Incremental construction of *n*D objects

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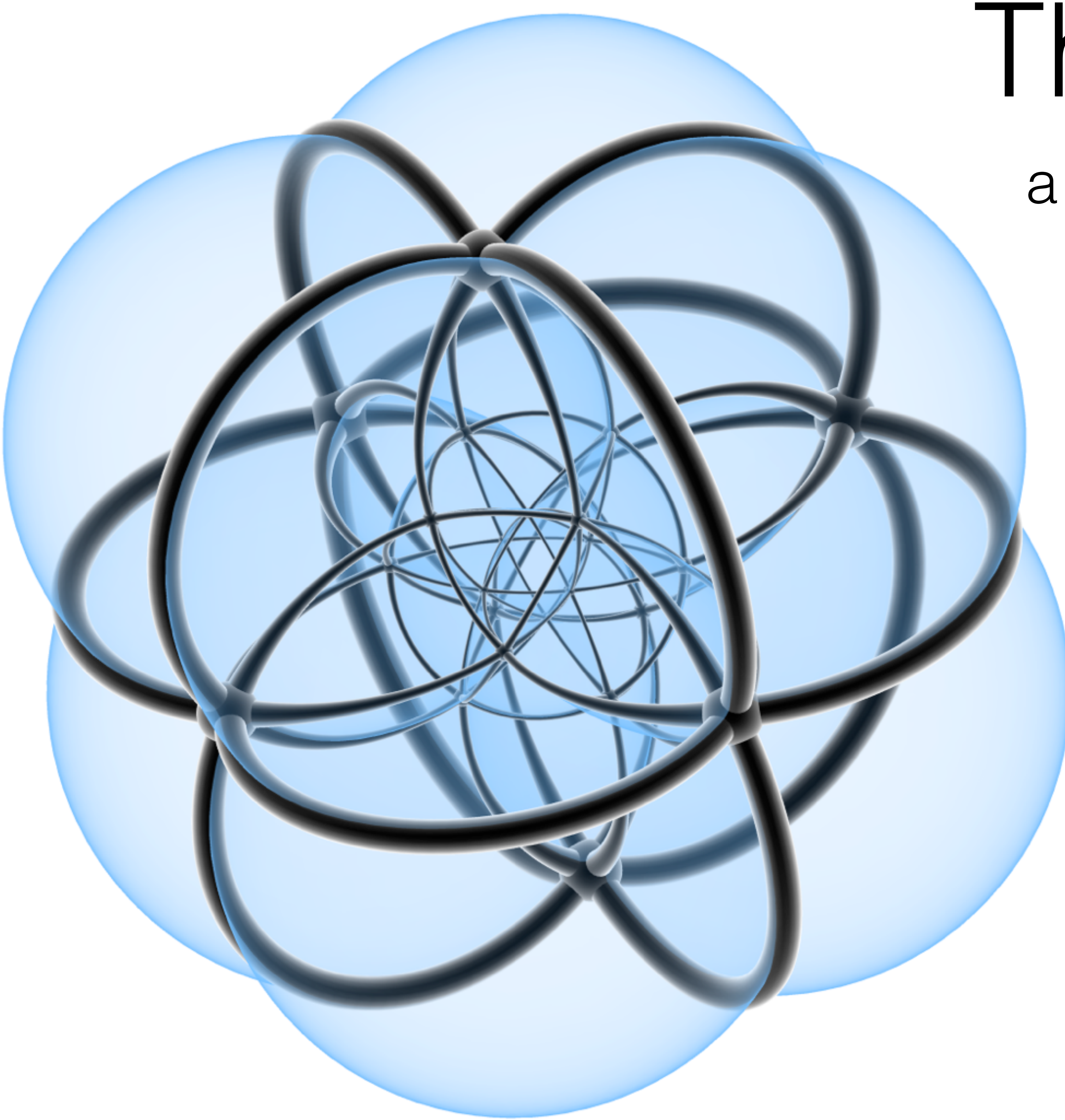
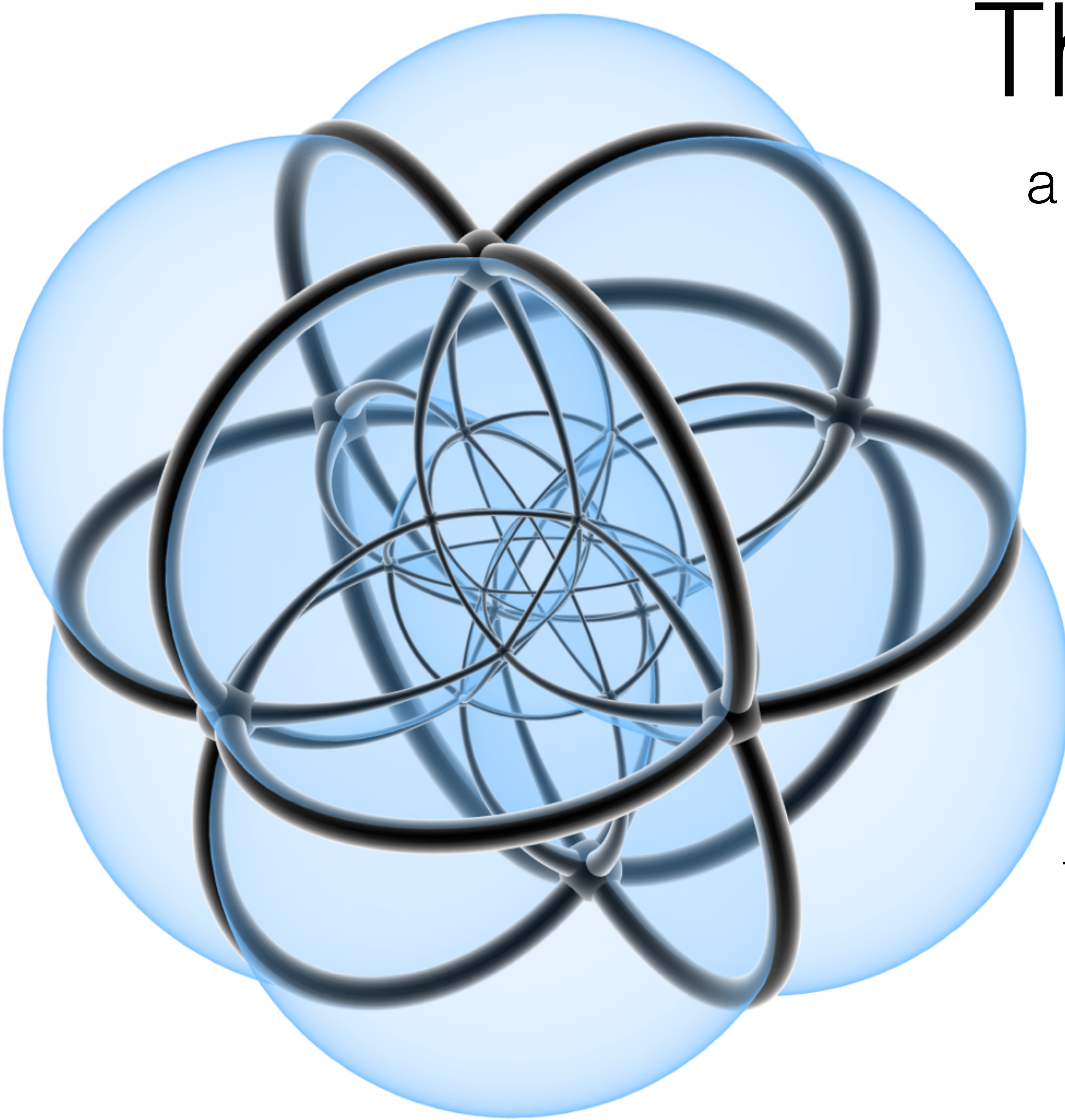
The 24-cell

a “simple” 4D object



The 24-cell

a “simple” 4D object

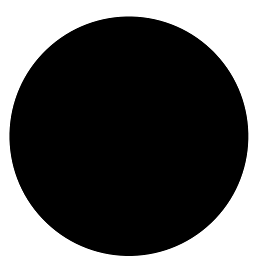


24 0D vertices
96 1D edges
96 2D faces
24 3D volumes
1 4D hypervolume

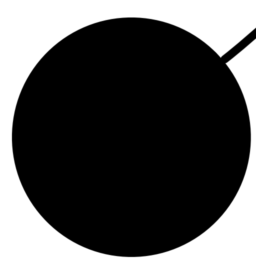
objects in more than 3D are complex!

Defining 0D-3D objects in *n*D space

0D: a vertex

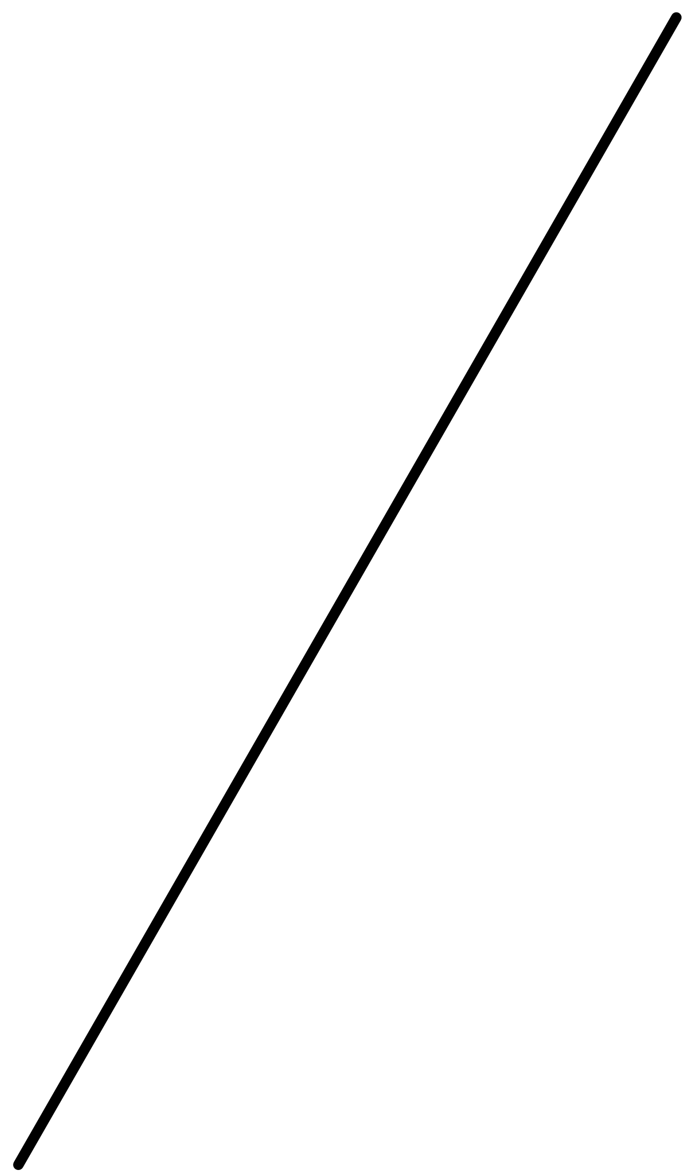


0D: a vertex

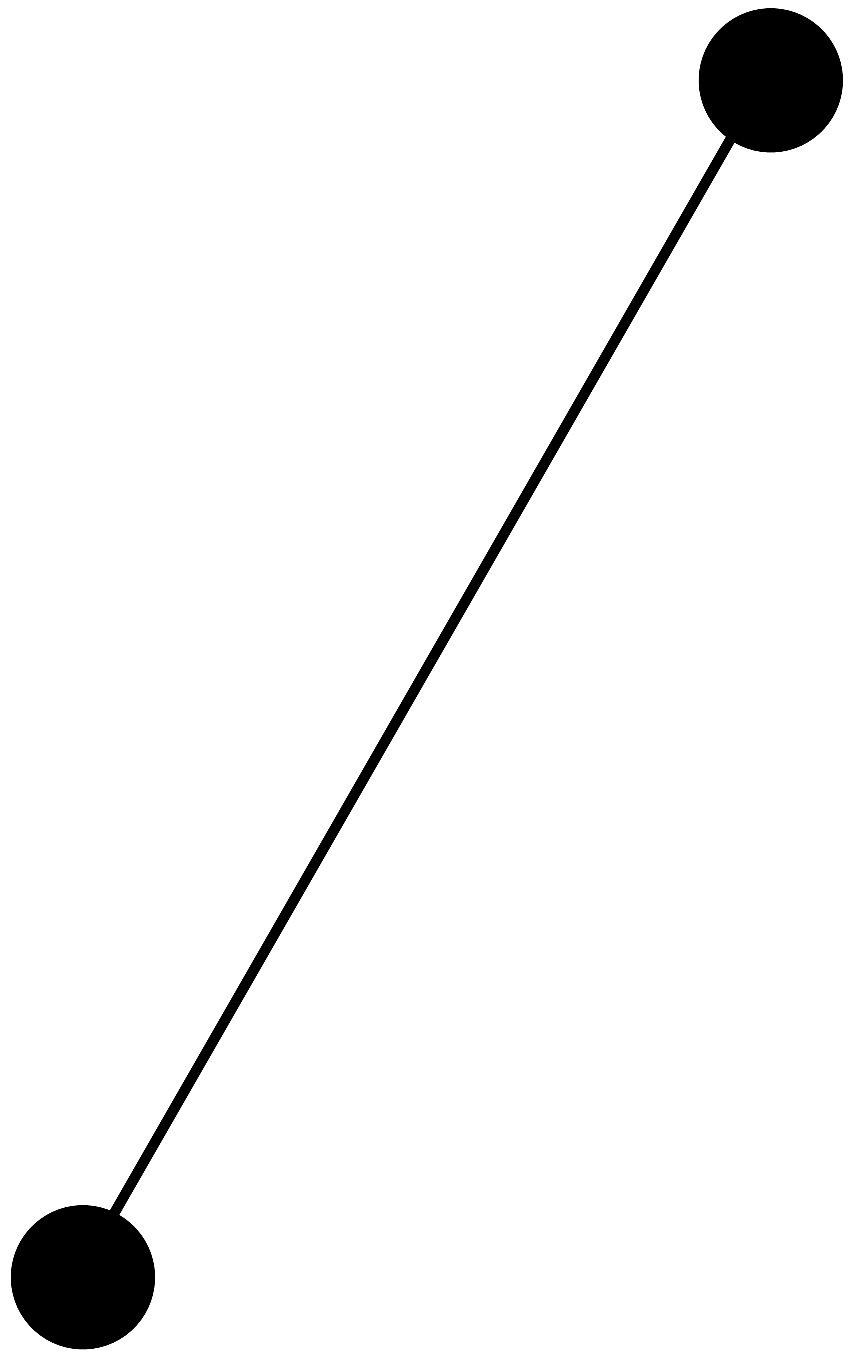


$$(x_0, x_1, \ldots)$$

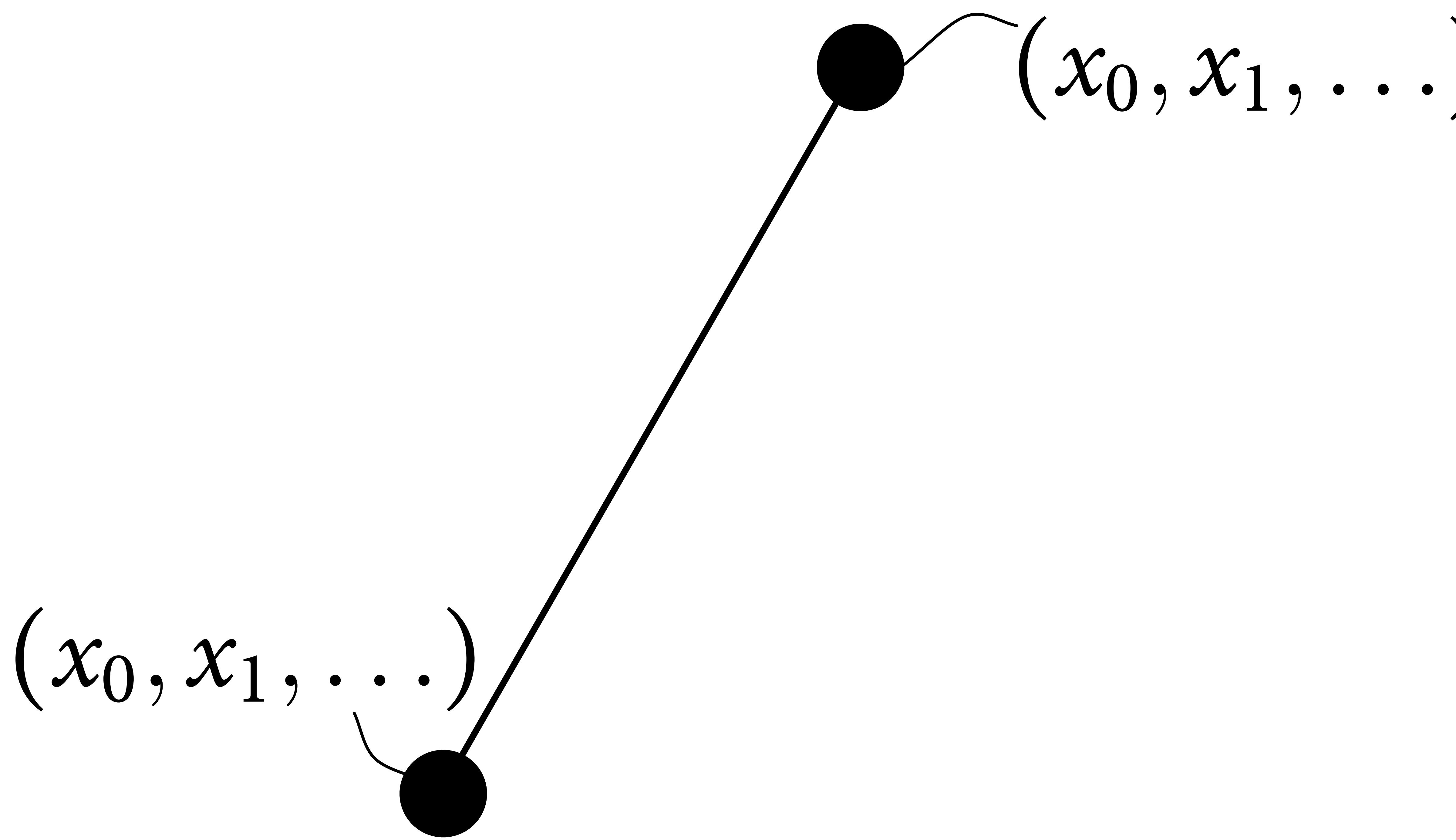
1D: an edge



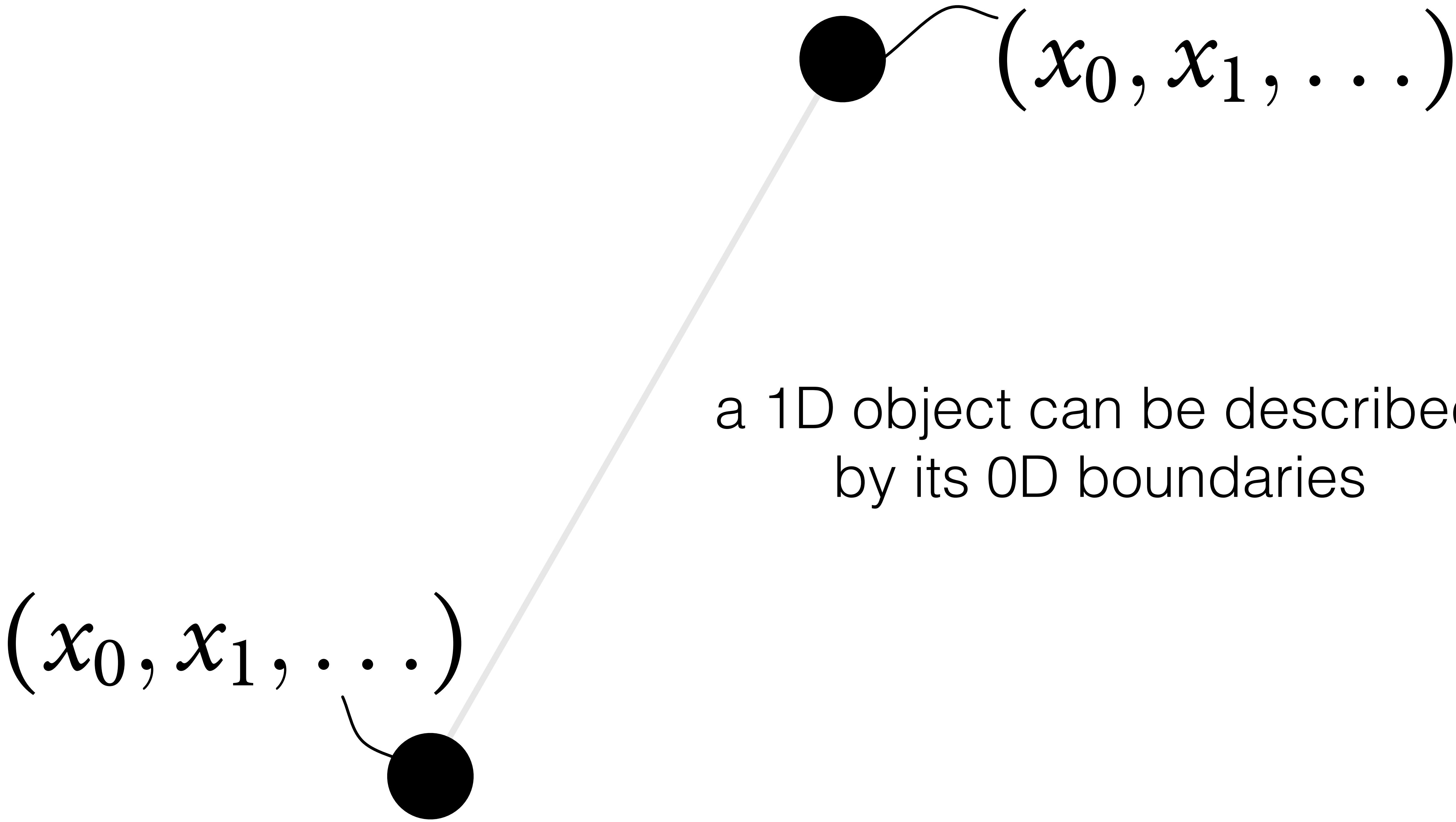
1D: an edge



1D: an edge



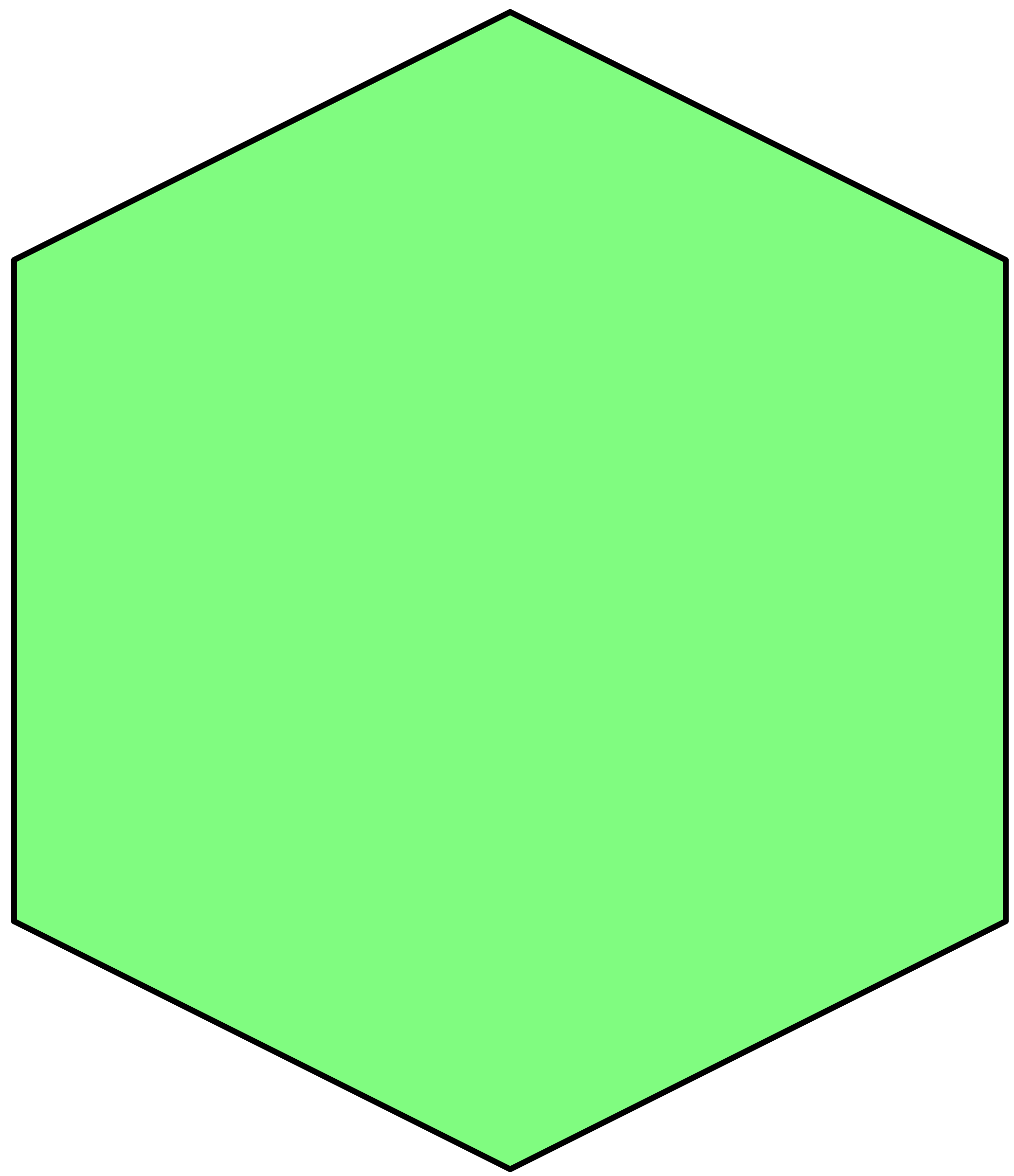
1D: an edge



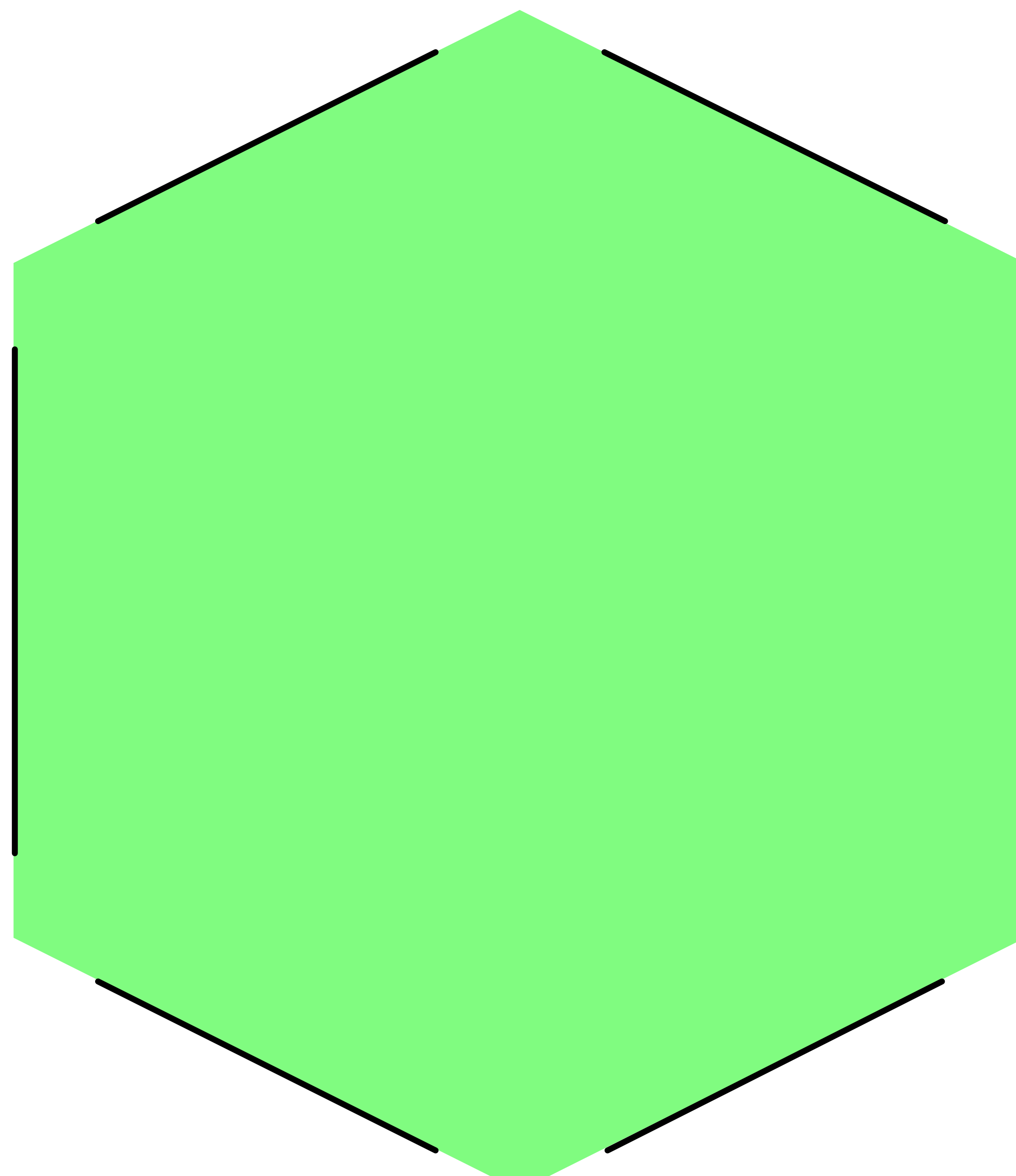
a 1D object can be described by its 0D boundaries

2D: a face

2D: a face

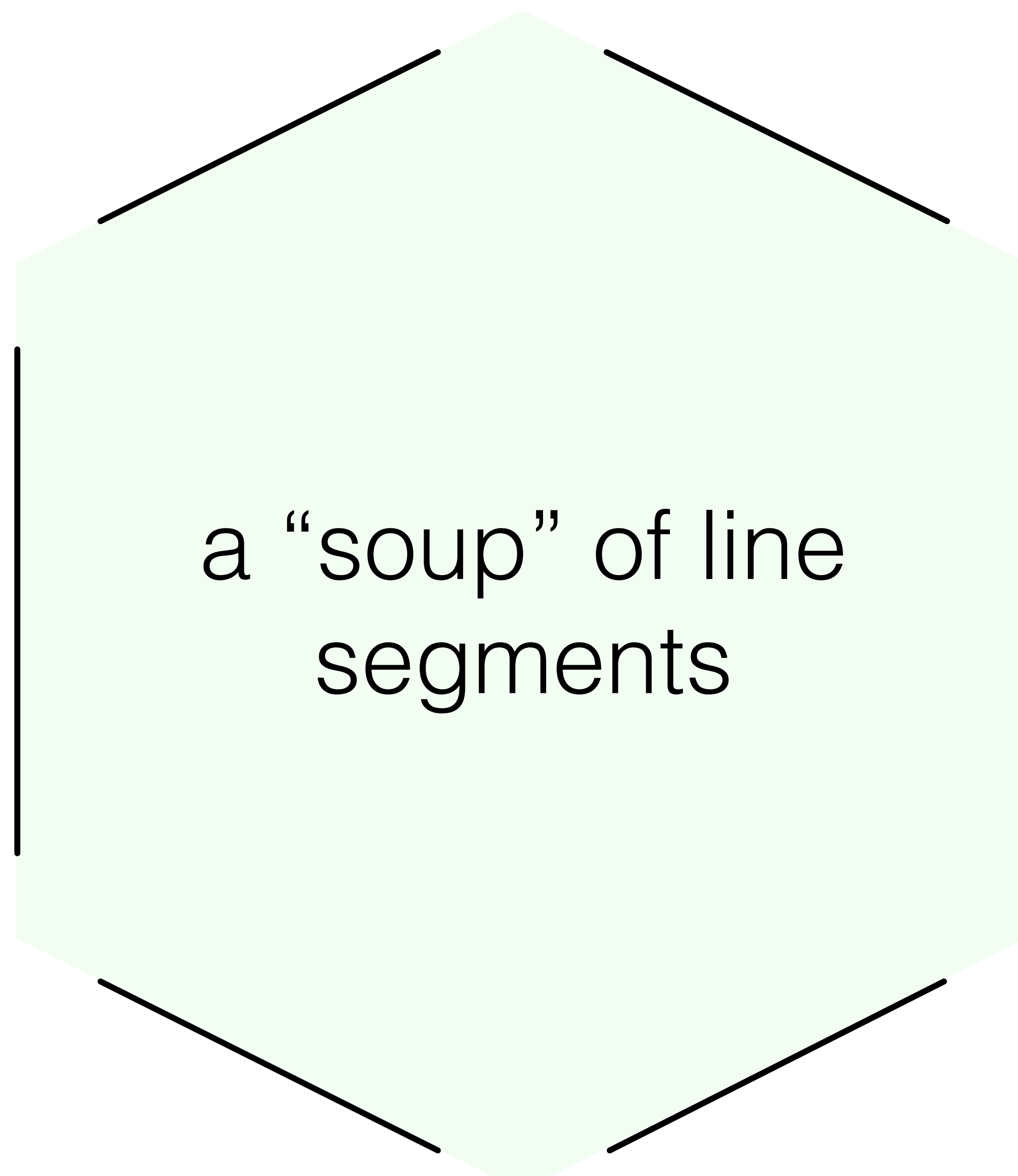


2D: a face

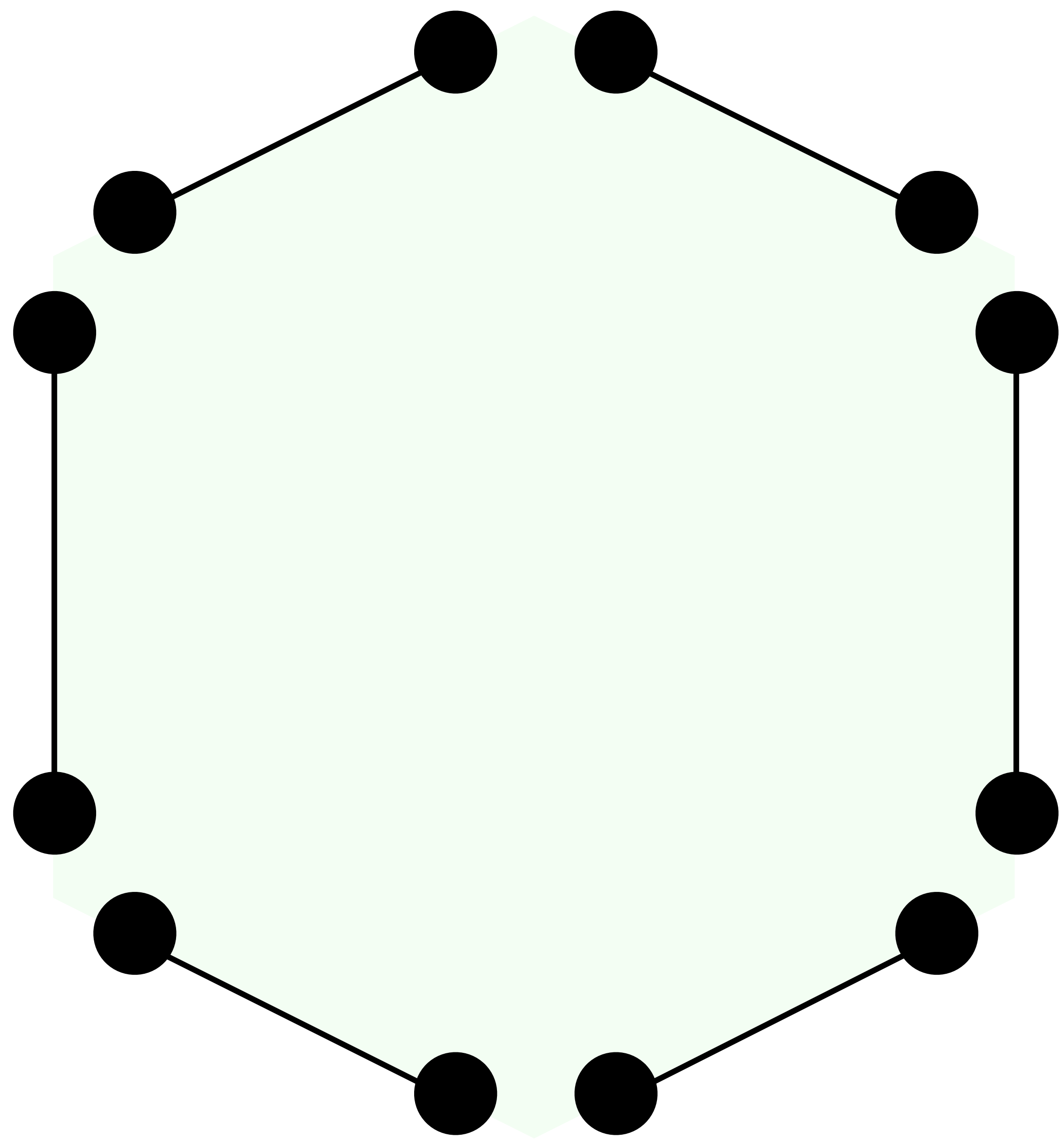


a 2D object can be described by **a set** of 1D objects that form its boundary

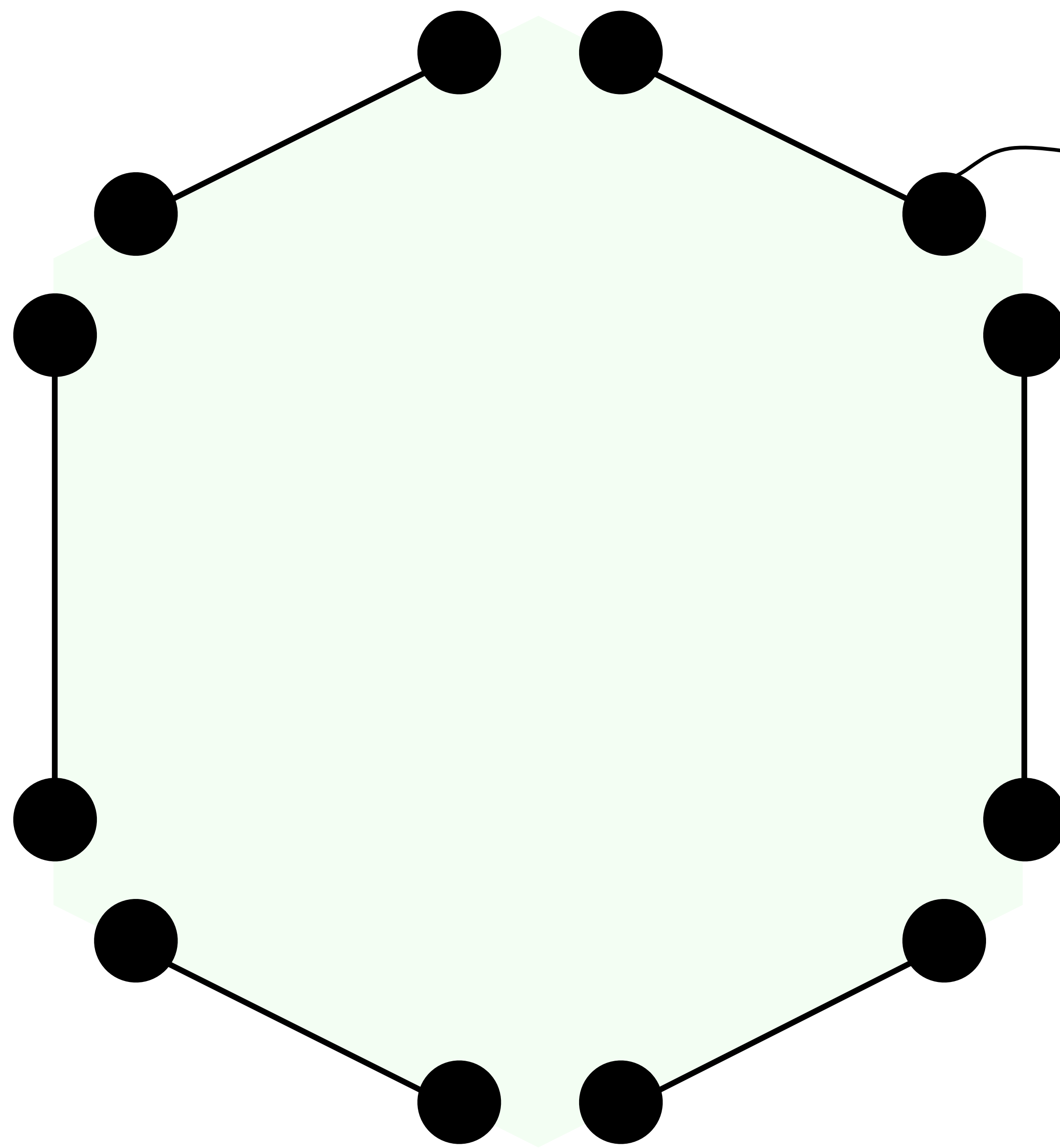
2D: a face



2D: a face



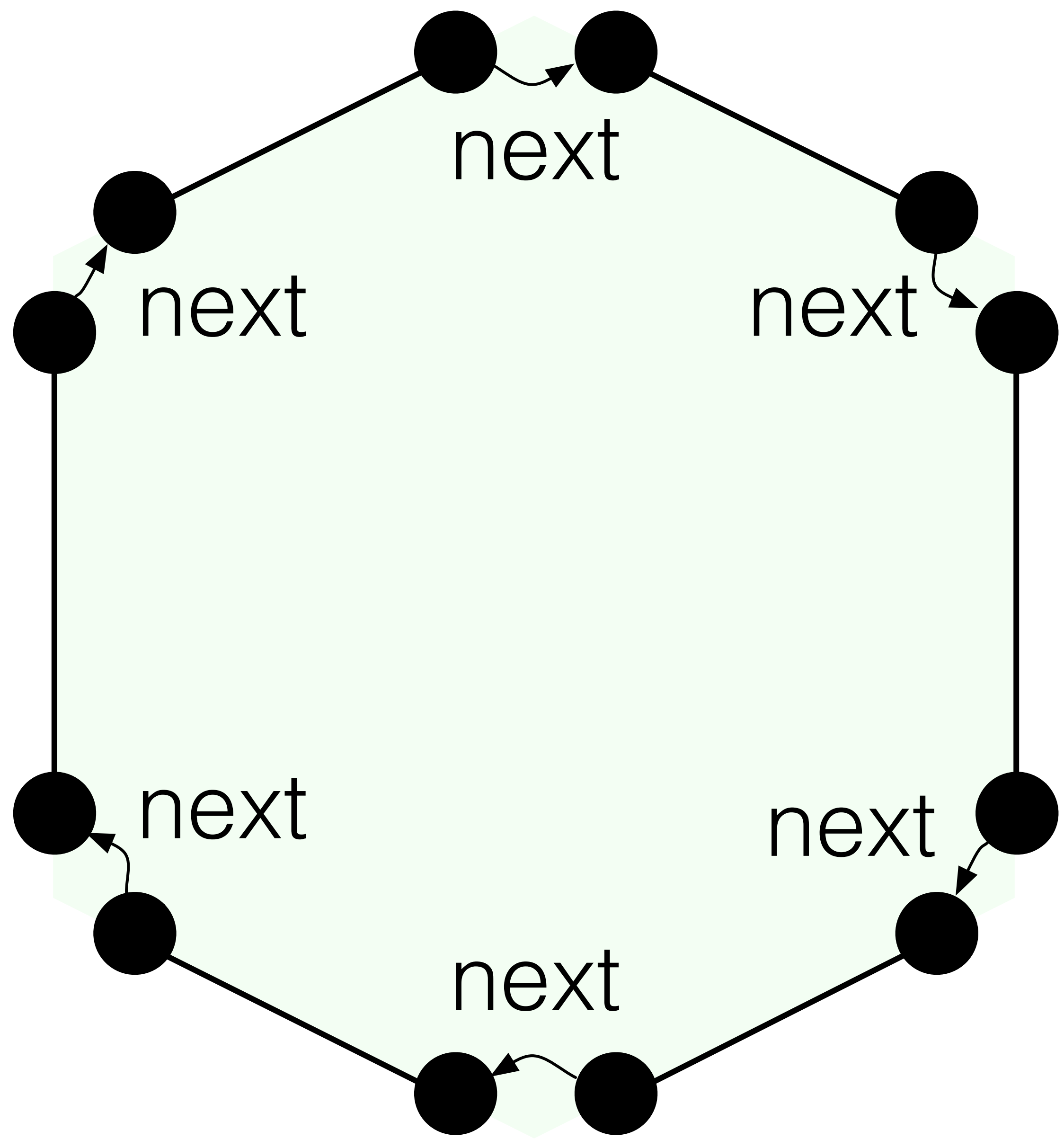
2D: a face



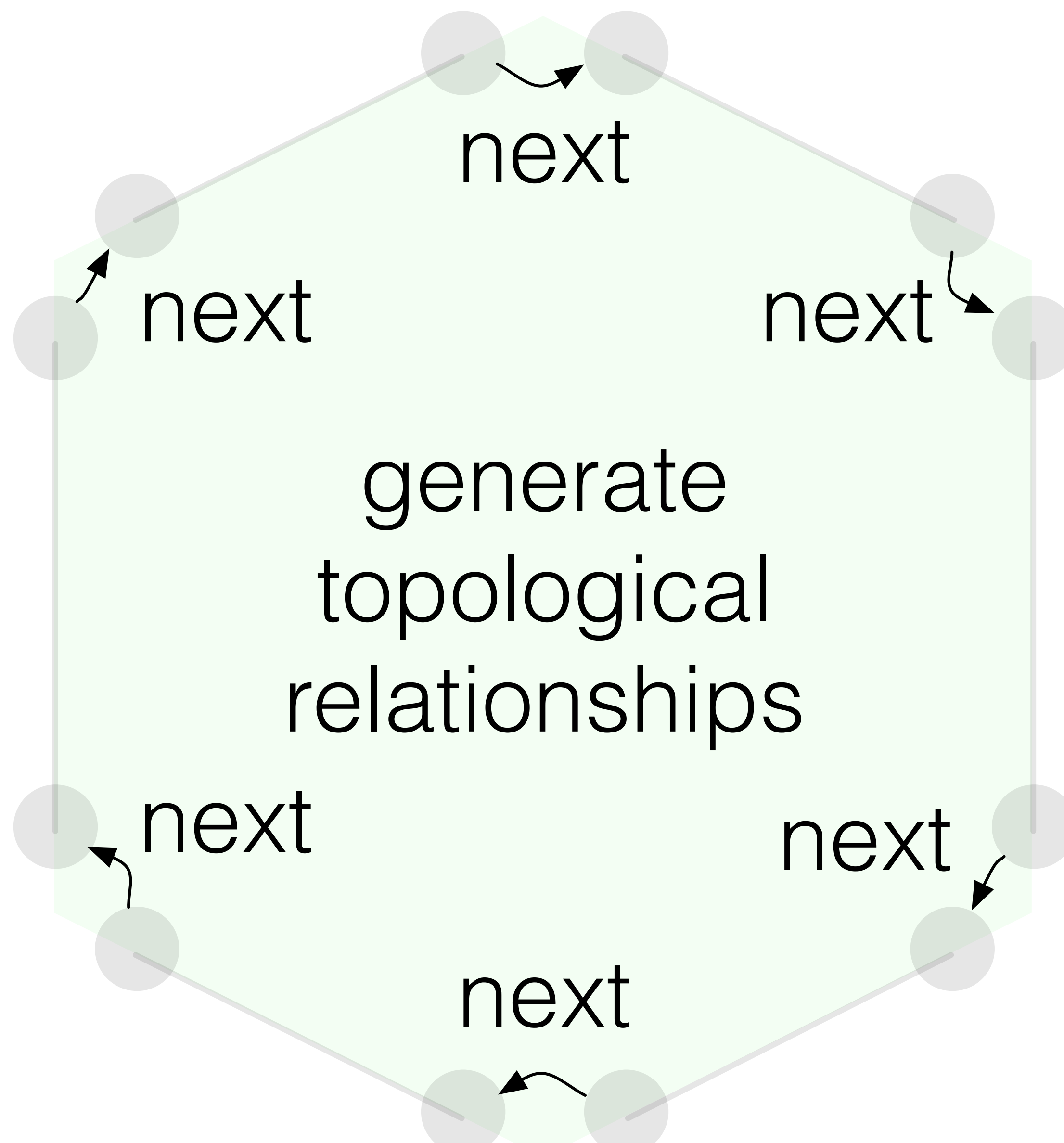
$(x_0, x_1, \ldots)$

find common 0D vertices on the boundary of a set of 1D edges

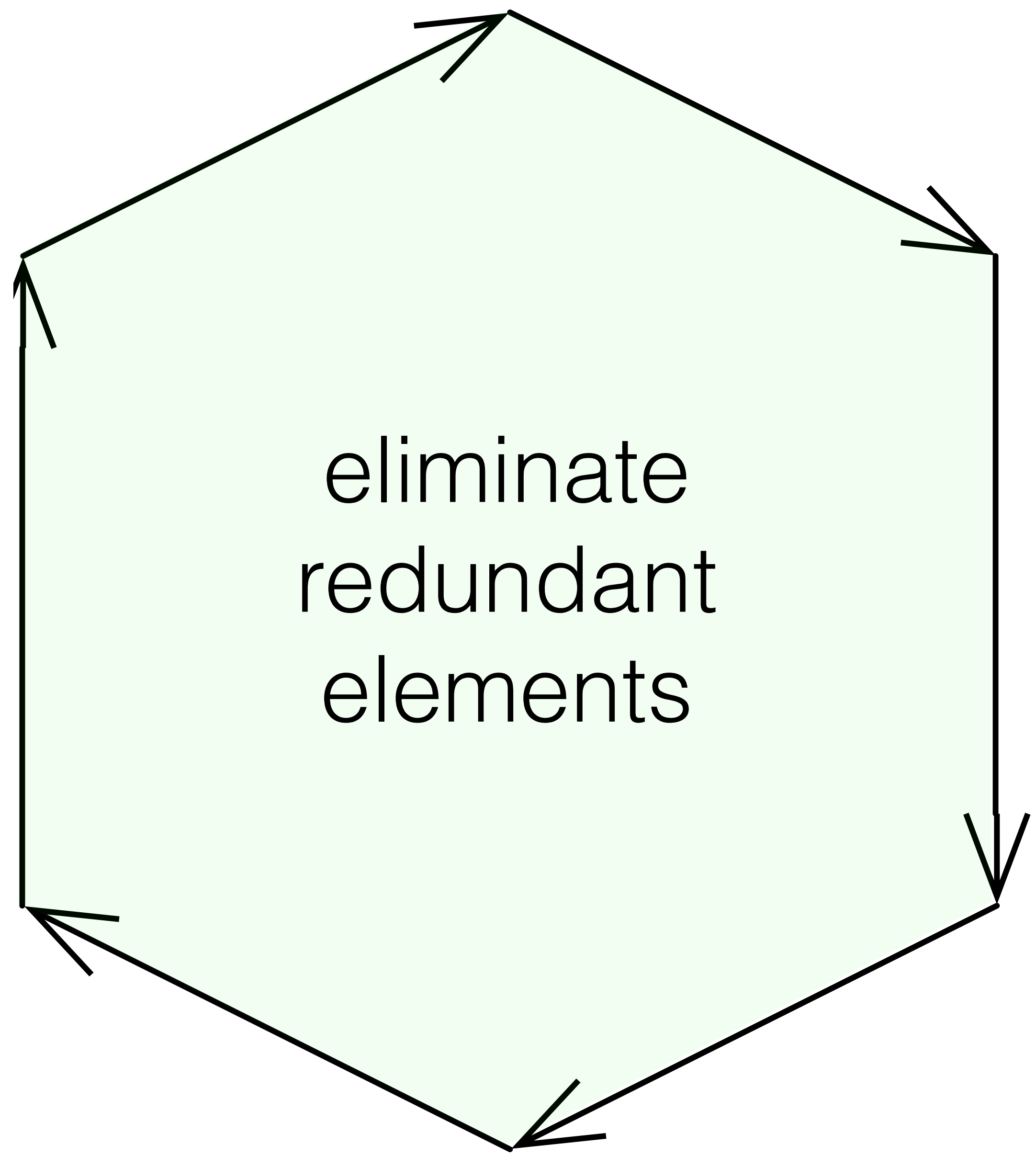
2D: a face



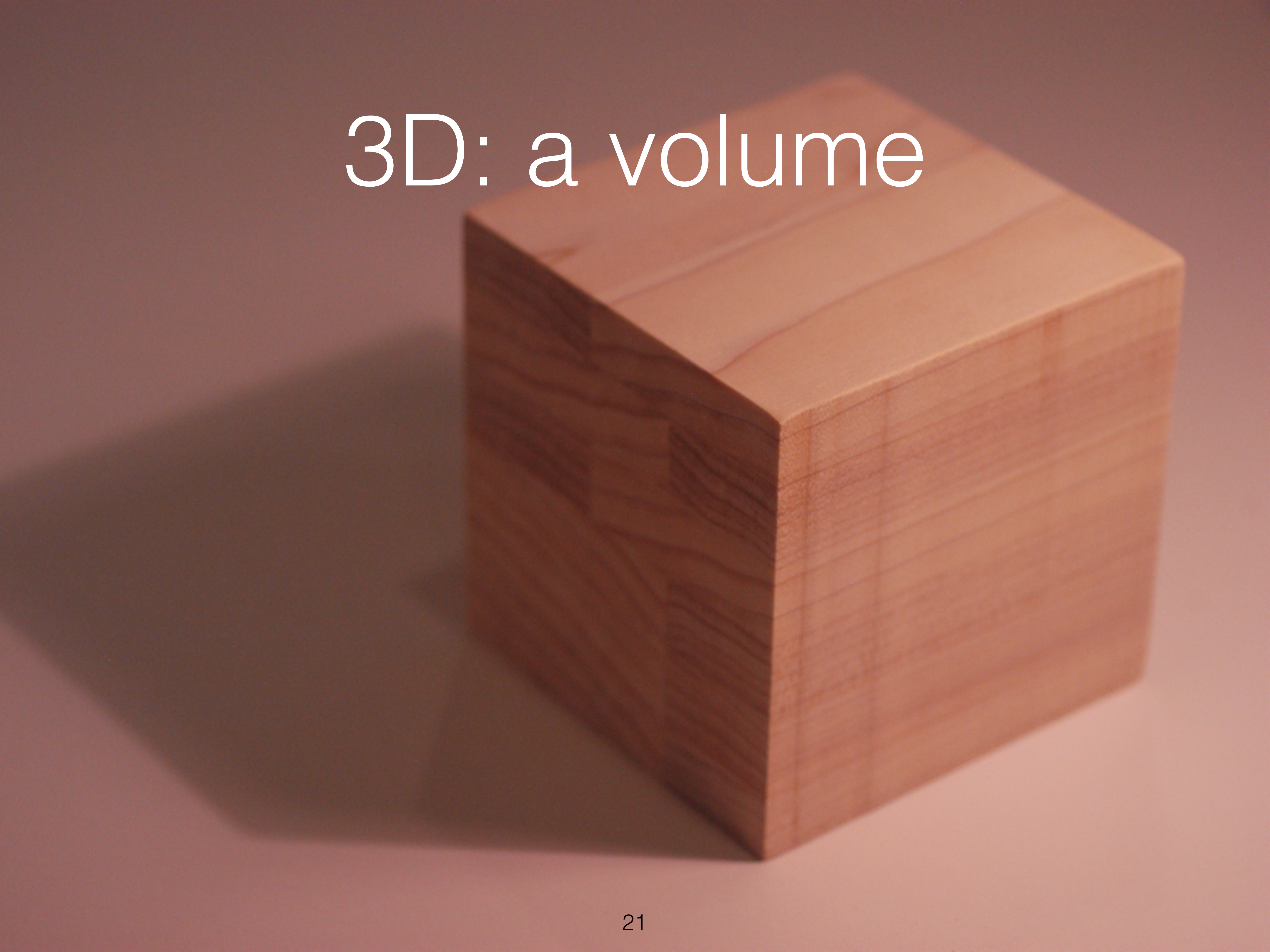
2D: a face


next
next
next
generate
topological
relationships
next
next
next

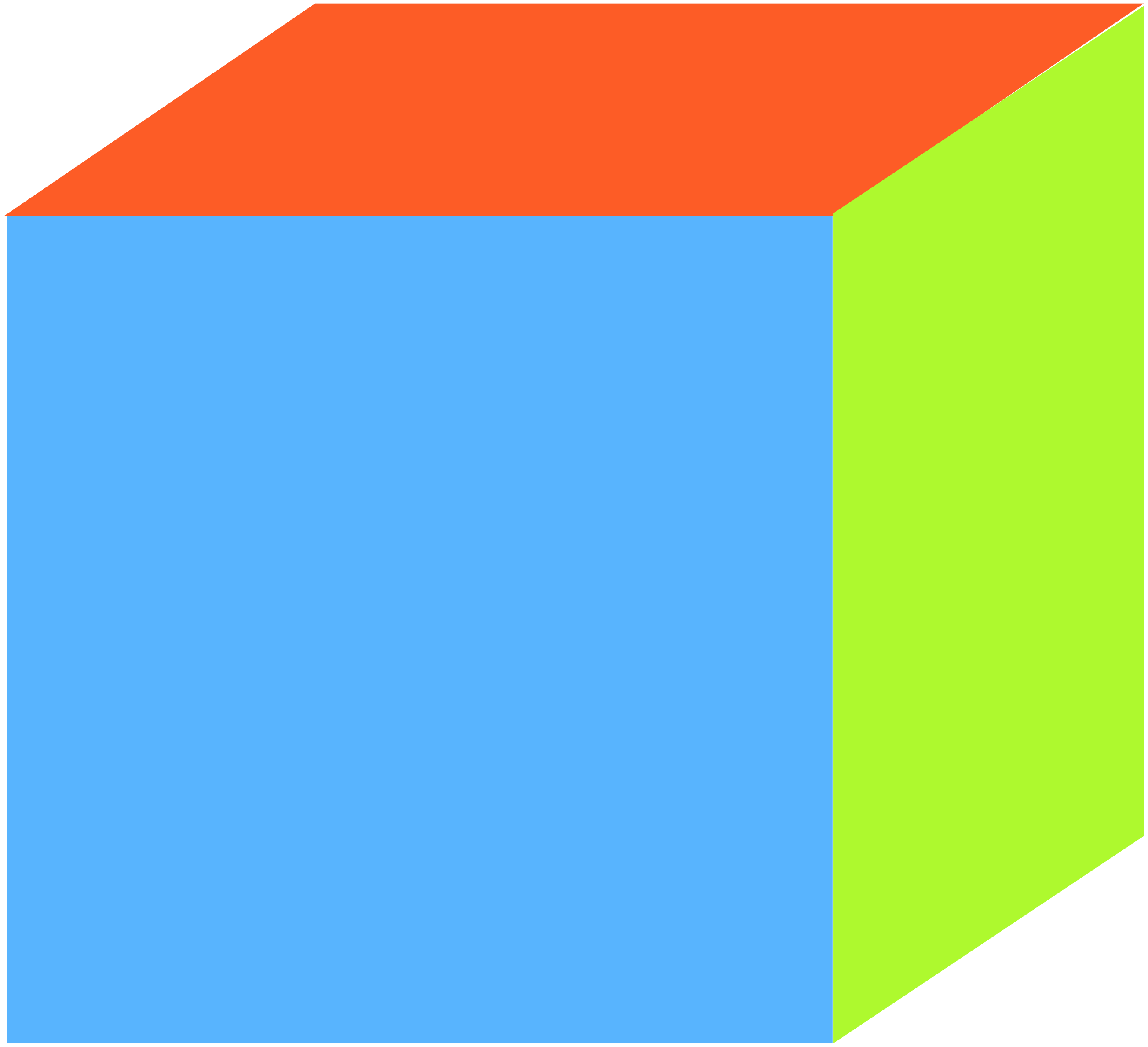
2D: a face



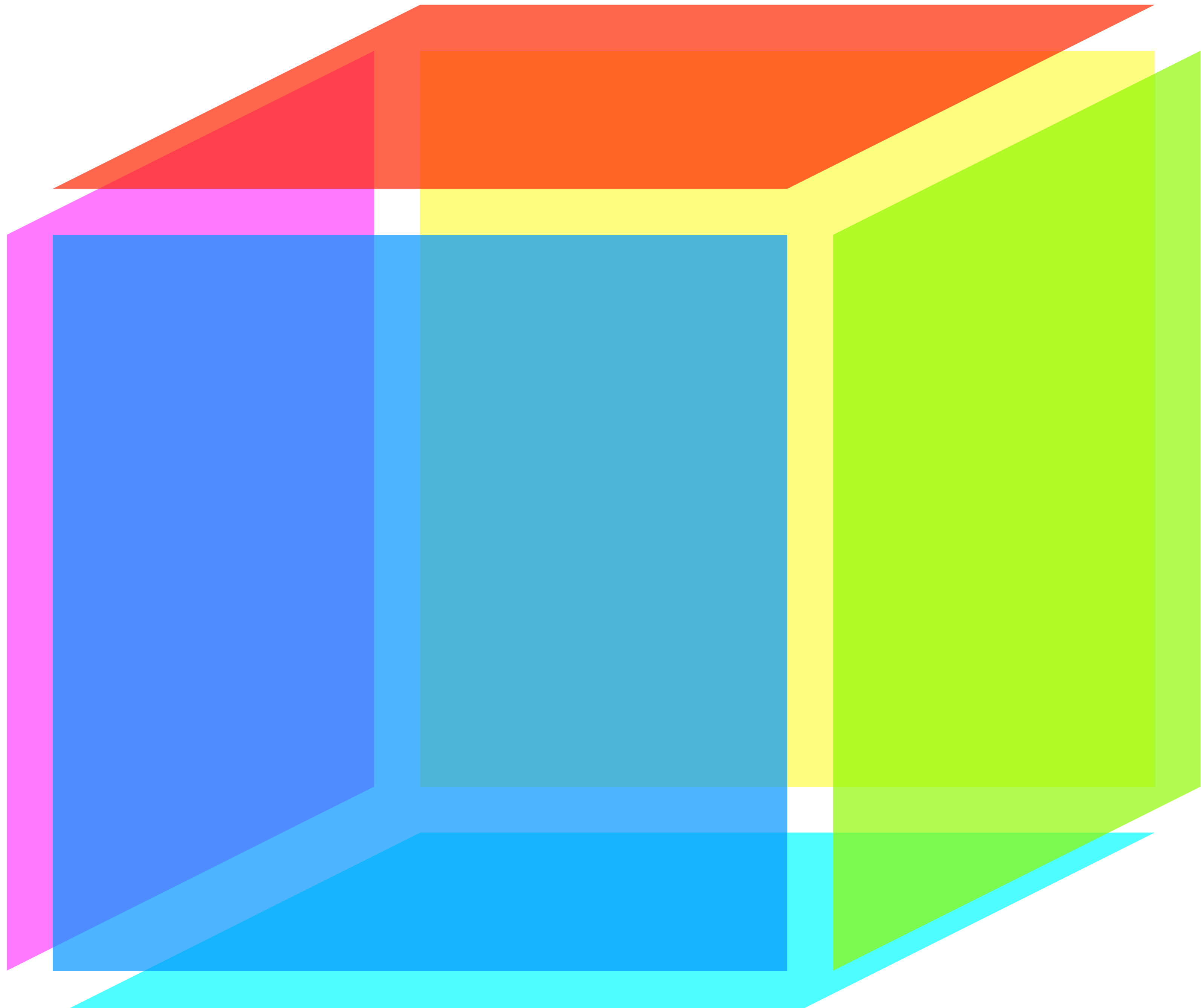
3D: a volume



3D: a volume

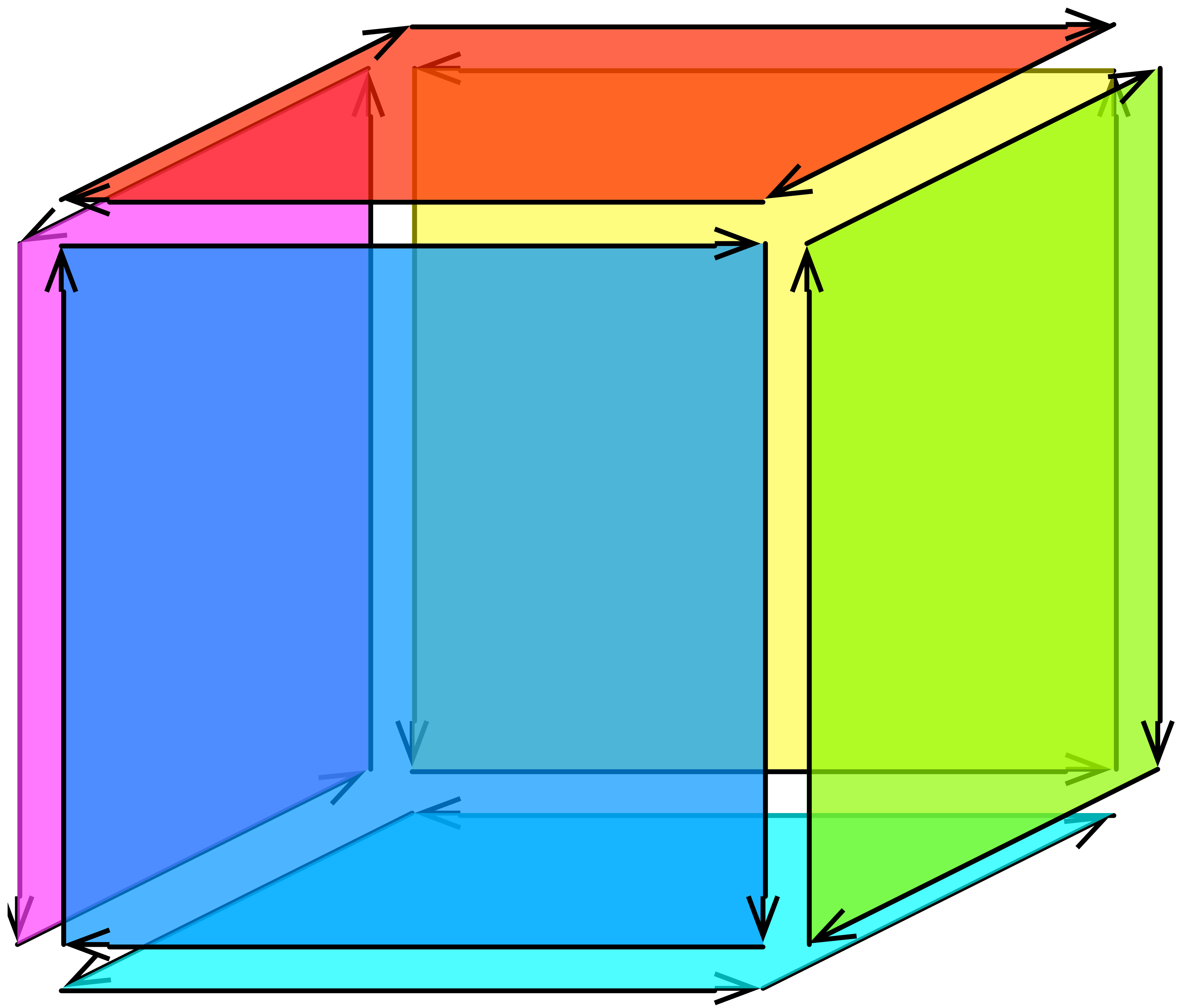


3D: a volume

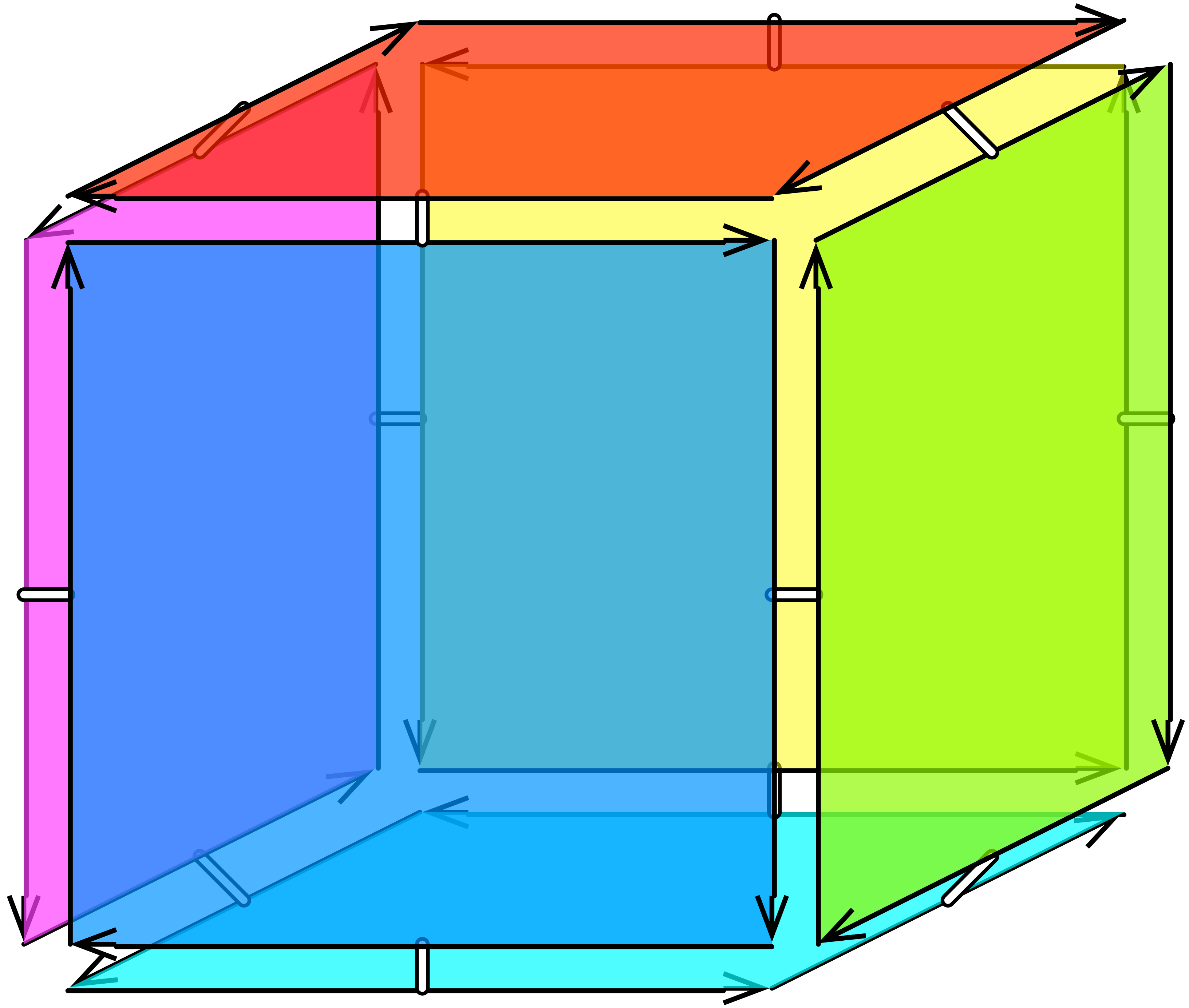


a “soup”
of faces

3D: a volume



3D: a volume

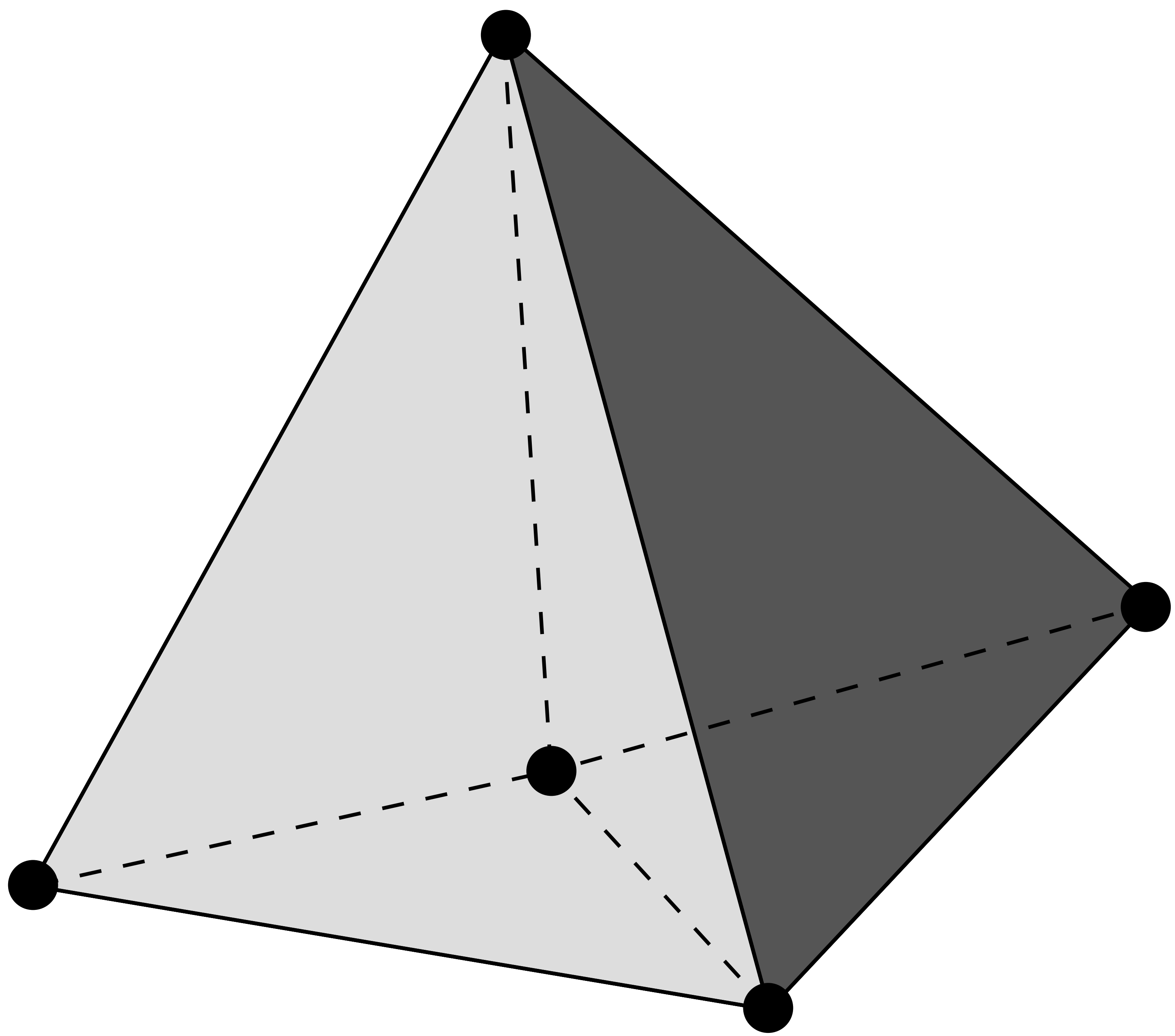


find
common
1D edges
on the
boundary of
a set of 2D
faces

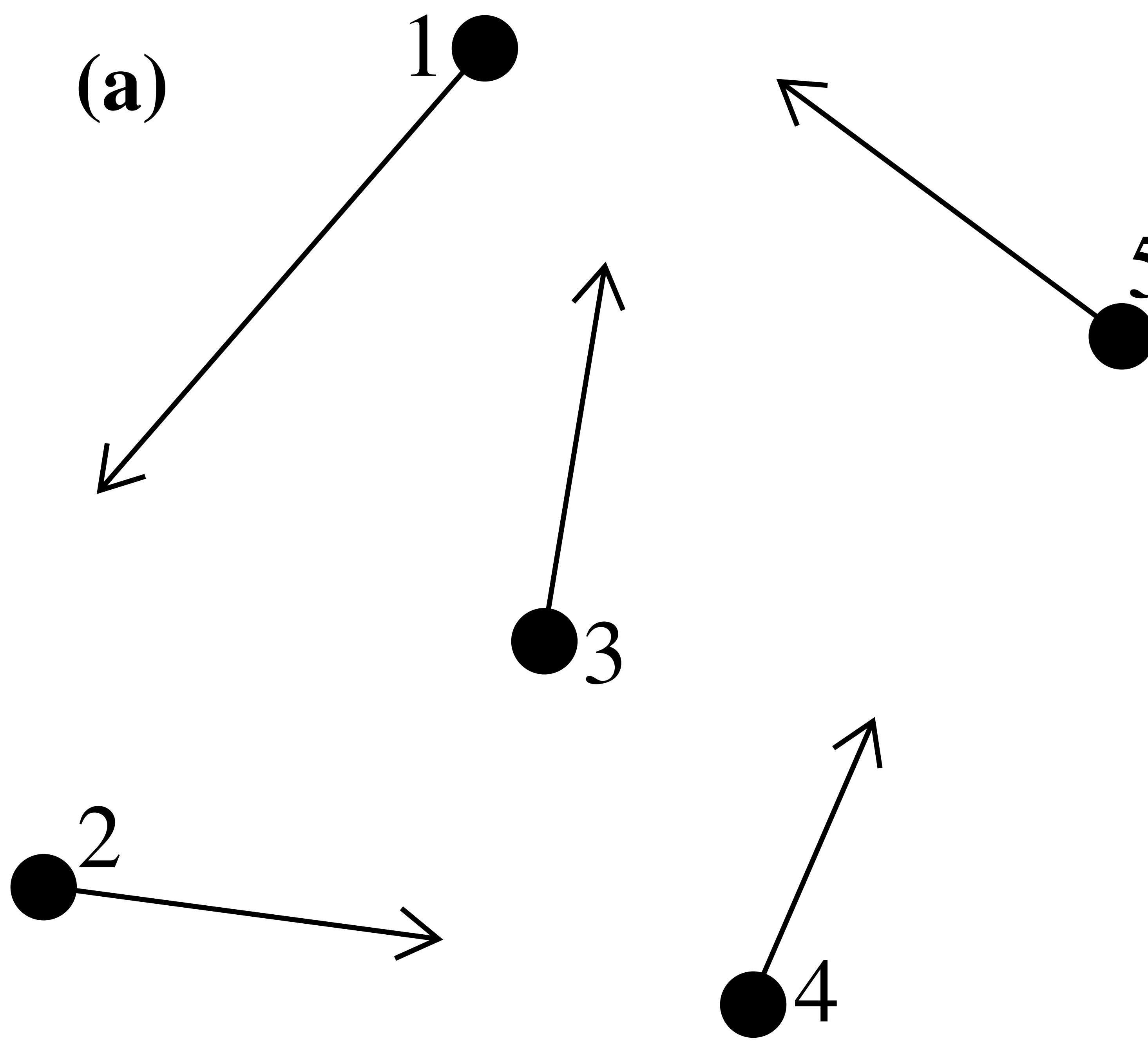
Incremental construction

- Start from a set of 0D vertices
- Connect them to create 1D edges
- Connect 1D edges, forming 2D faces by finding common 0D vertices
- Connect 2D faces, forming 3D volumes by finding common 1D edges
- Connect nD cells, forming (n+1)D cells by finding common (n-1)D cells

Building two tetrahedra



Building two tetrahedra



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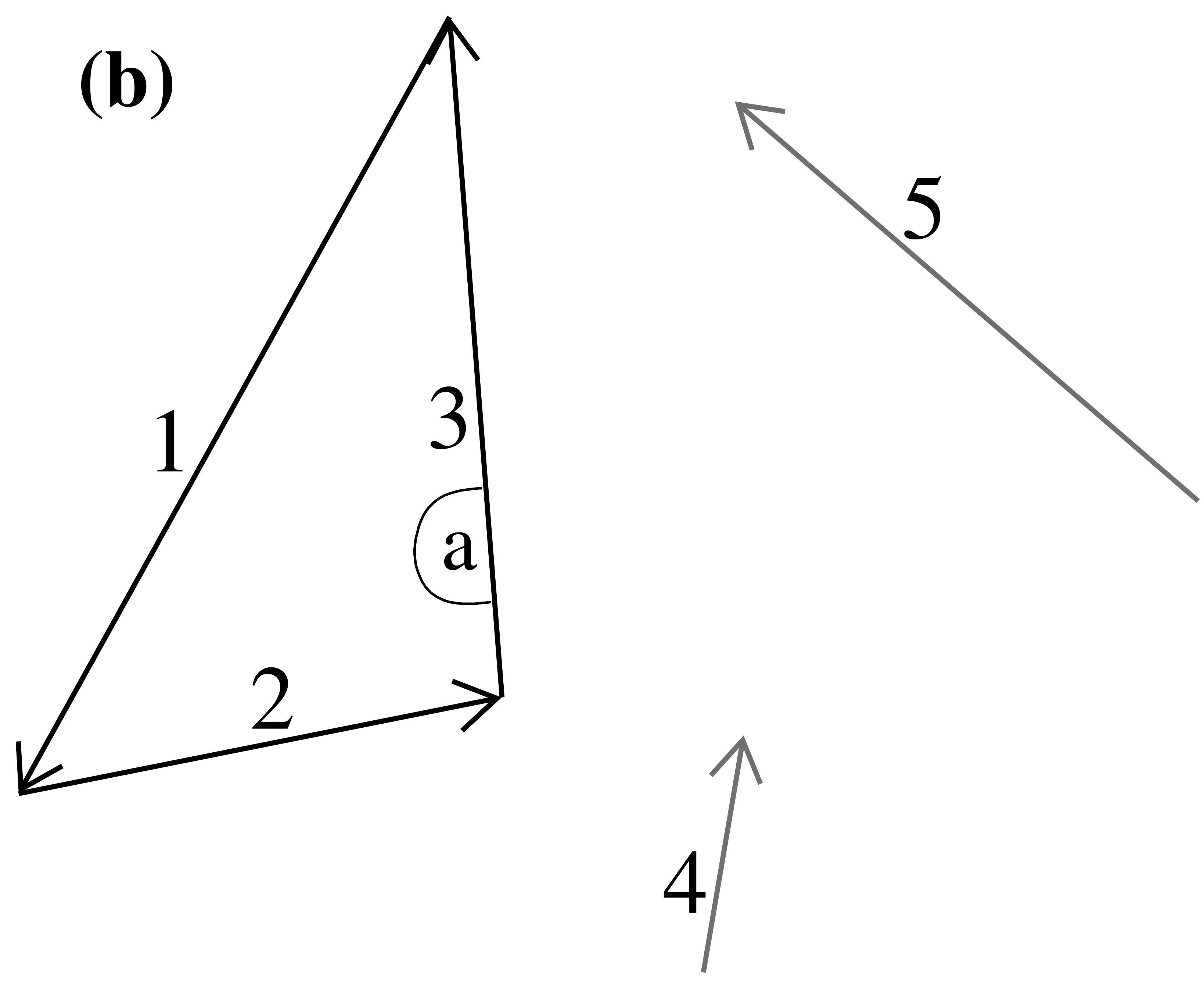

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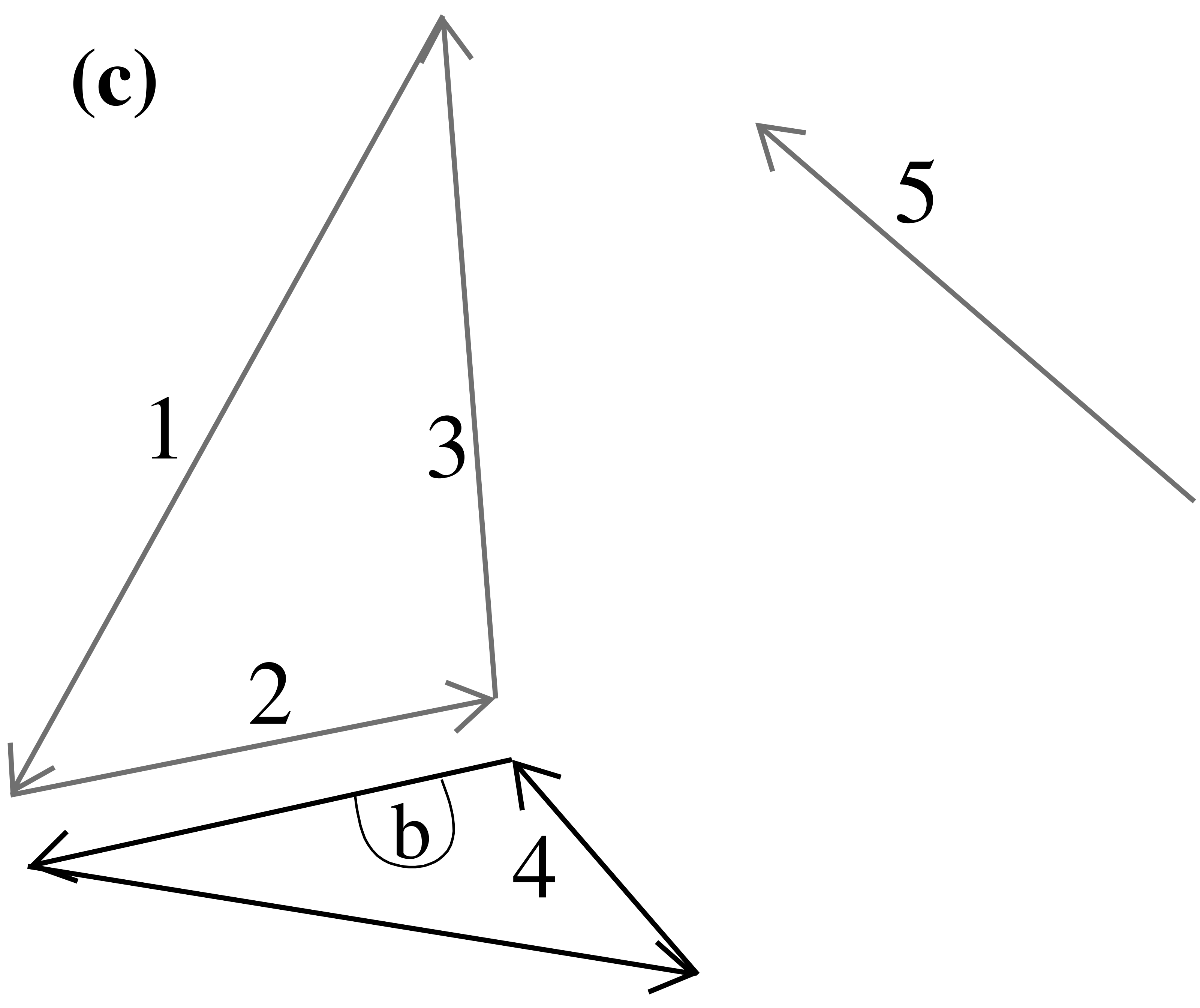

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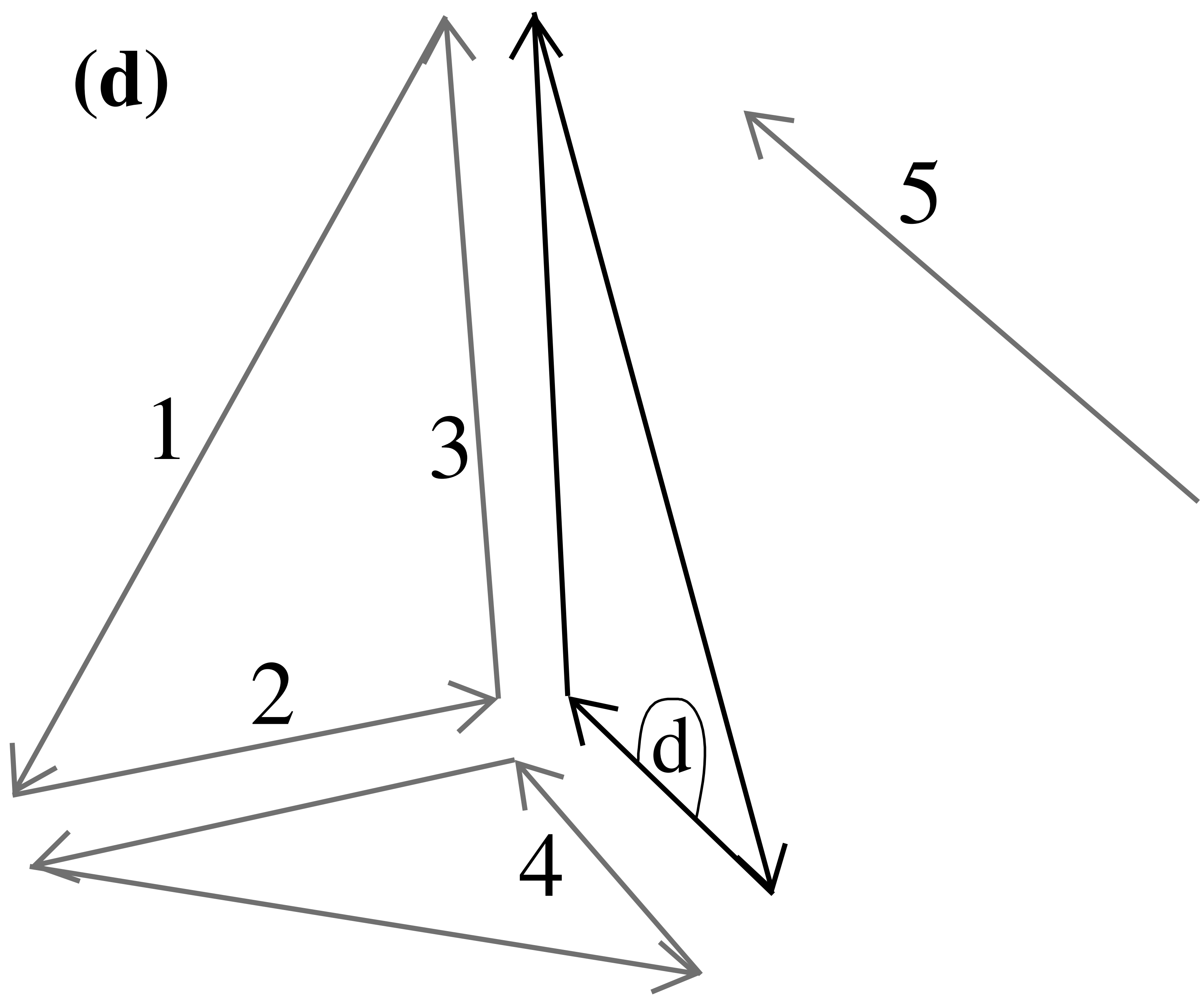

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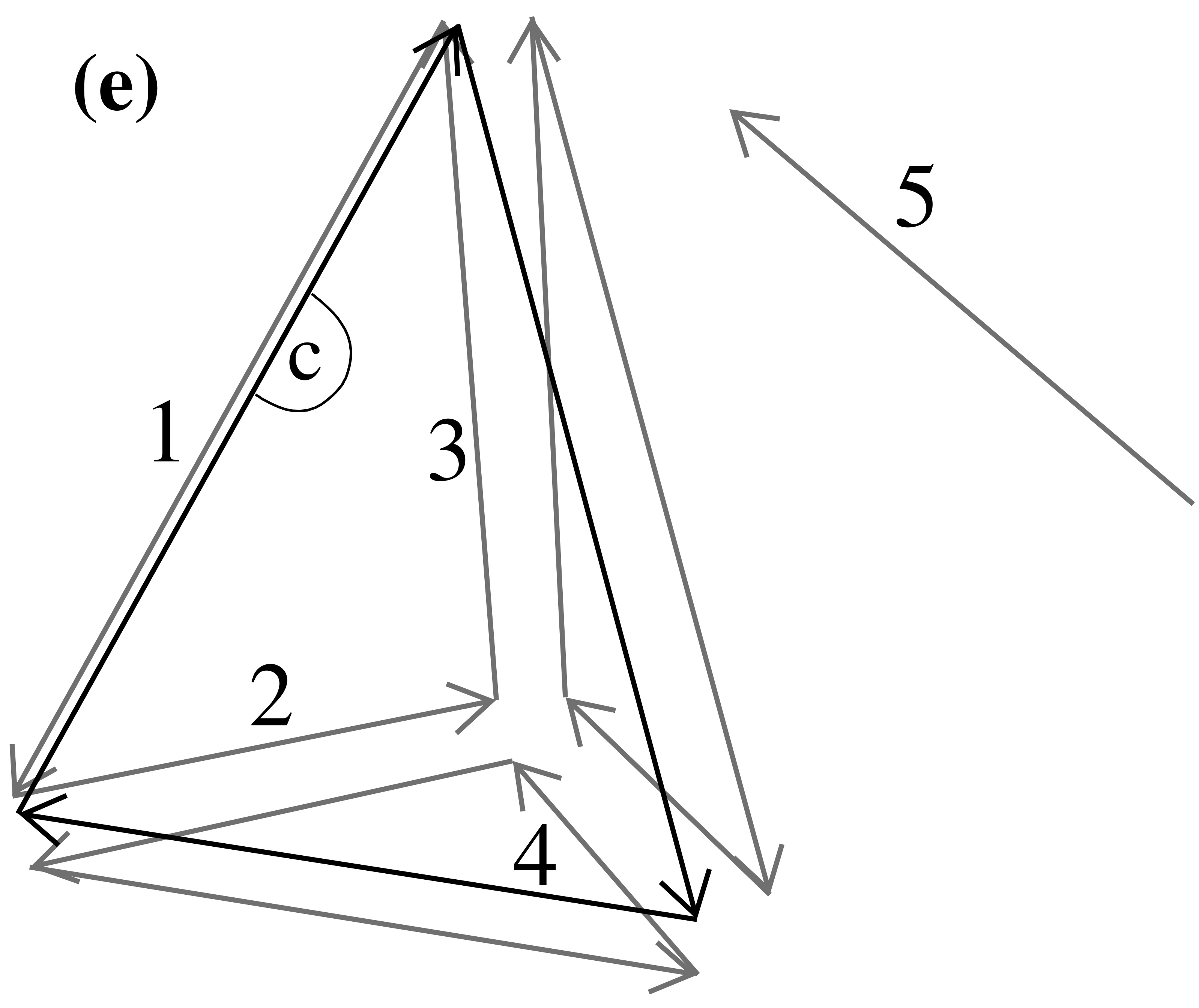

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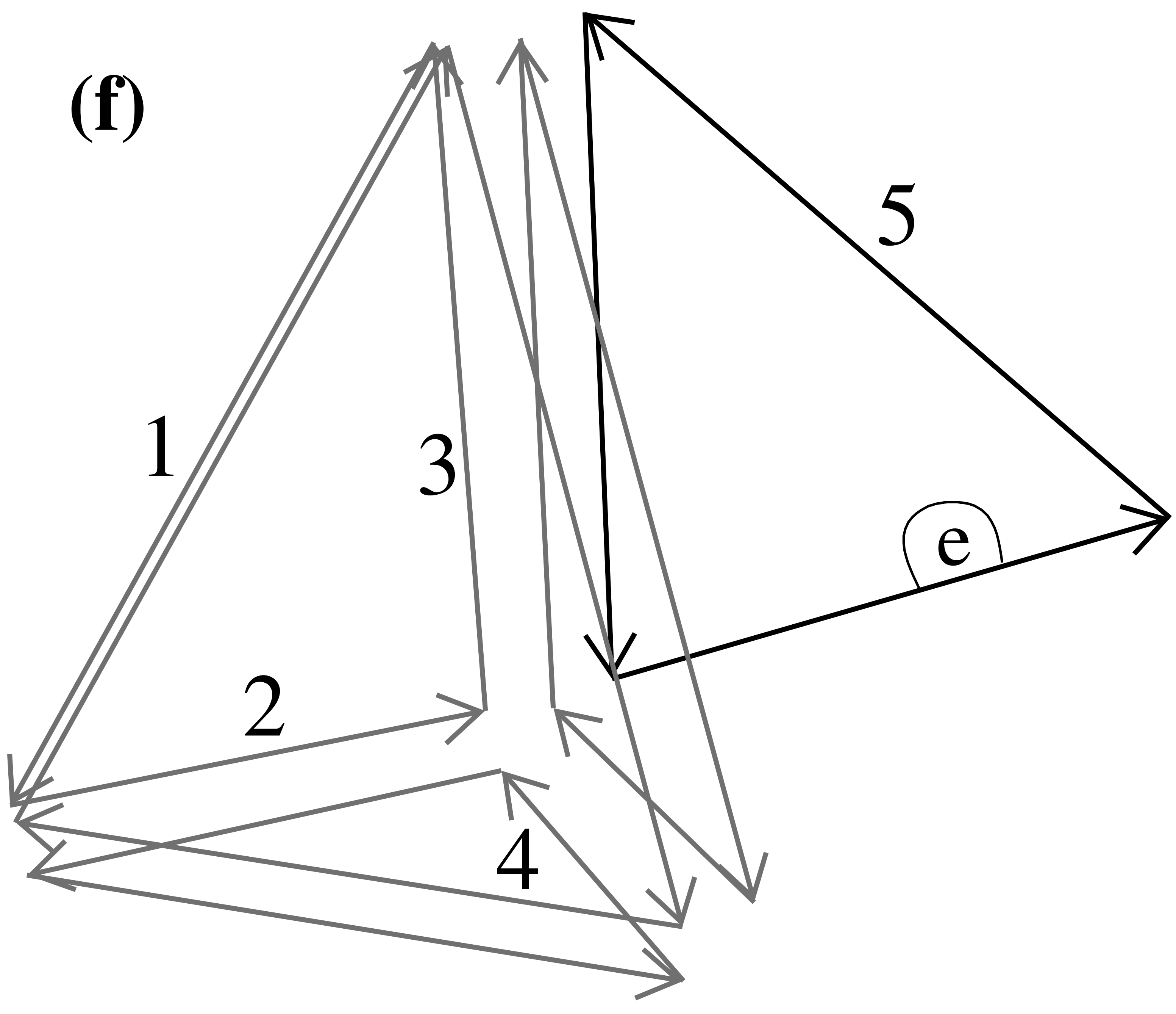

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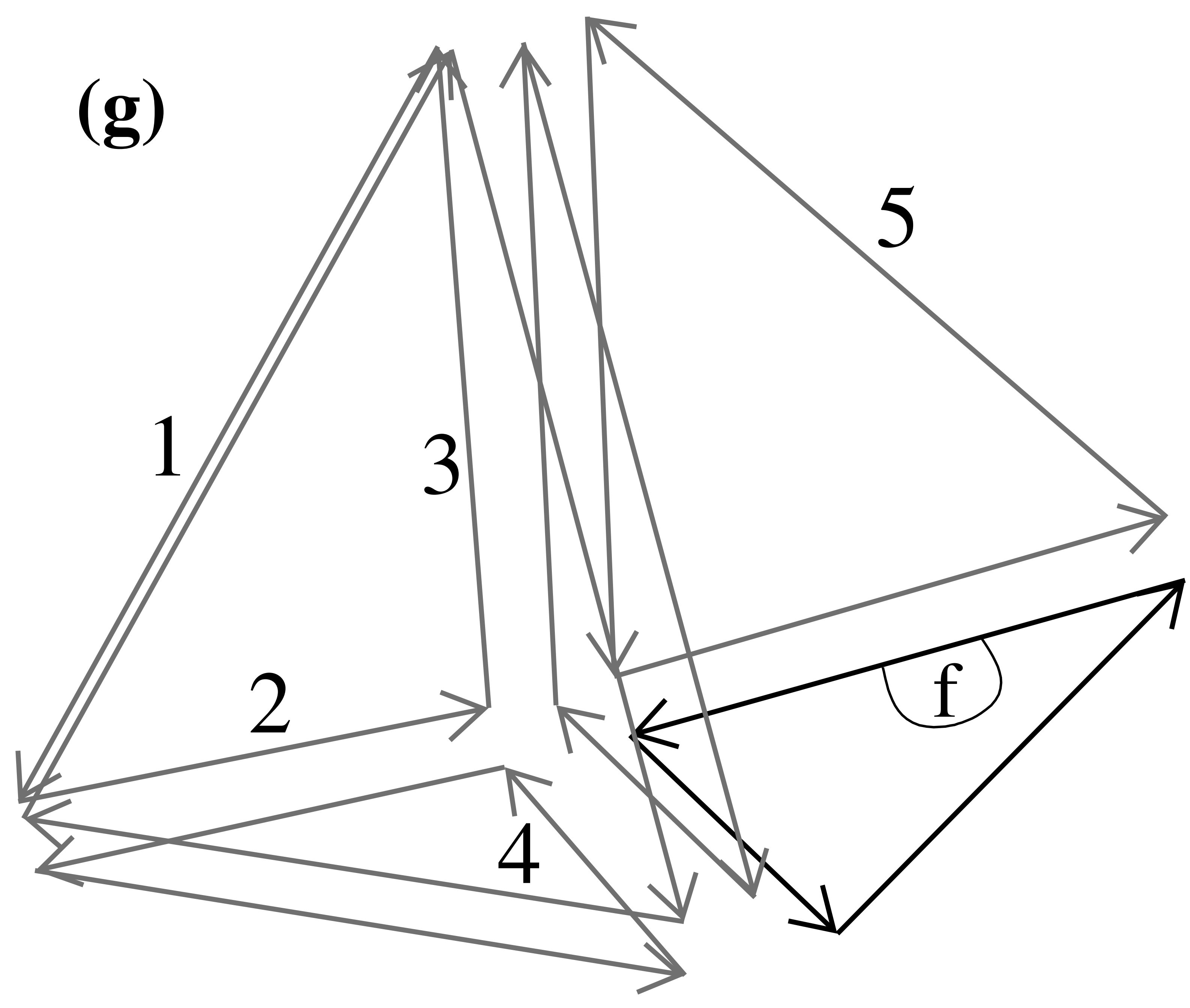

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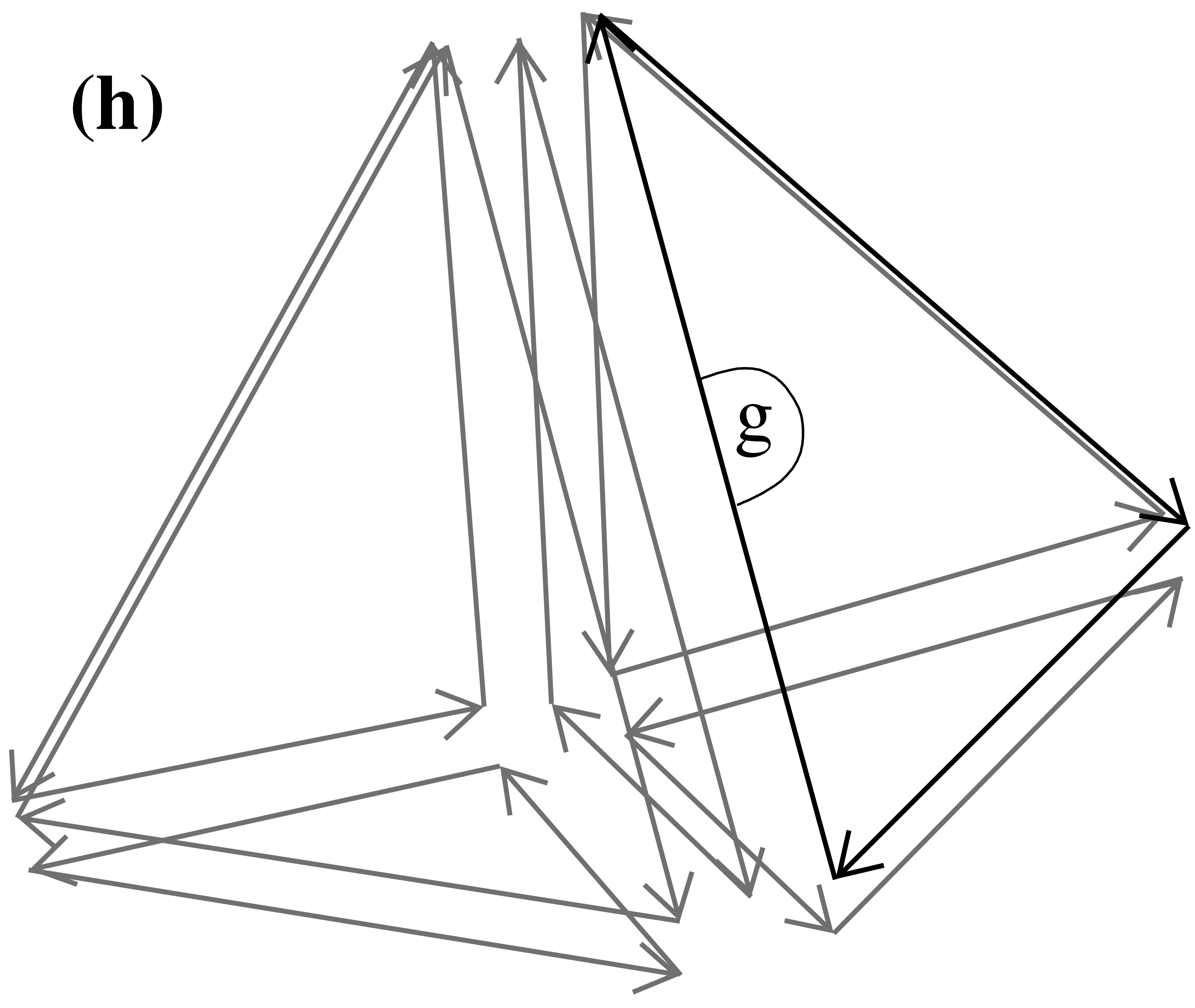

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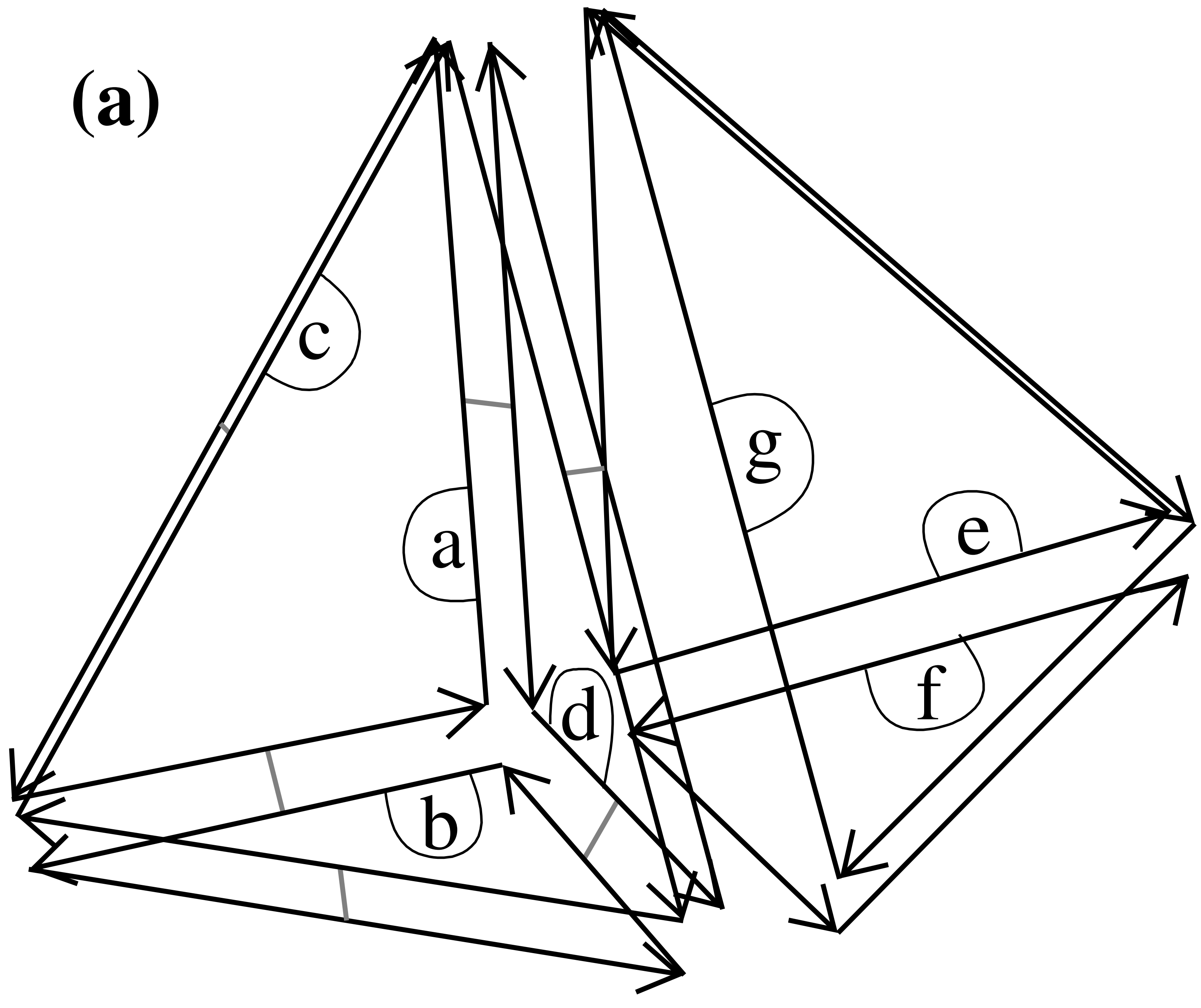

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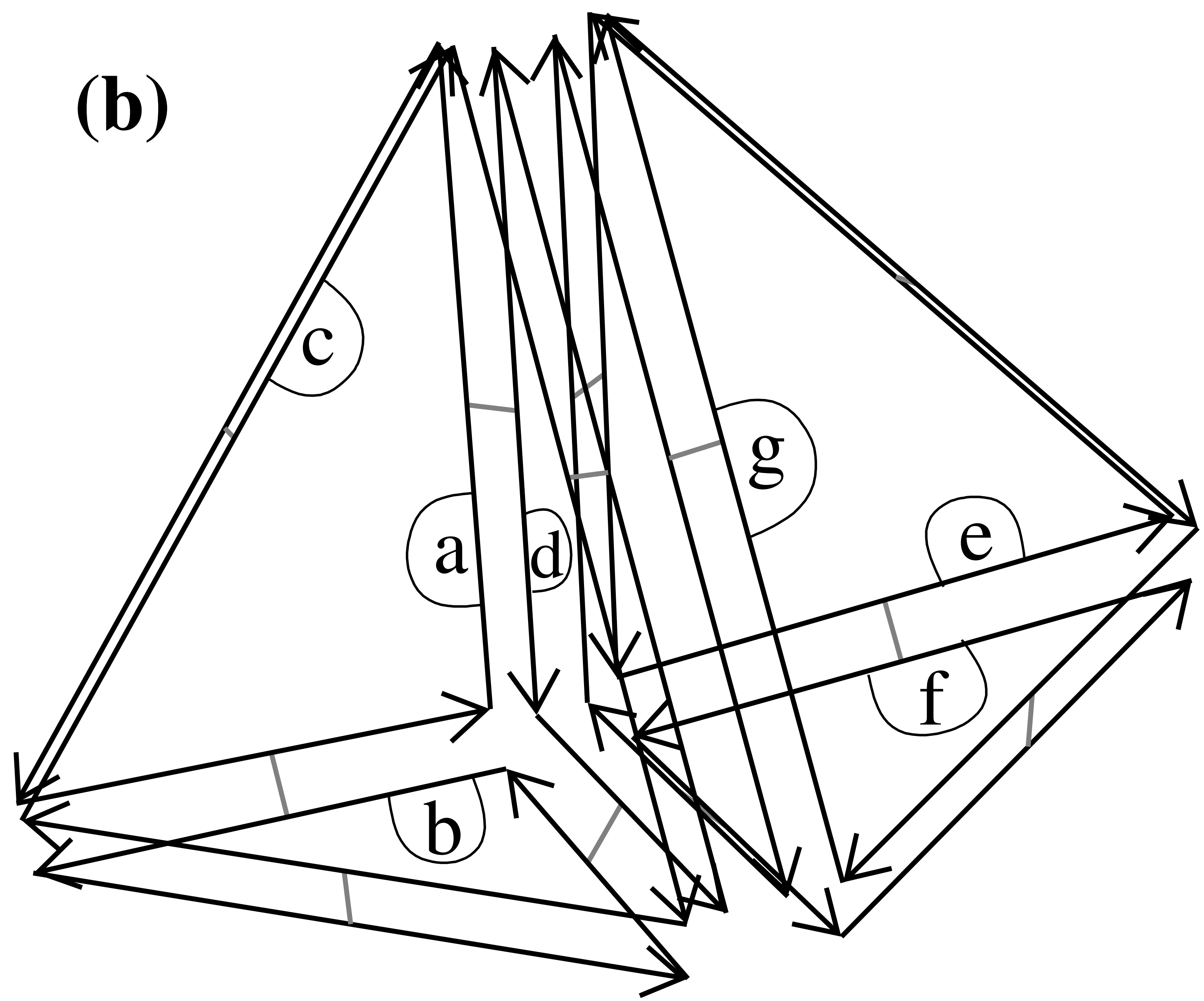
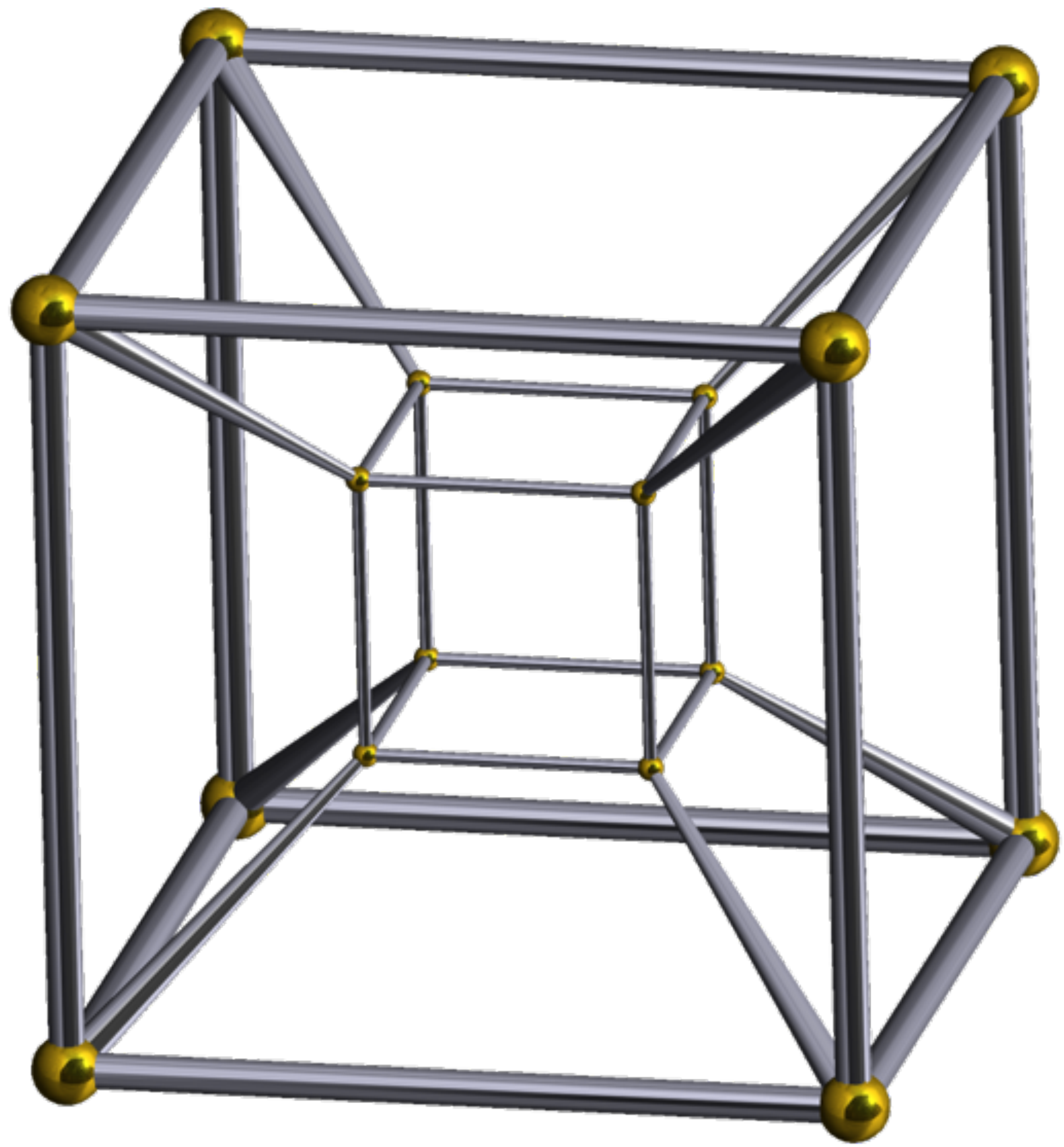
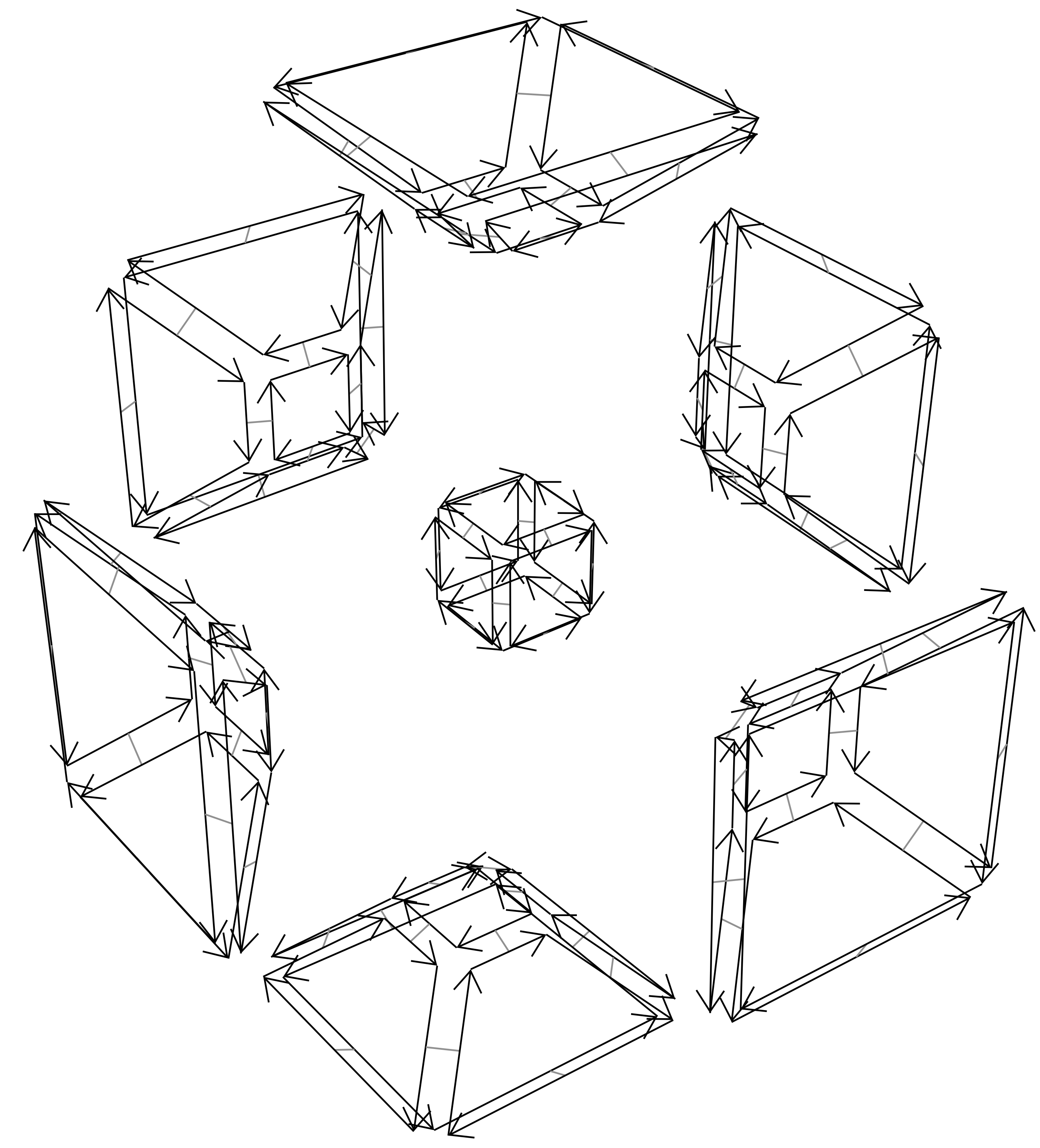


done!

Build a tesseract

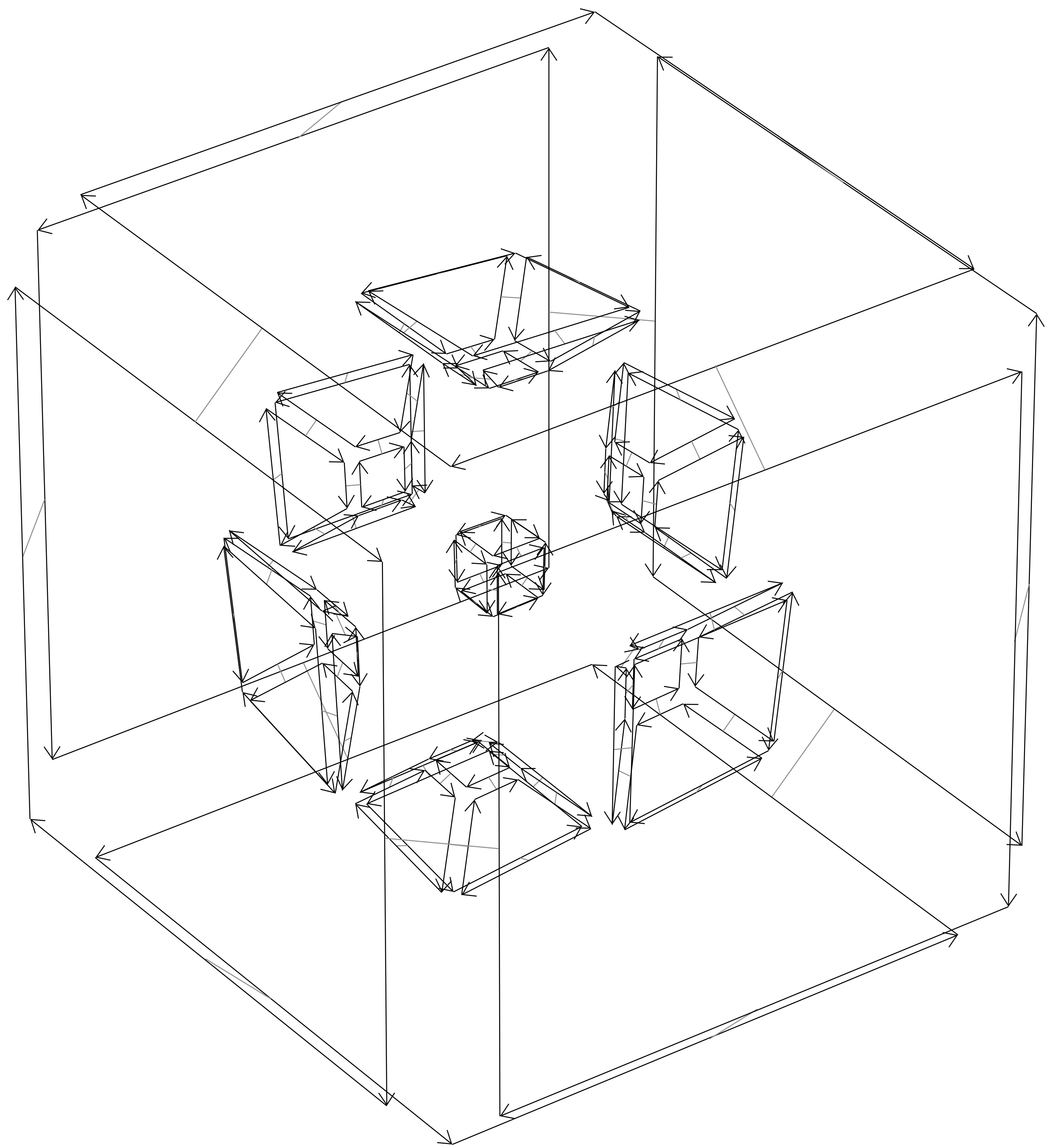


Build a tesseract



build each
cube
separately

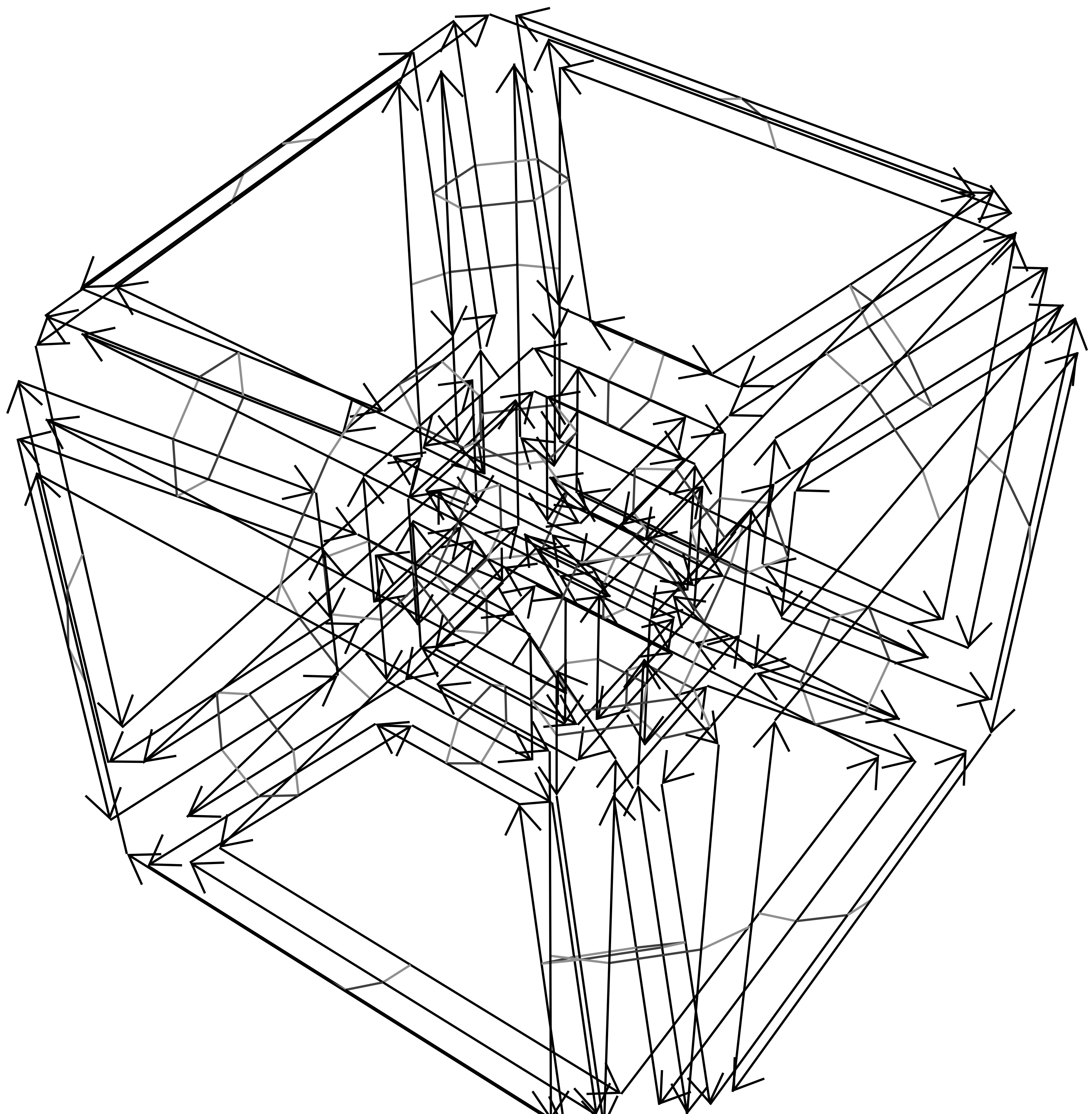
Build a tesseract



build each cube separately,

then join them

Build a tesseract



done!

Thank you!

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tudelft.nl/kenohori

Images from:

- http://commons.wikimedia.org/wiki/File:Stereographic_polytope_24cell_faces.png
- http://blogs.lt.vt.edu/foundationdesignlab/category/materials/
- http://commons.wikimedia.org/wiki/File:Schlegel_wireframe_8-cell.png

More info

Ken Arroyo Ohori, Guillaume Damiand and Hugo Ledoux. **Constructing an n-dimensional cell complex from a soup of (n-1)-dimensional faces**. In Prosenjit Gupta and Christos Zaroliagis (eds.), *Applied Algorithms*, Volume 8321 of Lecture Notes in Computer Science, Springer International Publishing Switzerland, January 2014, pp. 37–48.