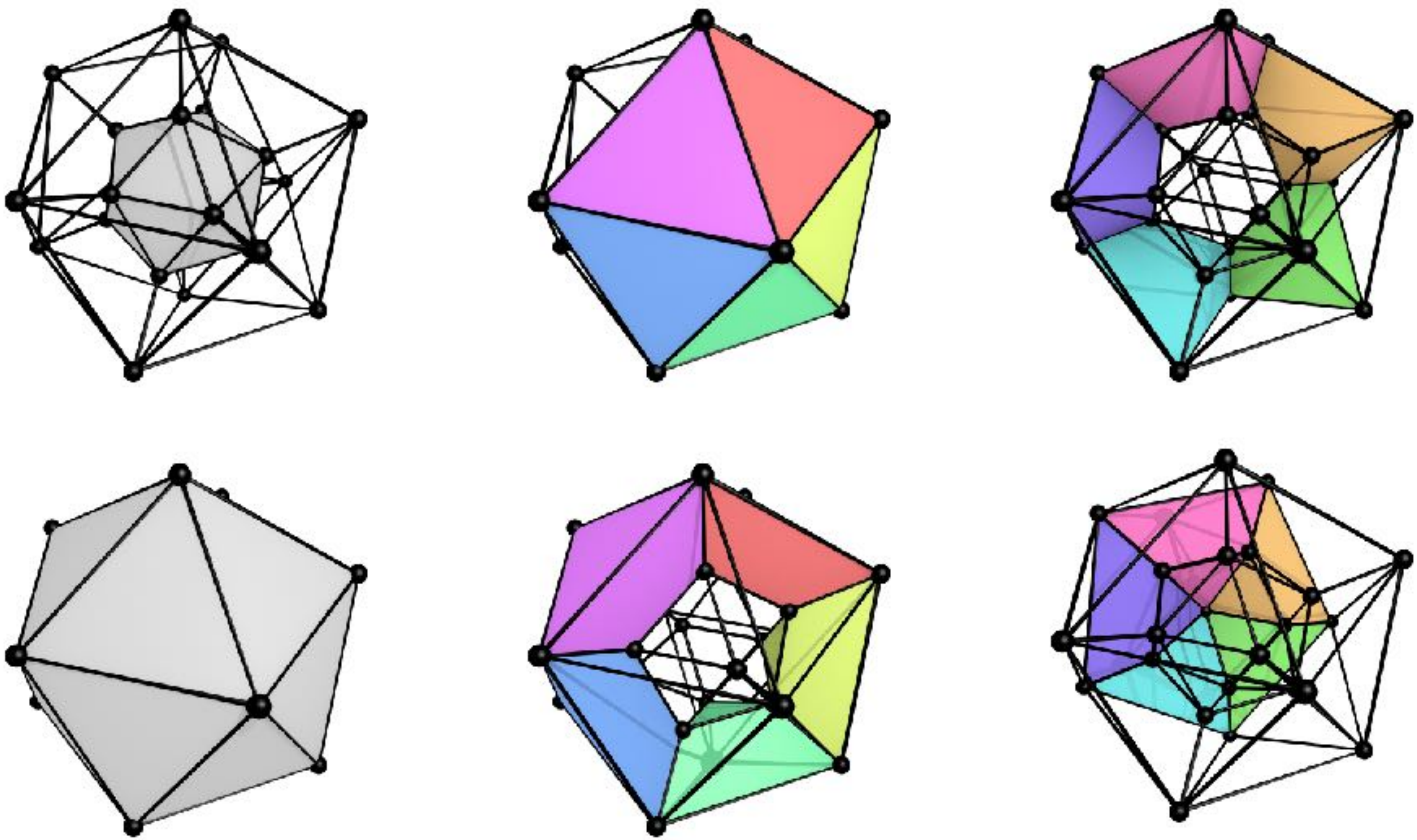
Extrusion (*n*D)

- Unlike arbitrary *n*D objects, *n*D prismatic polytopes can be easily created based on *n*D extrusion using existing algorithms.

- Among others:

  - **A dimension-independent extrusion algorithm using generalised maps.** Ken Arroyo Ohori, Hugo Ledoux and Jantien Stoter. *International Journal of Geographical Information Science* 29(7), July 2015, pp. 1166–1186.

Extrusion (3D to 4D)



Our proposed approach

1. Creating simple *n*D objects based on *n*D prismatic polytopes

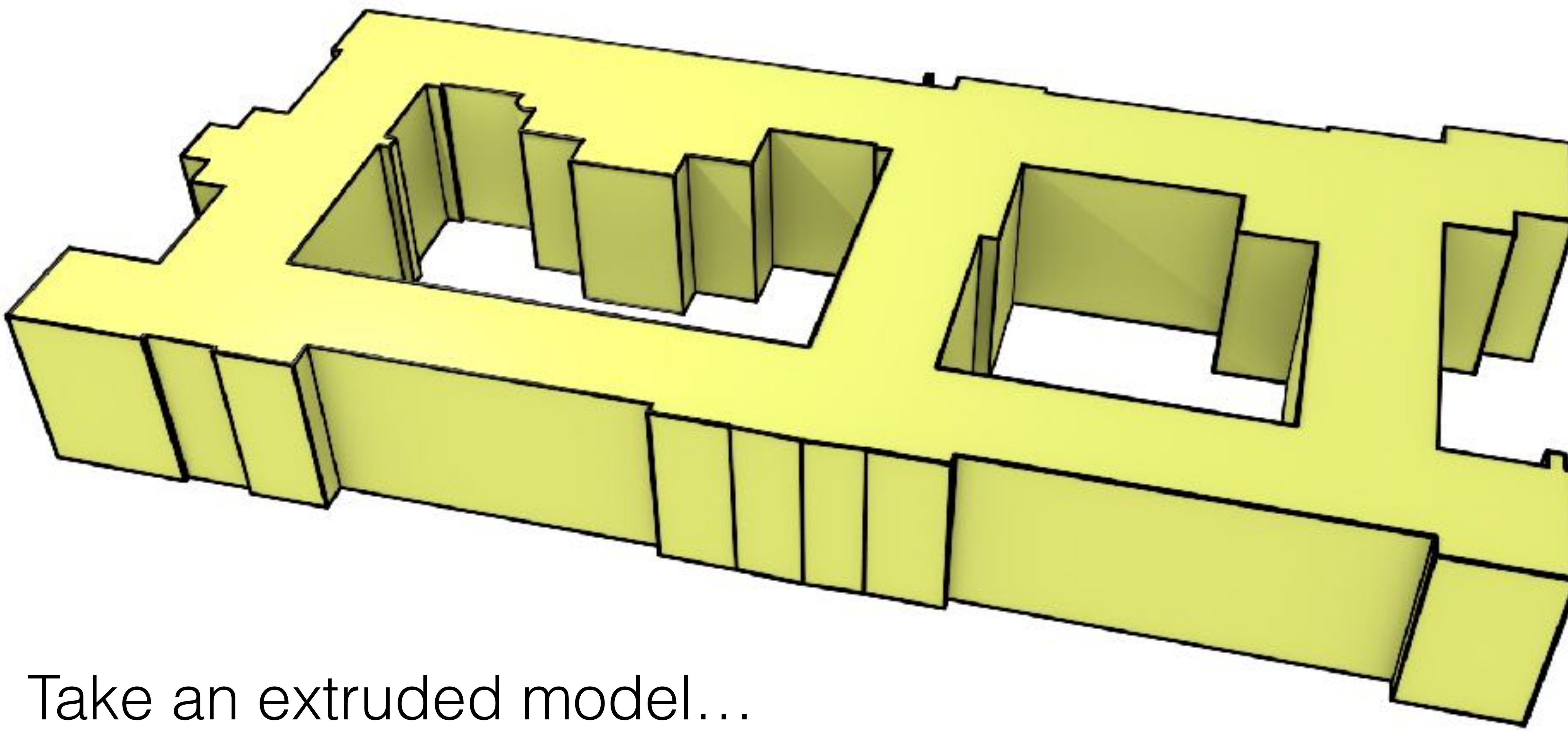
2. Defining simple modification operations at the vertex level

3. Simple postprocessing to fix the errors introduced in the model

Extrusion (*n*D)

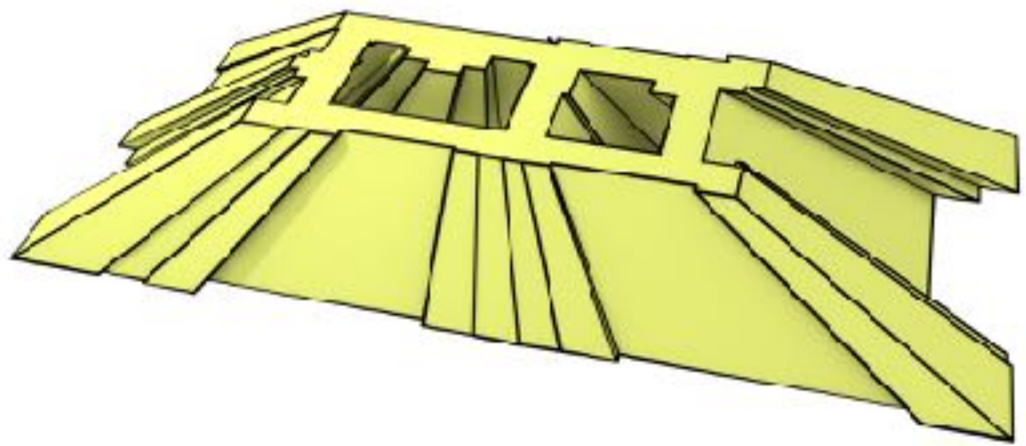
- By applying a modification operation to the vertices of the top or bottom facet of a prismatic polytope, they can be used to model many common geographic phenomena.
- Shown today, objects that are:
  - moving and/or changing shape in time, or
  - being generalised as their LOD is reduced.

Model moving objects

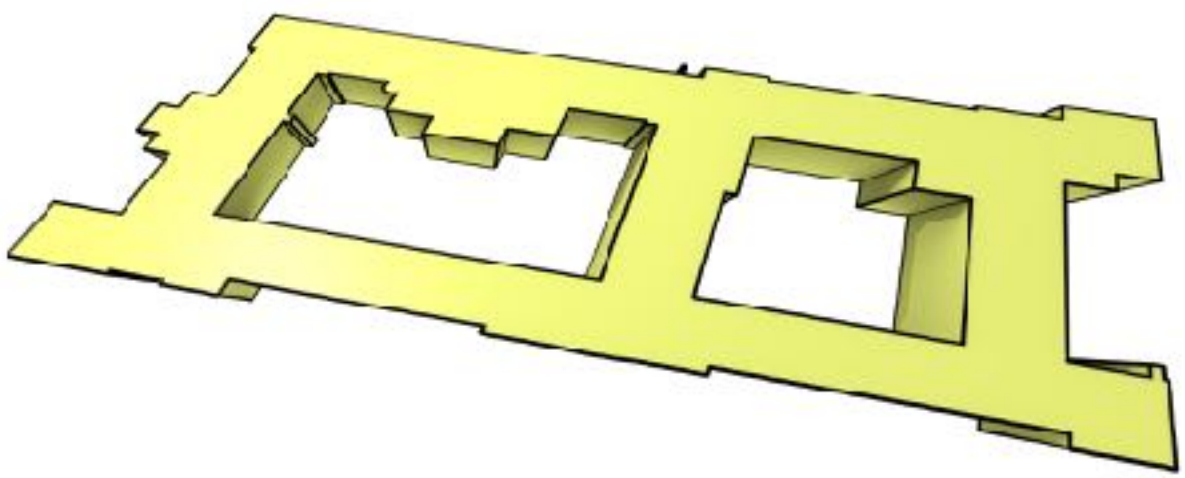


Take an extruded model…

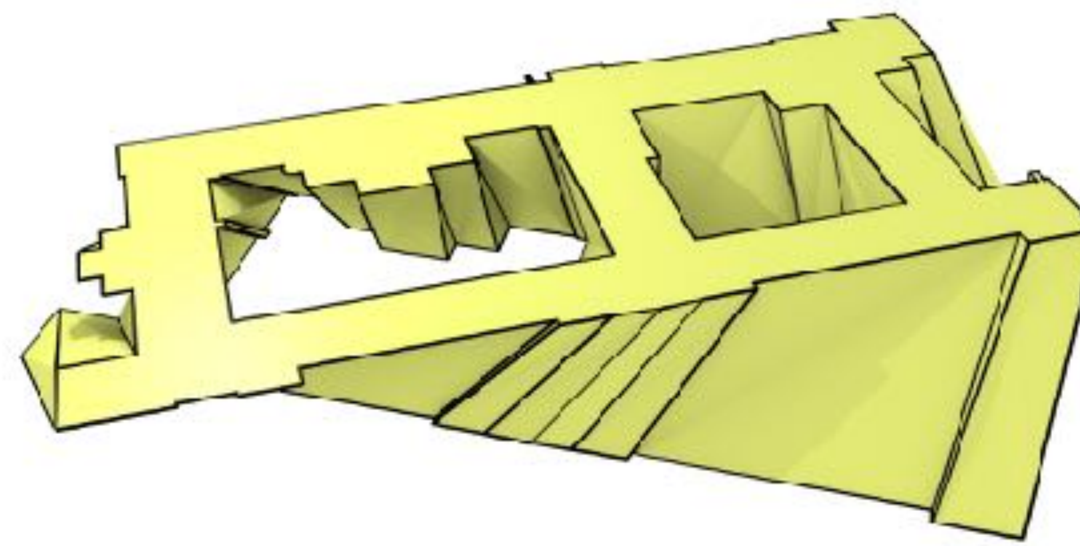
Model moving objects



Scale

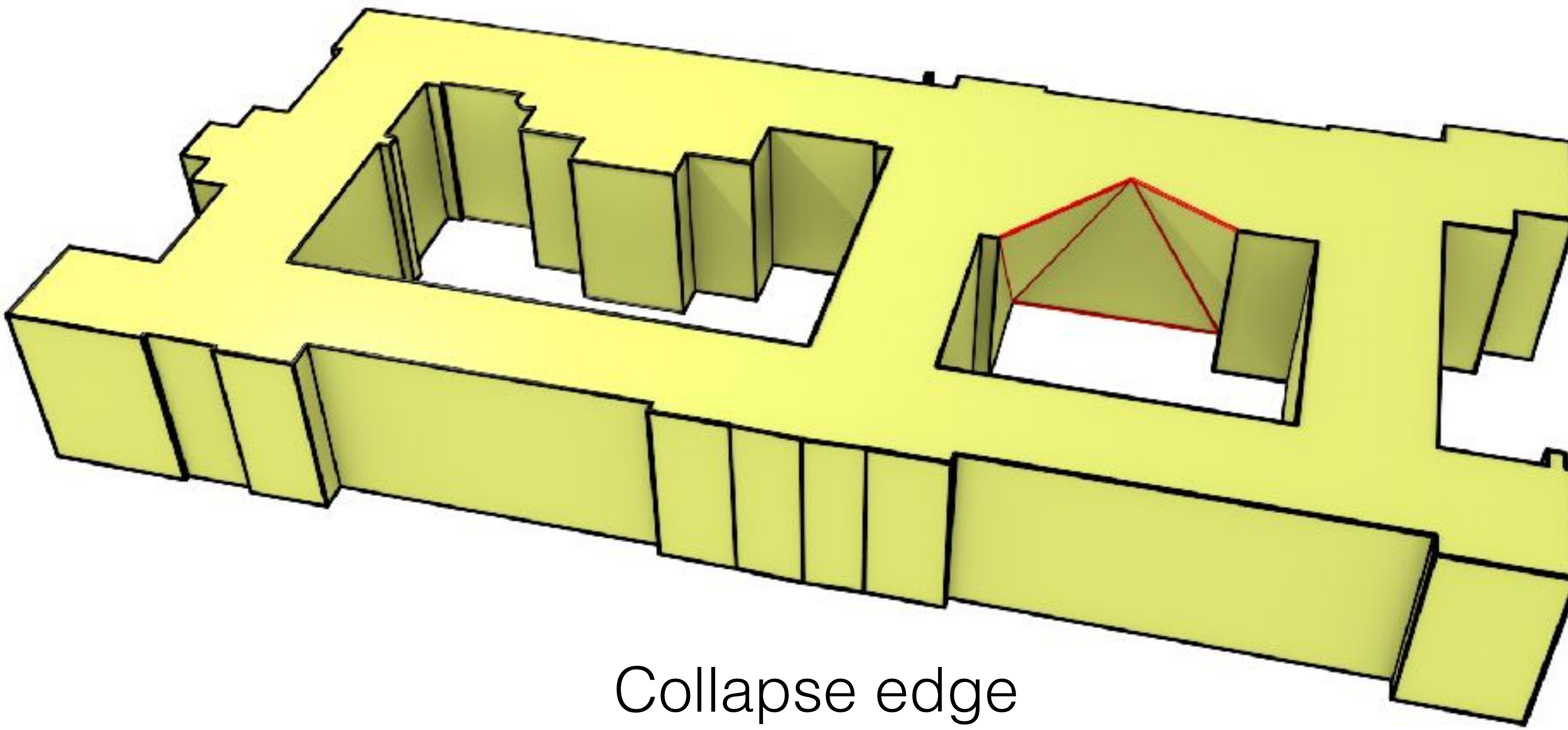


Translation



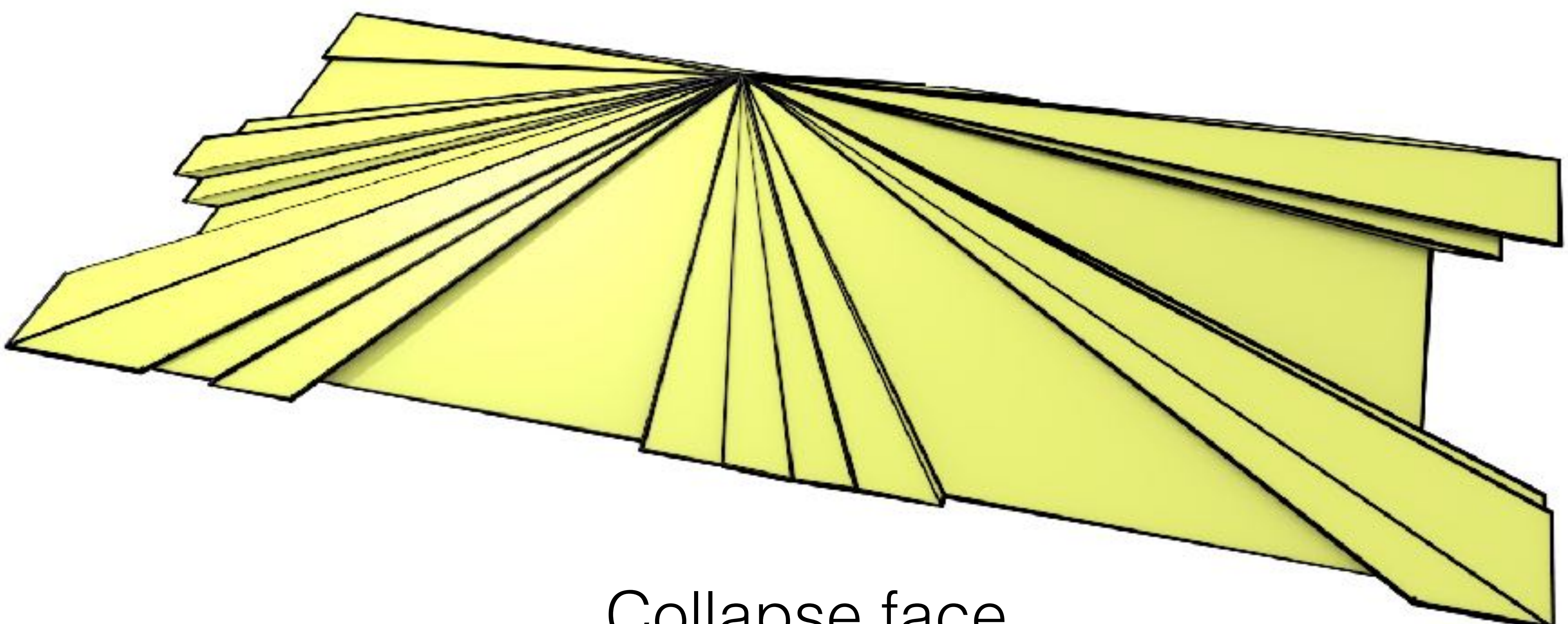
Rotation

Generalisation through collapse



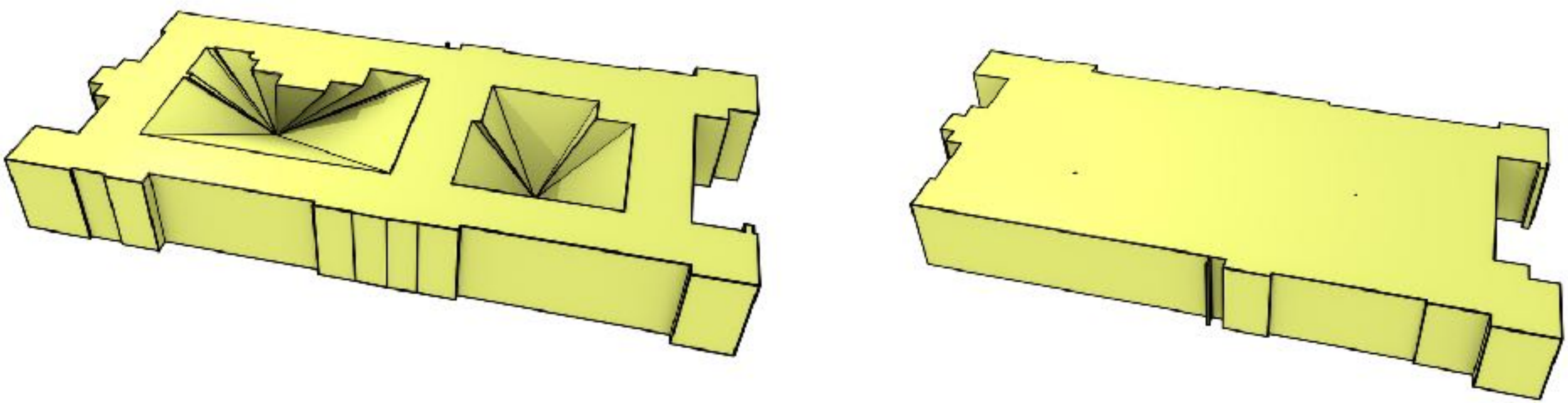
Collapse edge

Generalisation through collapse



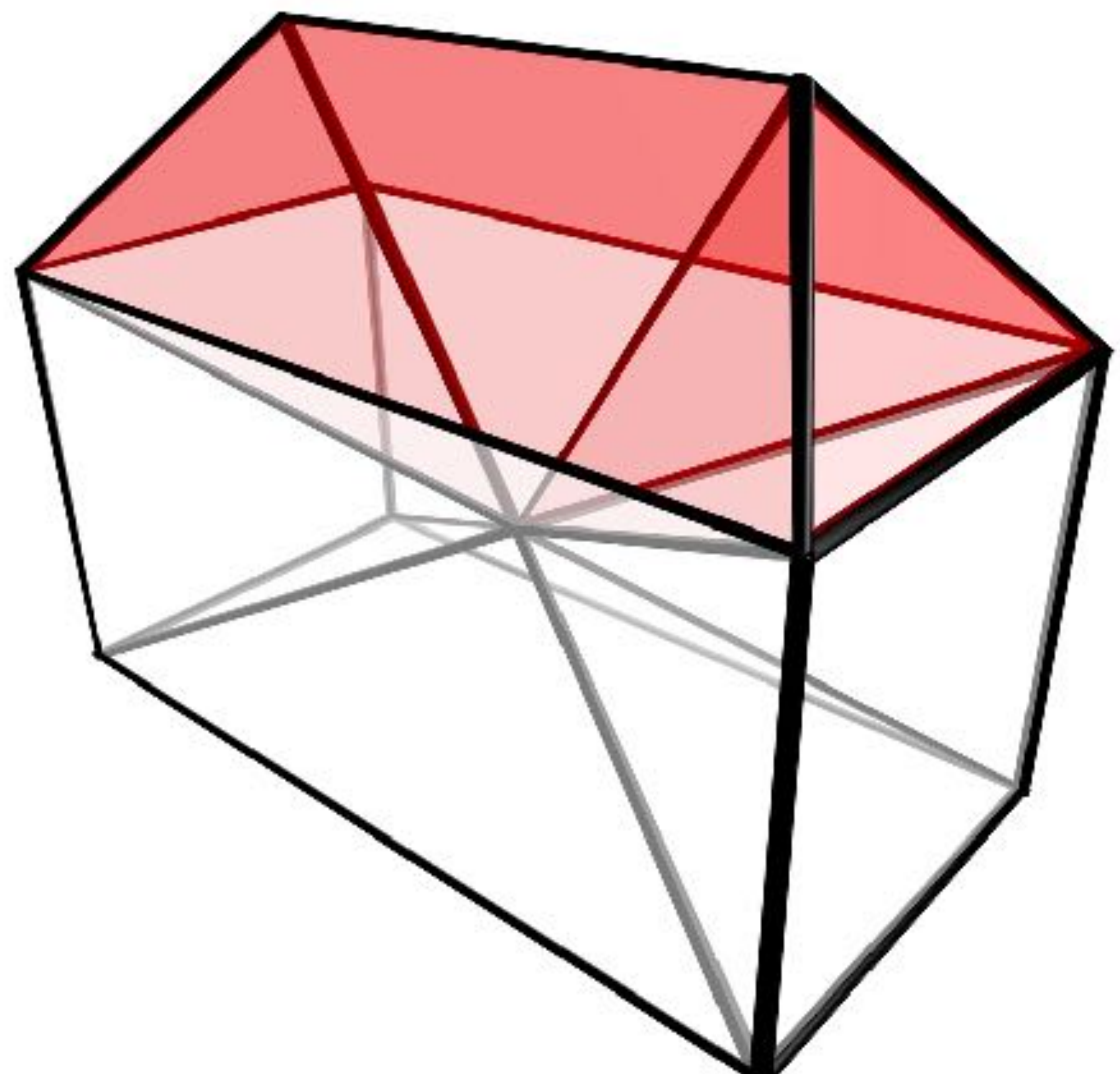
Collapse face

Generalisation through collapse



Collapse holes

Generalisation through collapse



Collapse 4-cell

Caveats (examples)

- A rotation applied to the vertices of the top or bottom facet of a prismatic polytope causes its side facets to deform.

- If a lower-dimensional cell that is collapsed is a face of some higher-dimensional cells, the higher-dimensional cells might deform so that their vertices will not lie on a hyperplane.

In the paper

- How to implement:
  - math (transformations, combinatorics)
  - data structures
  - postprocessing for the two operations shown

DEFINING SIMPLE nD OPERATIONS BASED ON PRISMATIC nD OBJECTS

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ABSTRACT:

An alternative to the traditional approaches to model separately 2D/3D space, time, scale and other parametrisable characteristics in GIS lies in the higher-dimensional modelling of geographic information, in which a chosen set of non-spatial characteristics, e.g. time and scale, are modelled as extra geometric dimensions perpendicular to the spatial ones, thus creating a higher-dimensional model. While higher-dimensional models are undoubtedly powerful, they are also hard to create and manipulate due to our lack of an intuitive understanding in dimensions higher than three. As a solution to this problem, this paper proposes a methodology that makes nD object generation easier by splitting the creation and manipulation process into three steps: (i) constructing simple nD objects based on nD prismatic polytopes—analogous to prisms in 3D—, (ii) defining simple modification operations at the vertex level, and (iii) simple postprocessing to fix errors introduced in the model. As a use case, we show how two sets of operations can be defined and implemented in a dimension-independent manner using this methodology: the most common transformations (i.e. translation, scaling and rotation) and the collapse of objects. The nD objects generated in this manner can then be used as a basis for an nD GIS.

1. INTRODUCTION

The traditional approaches to model 2D/3D space, time and scale in GIS are mostly based on adaptations to well-known 2D data structures, such as the DCEL (Muller and Preparata, 1978) and the quad-edge (Guibas and Stolfi, 1985). Many '3D' GIS internally represent objects using a 2.5D structure, essentially treating the third dimension as an attribute, or represent individual 3D objects only implicitly through the 2D surface that separates their interior from their exterior. Spatiotemporal GIS keep multiple representations of 2D structures (Armstrong, 1988), each at a different point in time, or a list of changes per object (Worboys, 1992; Peuquet, 1994), while multi-scale datasets generally consist of independent datasets at each scale with some identifiers that link equivalent objects between datasets (Friis-Christensen and Jensen, 2003; Stoter et al., 2014).

An alternative to this approach lies in the *higher-dimensional modelling of geographic information* (Arroyo Ohori, 2016), where a chosen set of non-spatial characteristics, e.g. time and scale (van Oosterom and Stoter, 2010), are modelled as true geometric dimensions in addition to the spatial ones. For instance, the complete history of a set of 3D objects in time and all of their possible representations at various scales can be modelled as a single 5D model. Mathematically, a higher-dimensional model corresponds to the definition of an n-dimensional cell complex (Section 2.1), which can be directly implemented in a computer using a variety of data structures.

Higher-dimensional models are undoubtedly space-intensive, but they are also very powerful: they provide a simple and consistent way to store the geometry, attributes and topological relationships between any objects of any dimension. However, *one of the main problems of higher-dimensional models is that they are not intu-*

for the creation and manipulation of nD models. Prismatic polytopes essentially represent objects that are unchanged along a single dimension. By applying a modification operation to the vertices of the top or bottom facet of a prismatic polytope, they can be used to model many common geographic phenomena, such as objects that are moving and/or changing shape in time, or being generalised as their LOD is reduced. Moreover, unlike arbitrary nD objects, nD prismatic polytopes can be easily created based on nD extrusion, as is explained in Section 2.2.

Our methodology (Section 3) splits the creation or modification of a set of nD objects into three steps: (i) creating simple nD objects based on nD prismatic polytopes, (ii) defining simple modification operations at the vertex level, and (iii) simple postprocessing to fix the errors introduced in the model.

Within this paper, we describe in detail two concrete examples of operations defined based on our general methodology: the most common transformations applied to GIS objects (i.e. translation, scaling and rotation) in Section 4.1, and collapsing cells in Section 4.2. We finish the paper with a discussion on the possibilities of these and other nD operations based on the same methodology in Section 5.

2. RELATED WORK

2.1 nD cell complexes and their implementation

Hereafter follows a simple intuitive definition of n-dimensional cell complexes and their related terms as used in this paper. More correct (but harder) definitions are usually based on induction. See e.g. Fomenko (1990) or Hatcher (2002).

An n-dimensional cell complex is a structure made of connected

Summary

- Defining operations that are both dimension-independent and intuitive to use can be difficult, as we do not have the same intuitive understanding of the manipulation of higher-dimensional objects that we have in 2D and 3D.

- Our proposed solution to this problem is to define some of such operations on the basis of *prismatic polytopes*, as these *n*D objects are general enough to represent many phenomena commonly modelled in GIS but still have a simple geometry that is analogous to familiar shapes—2D rectangles and 3D prisms.

Summary

- Prismatic polytopes can be easily generated using dimension-independent extrusion, which has a simple definition and a relatively easy implementation in arbitrary dimensions.

- Starting from a set of non-overlapping *(n-1)*-dimensional objects, extrusion guarantees that its output consists of a set of valid non-intersecting *n*-dimensional objects.

Future work

- Implementation using CGAL gmaps or cmaps
- Other transformations (e.g. shears, reflections and homothetic transformations)
- Apply transformations to other facets of a prismatic polytope (i.e. not the bases)
- Reverse collapse to express the creation of an arbitrary cell

Thank you!

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