How to build an *n*-dimensional object?

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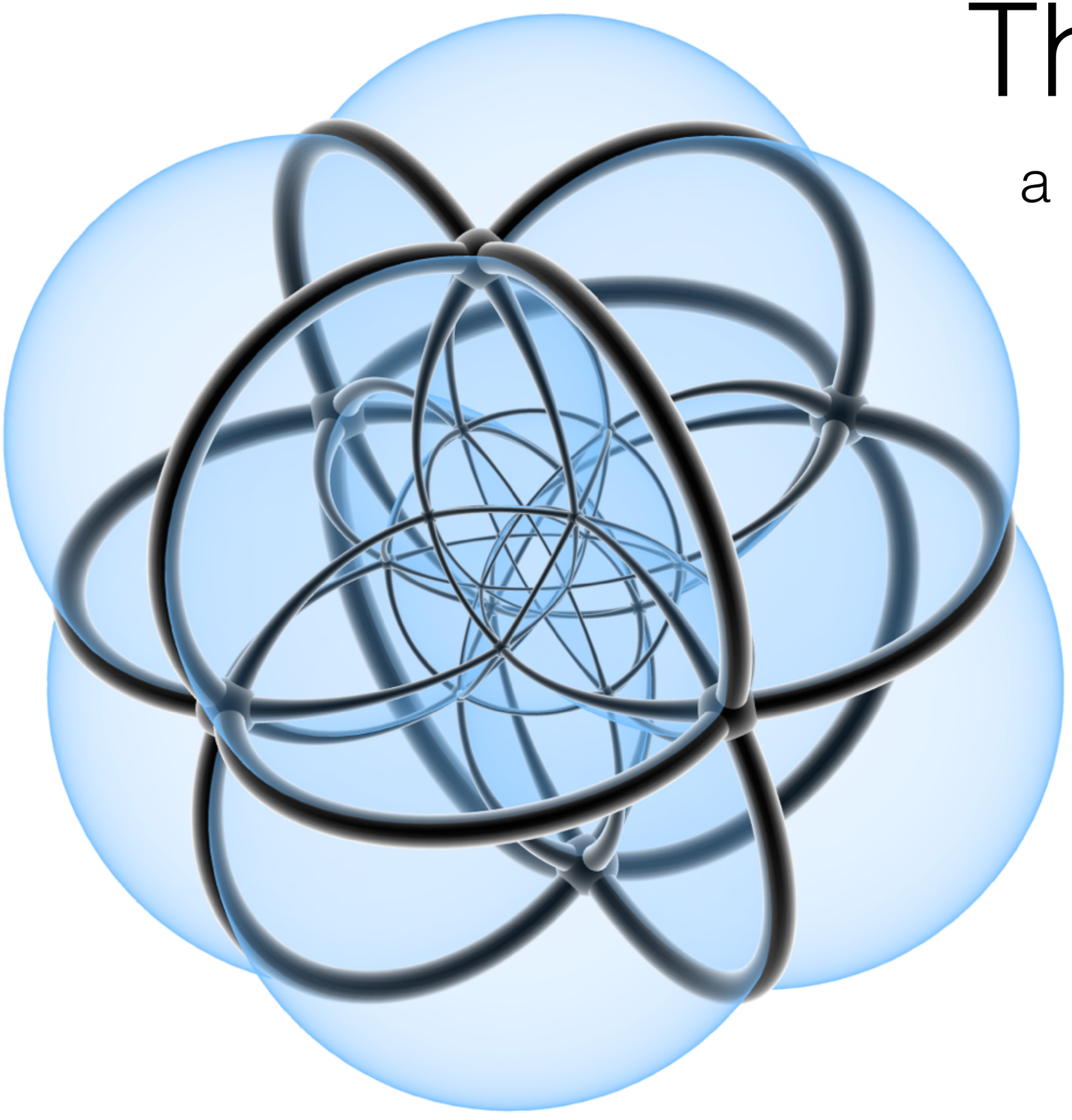
ABE010: Capita Selecta
20.11.2014

TU Delft
Delft University of Technology

The 24-cell

a “simple” 4D object



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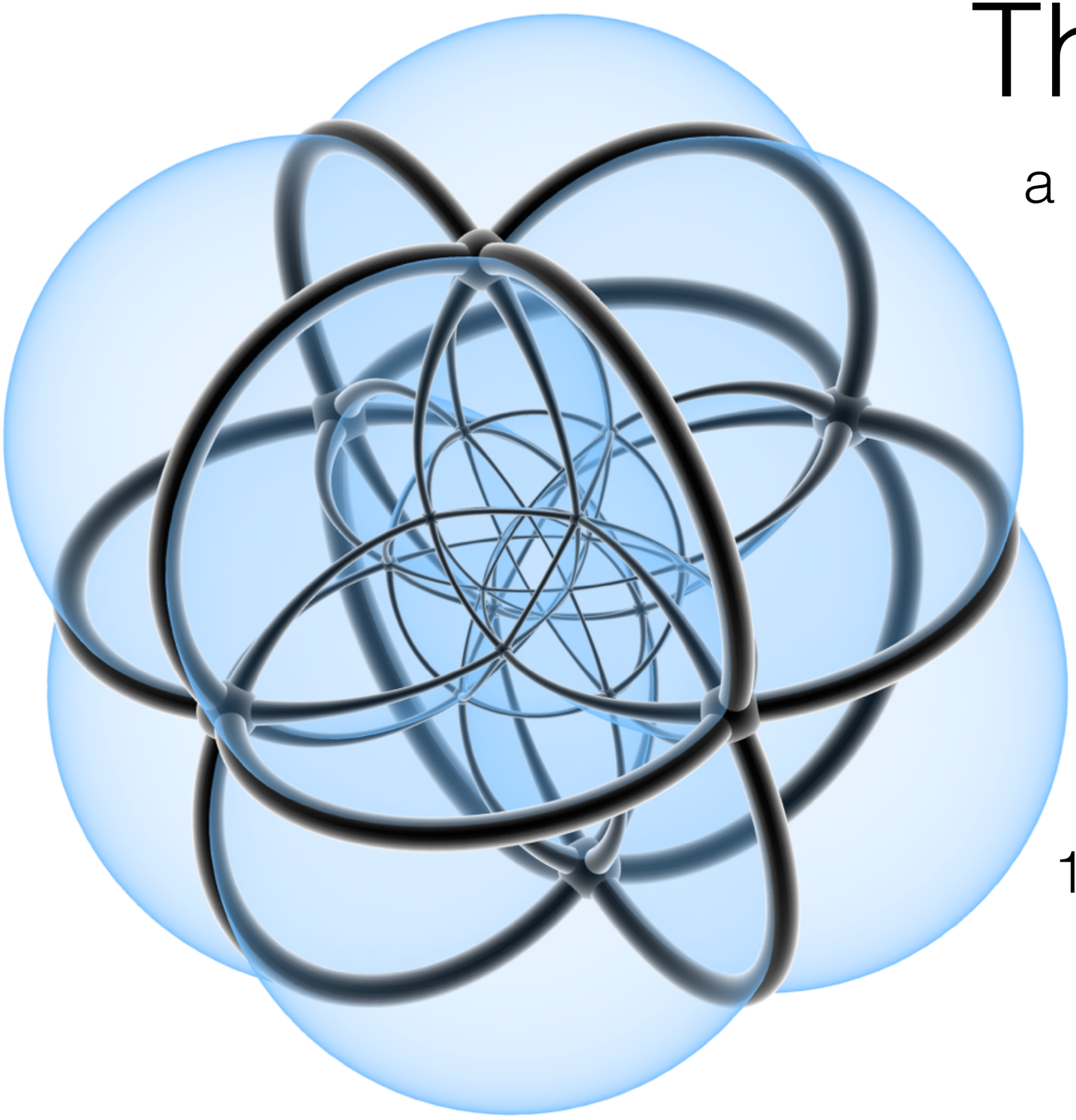
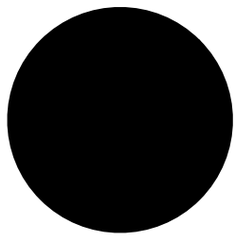


24 0D vertices
96 1D edges
96 2D faces
24 3D volumes
1 4D hypervolume

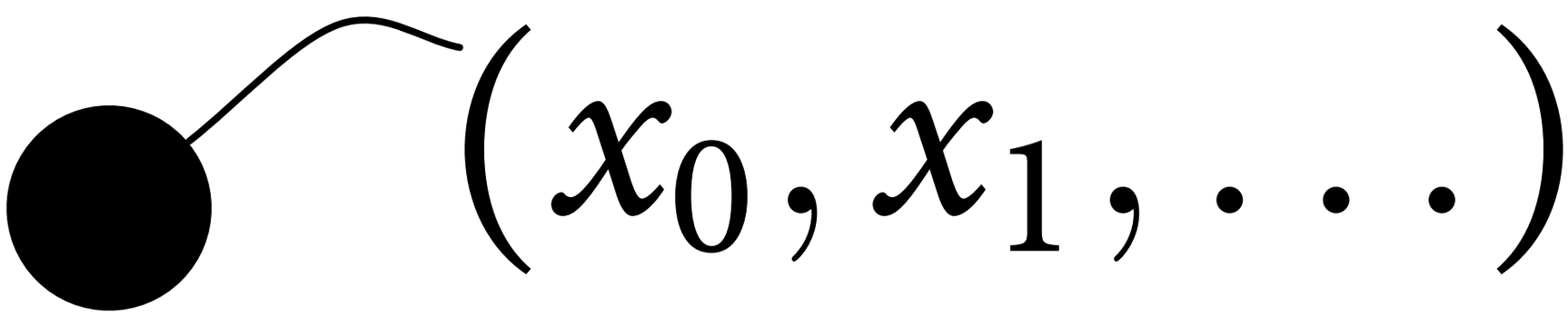
objects in more than 3D are complex!

Some background

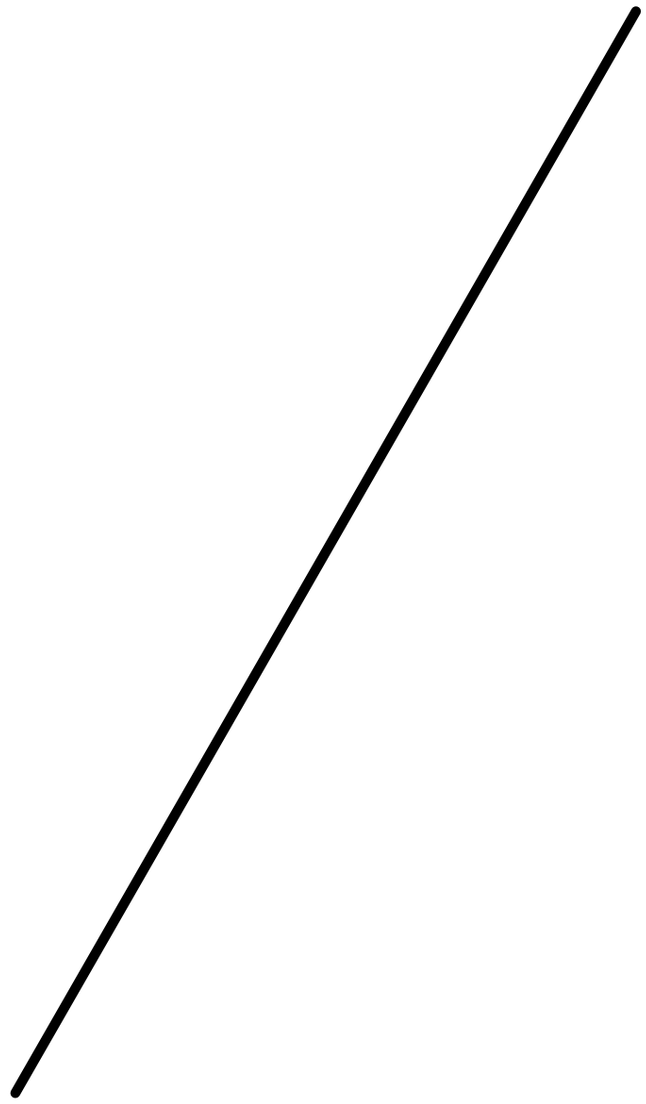
0D: a vertex



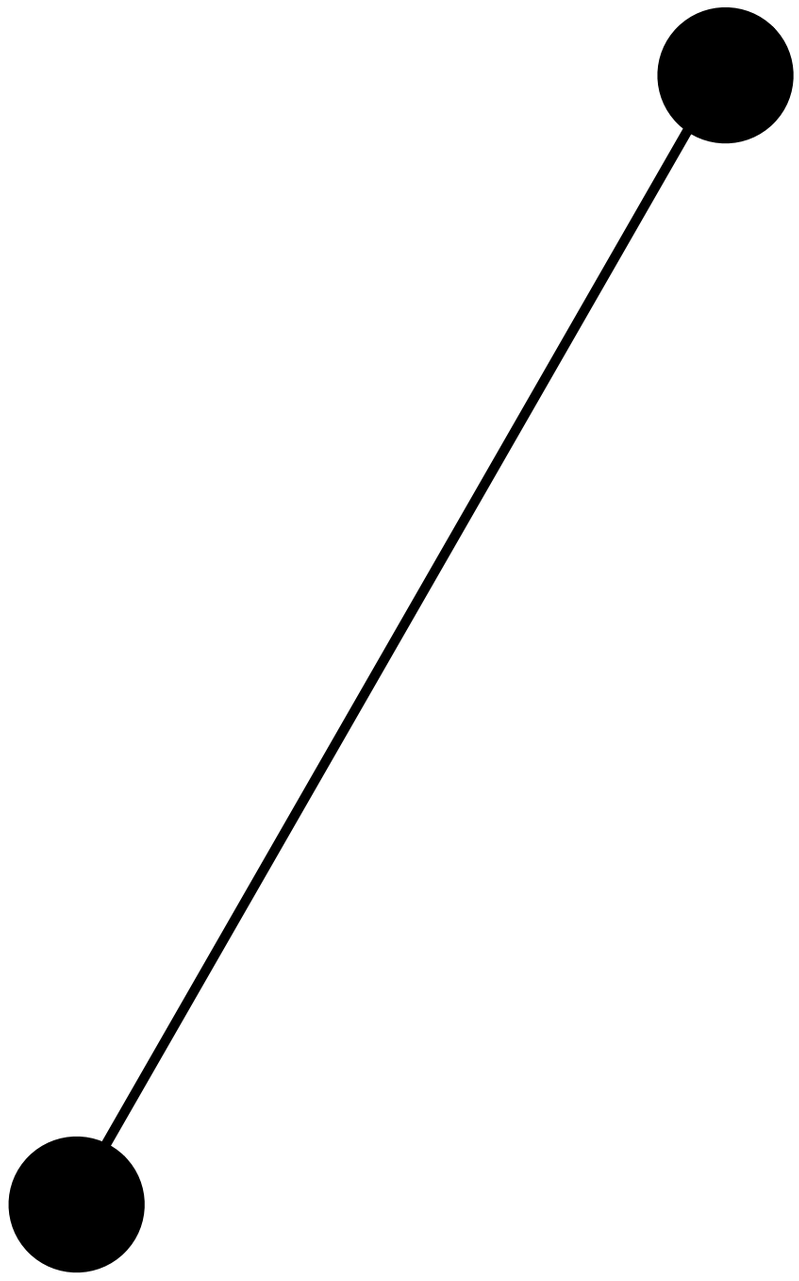
0D: a vertex



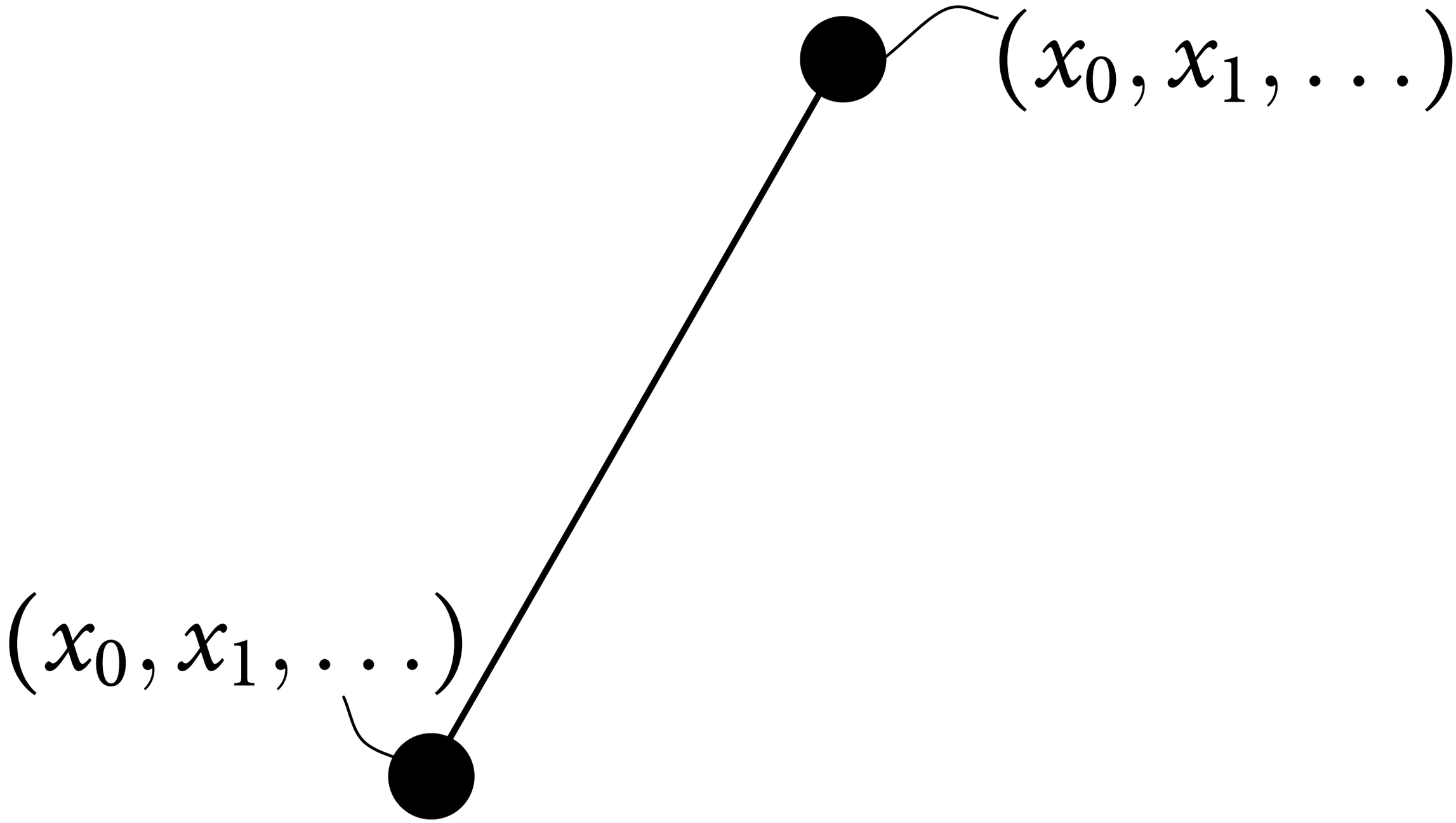
1D: an edge



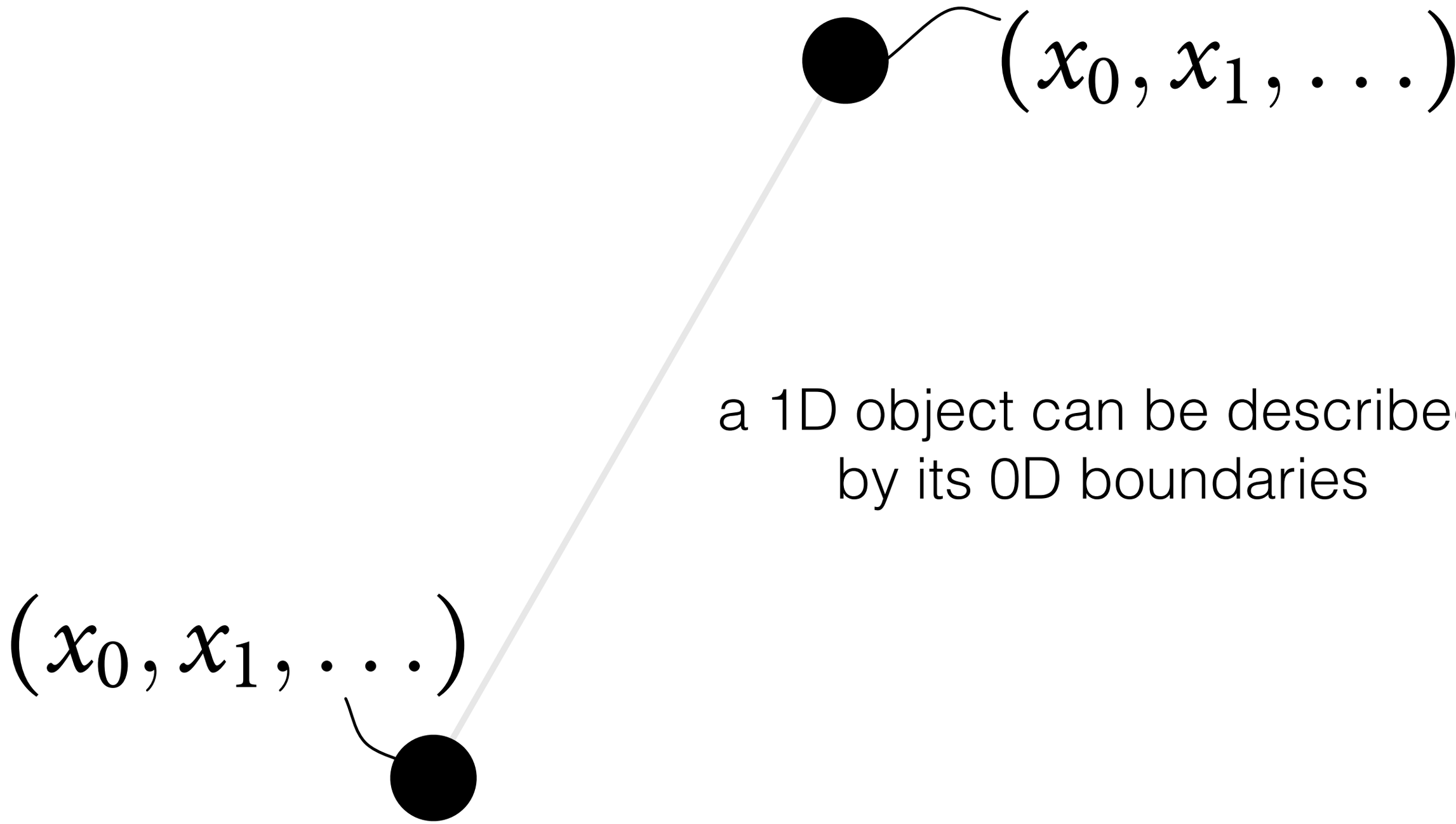
1D: an edge



1D: an edge

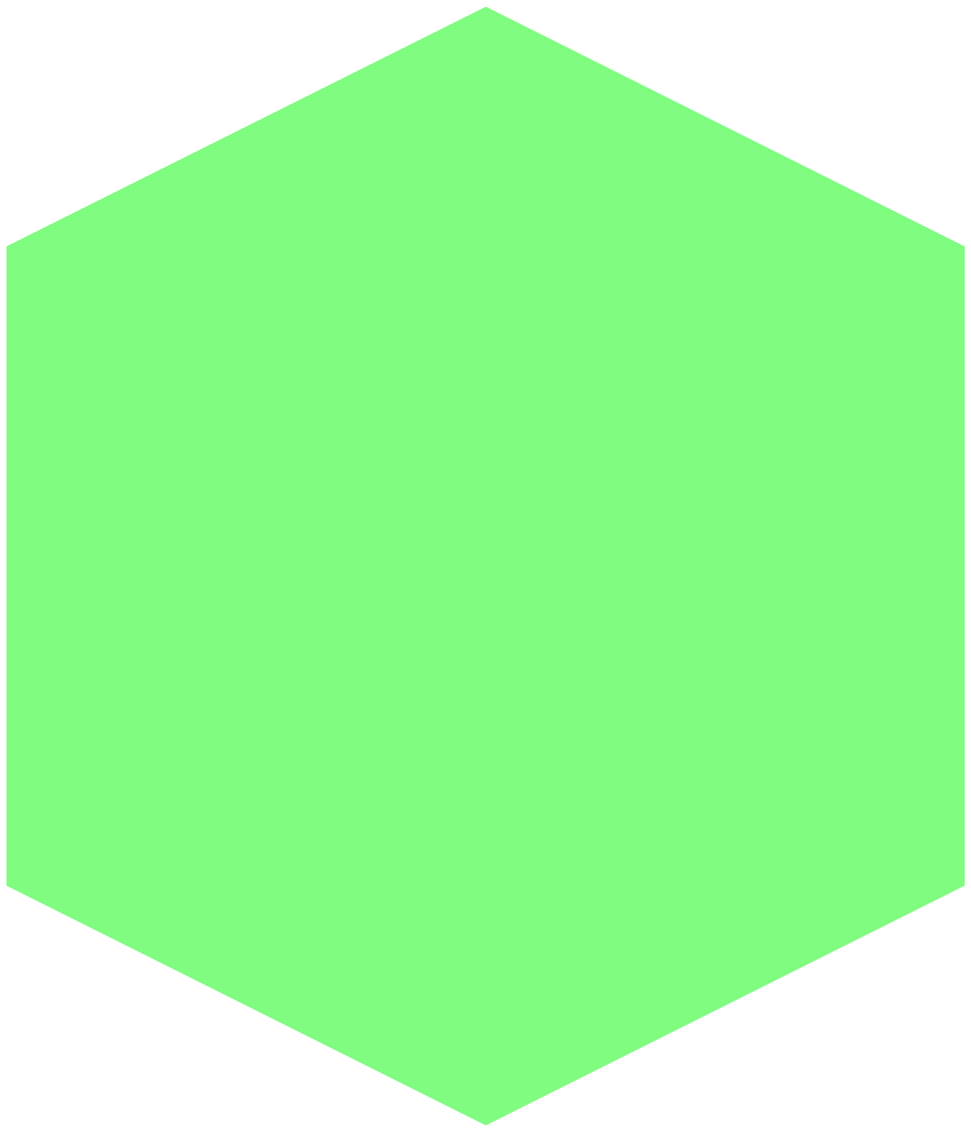


1D: an edge

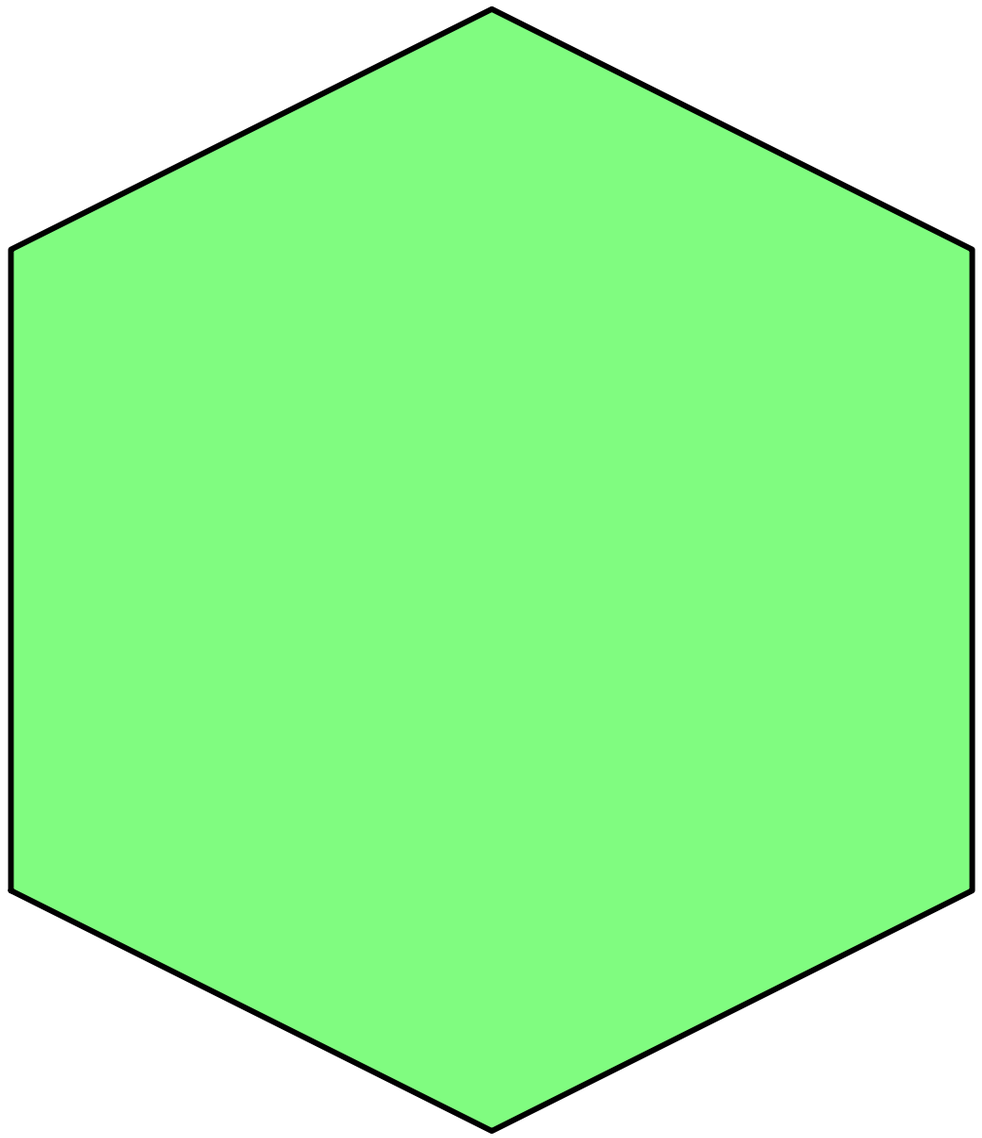


a 1D object can be described
by its 0D boundaries

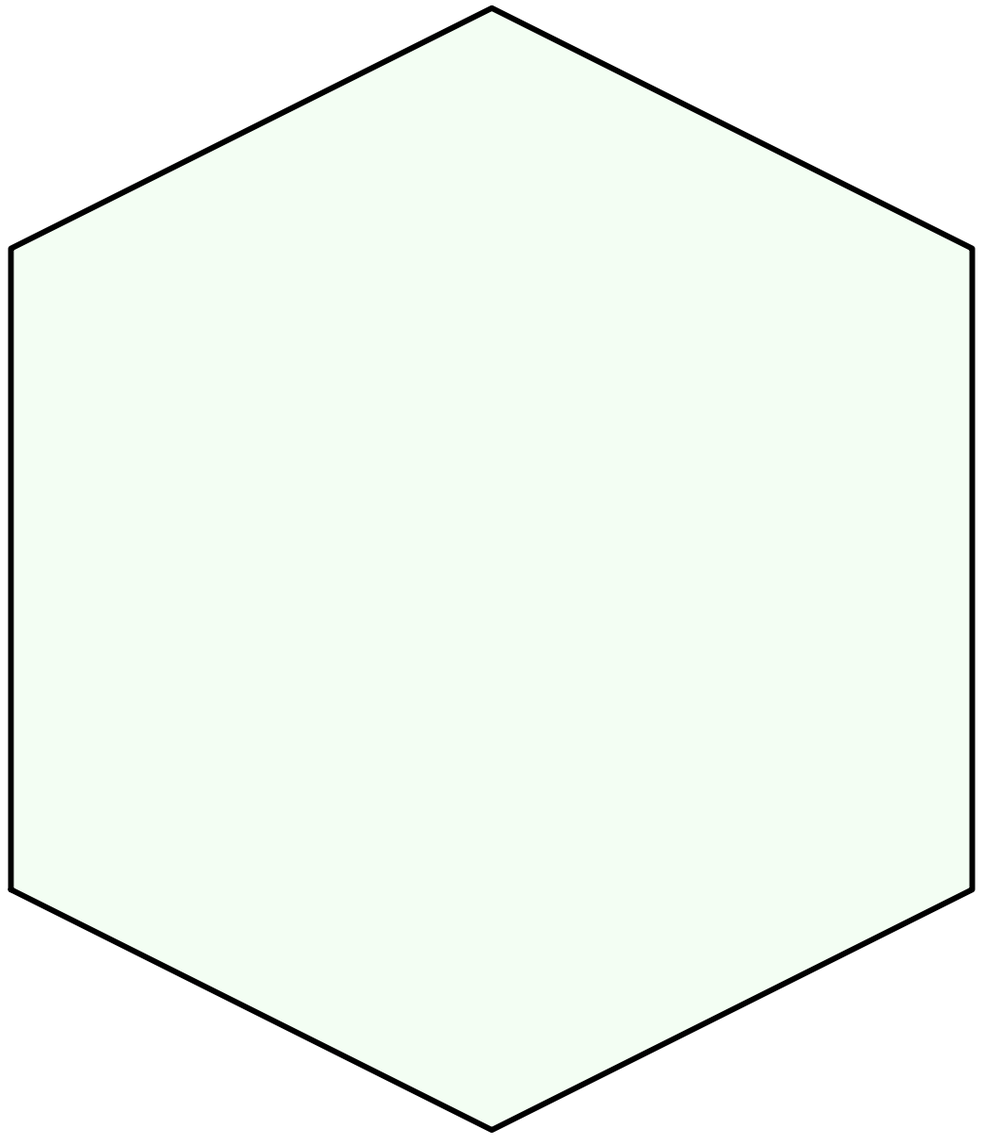
2D: a face



2D: a face

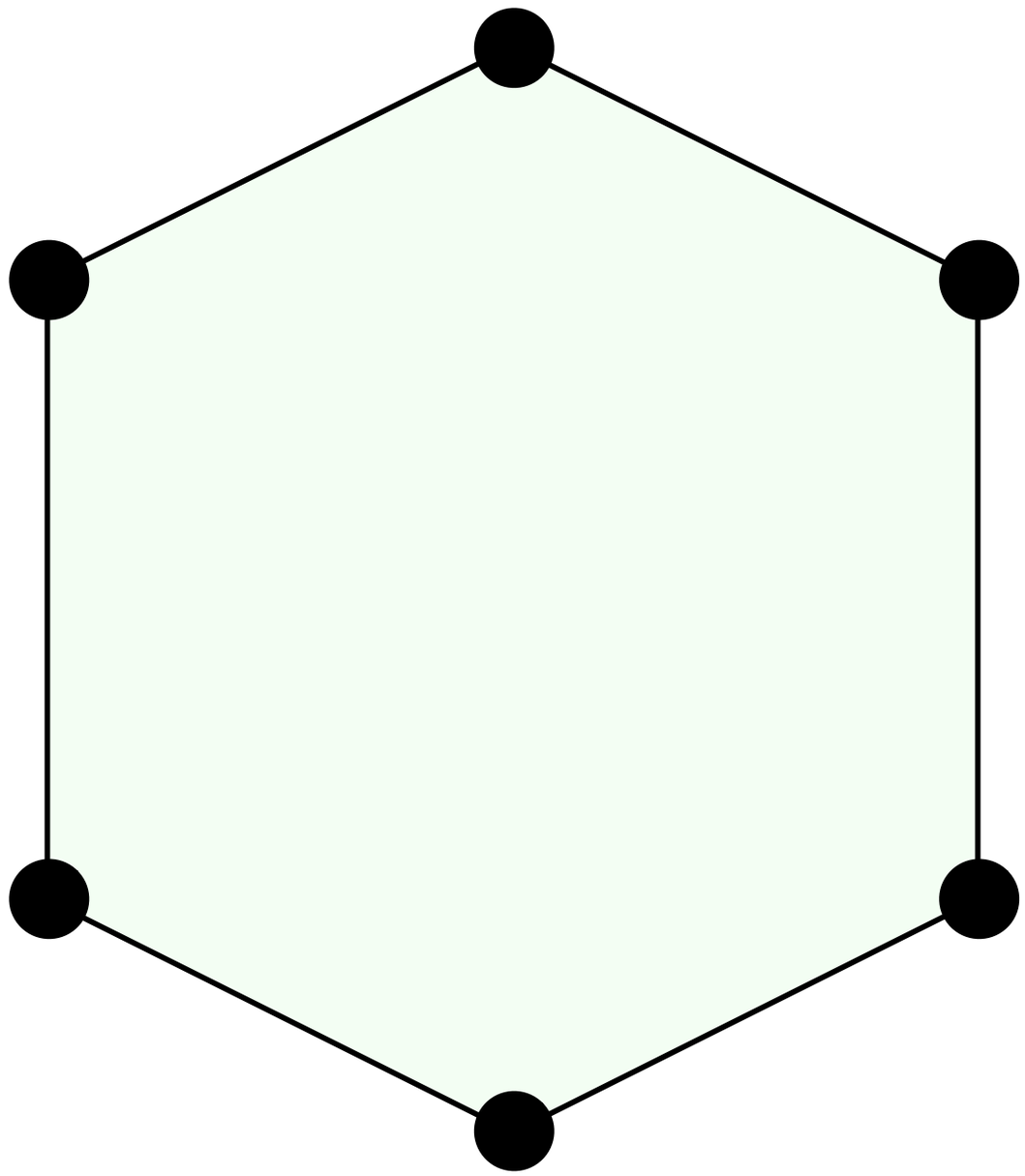


2D: a face

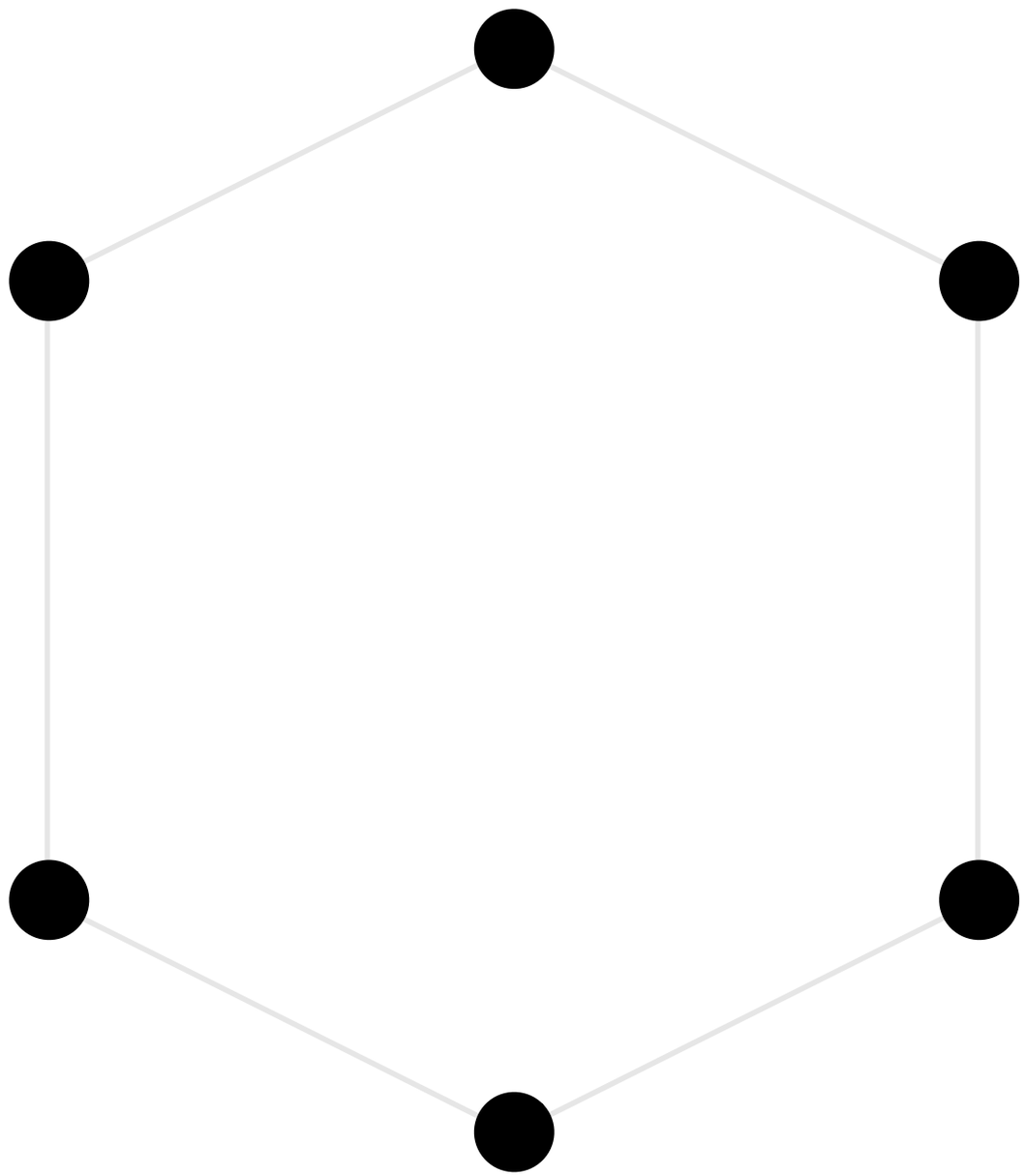


a 2D object can be described by its 1D boundary

2D: a face

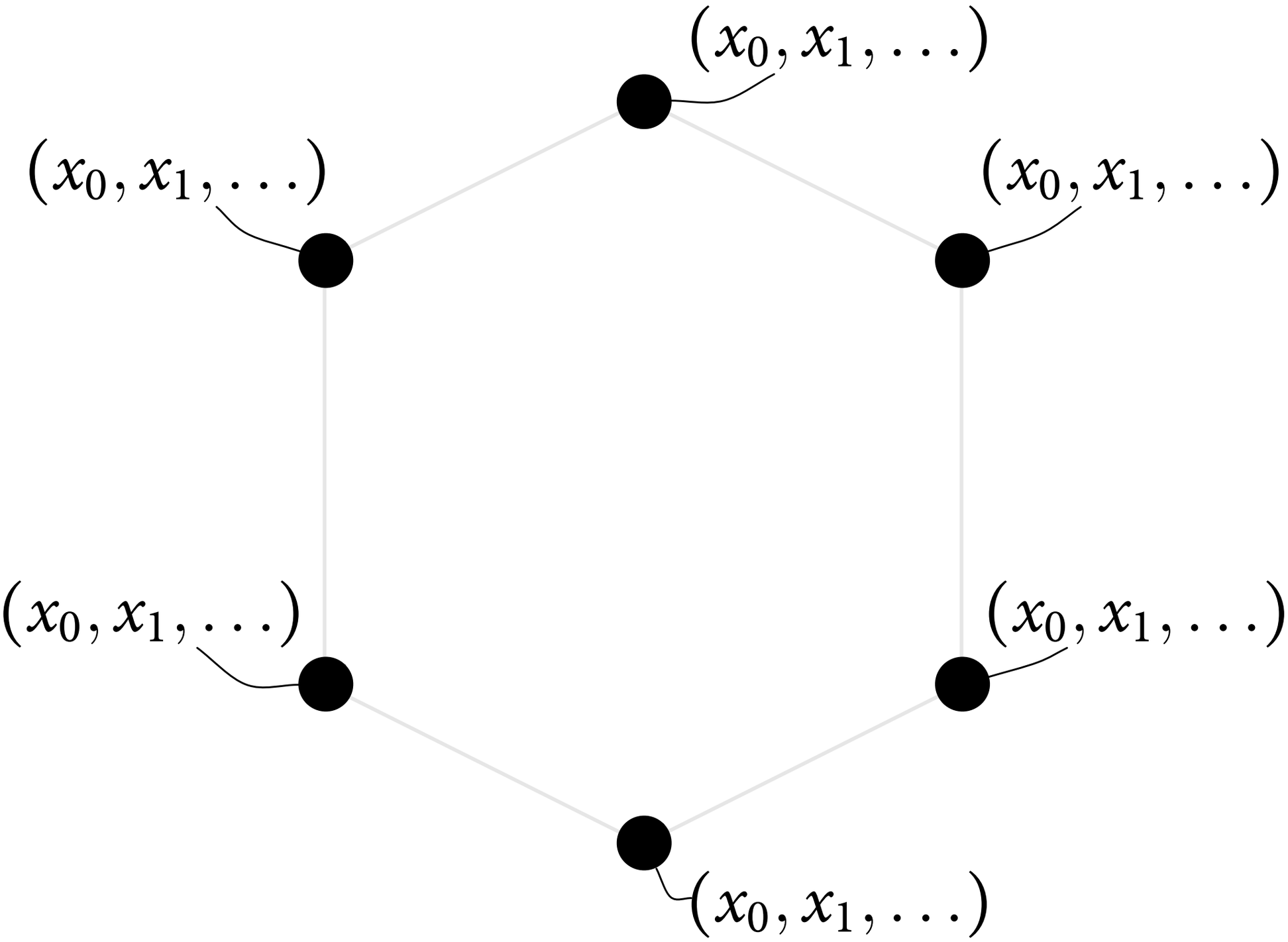


2D: a face



which can be described by its 0D boundaries

2D: a face


$(x_0, x_1, \ldots)$
$(x_0, x_1, \ldots)$
$(x_0, x_1, \ldots)$
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3D: a volume

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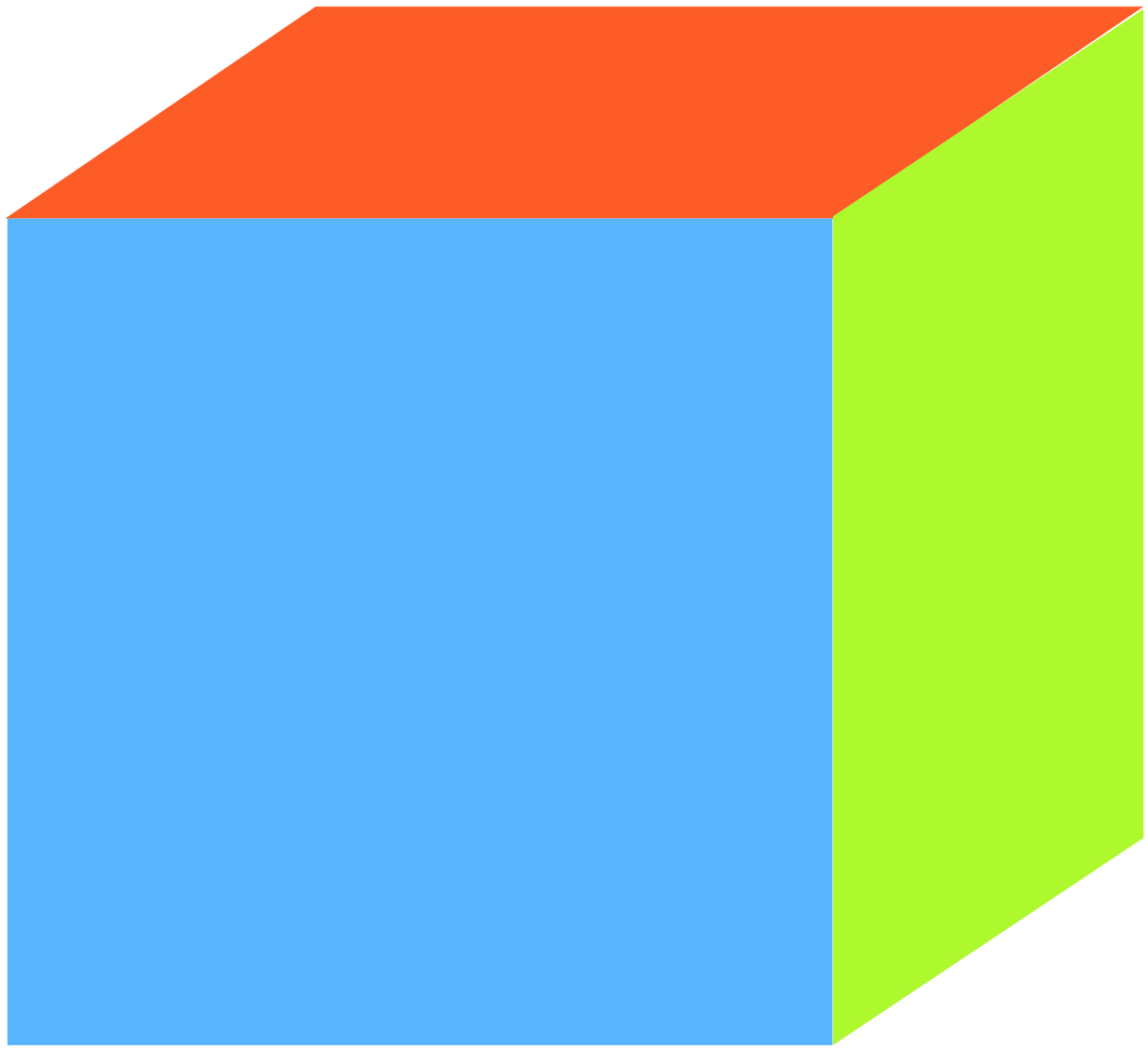
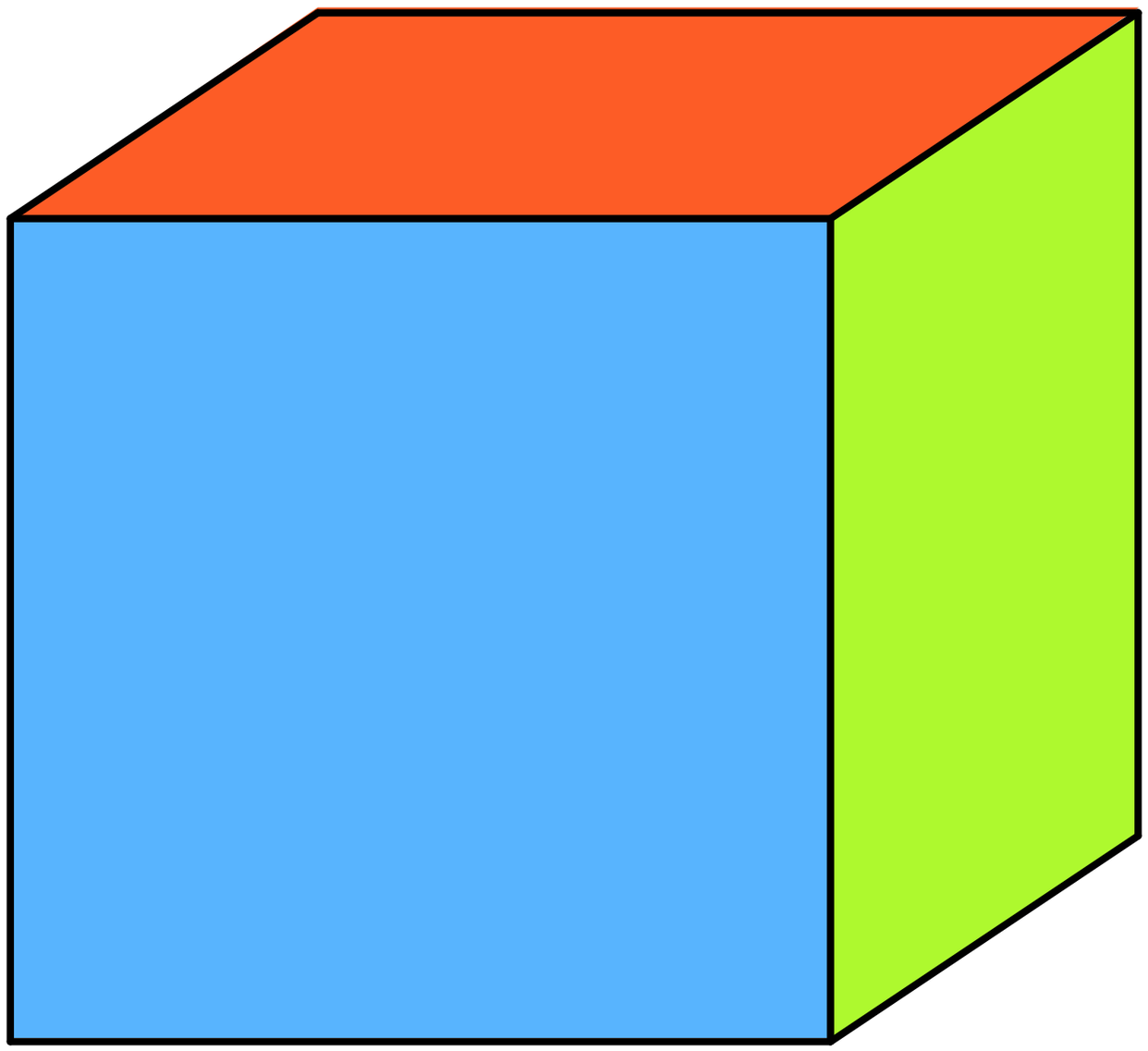
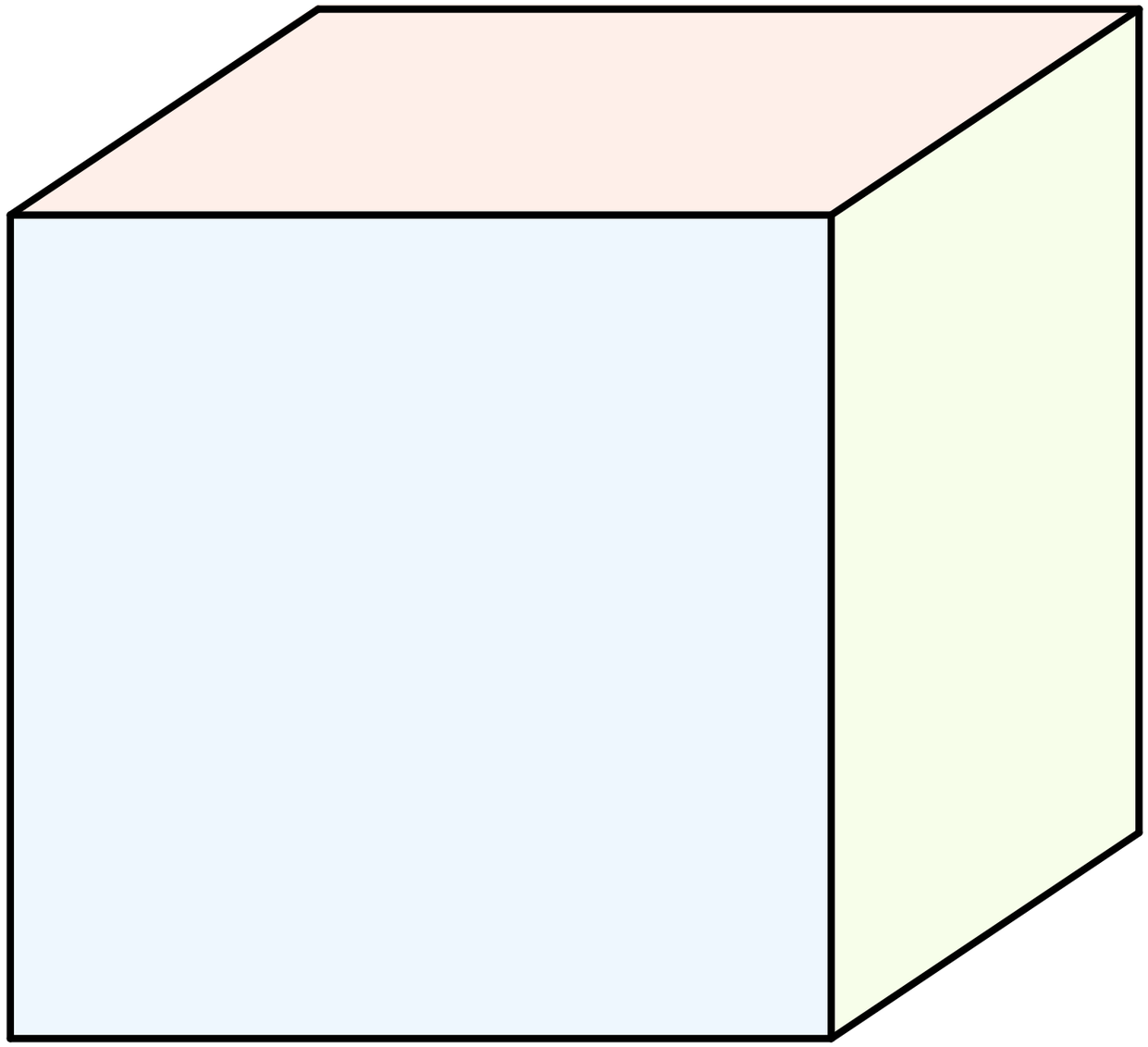


a 3D object can be described by its 2D boundary

3D: a volume

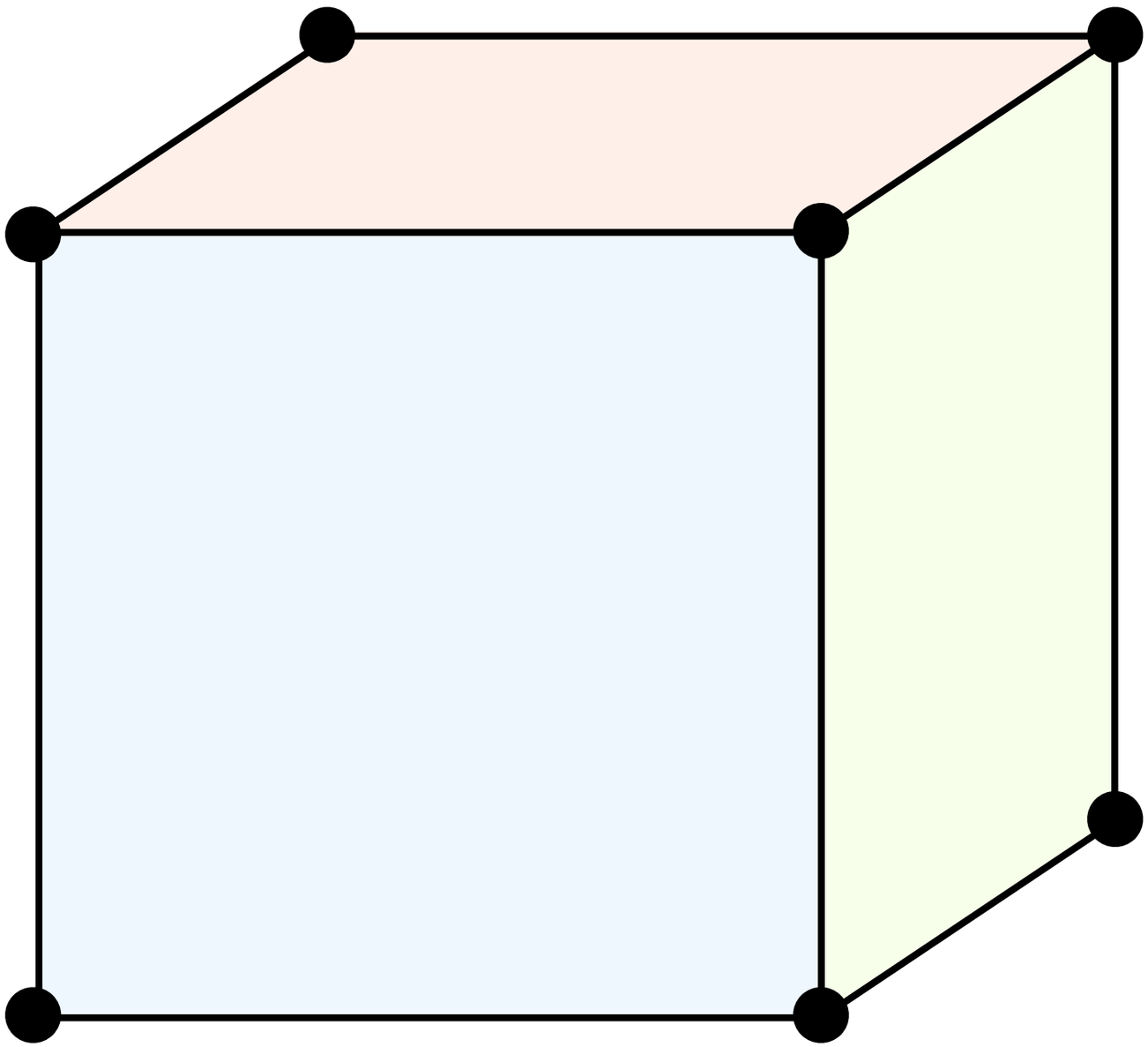


3D: a volume

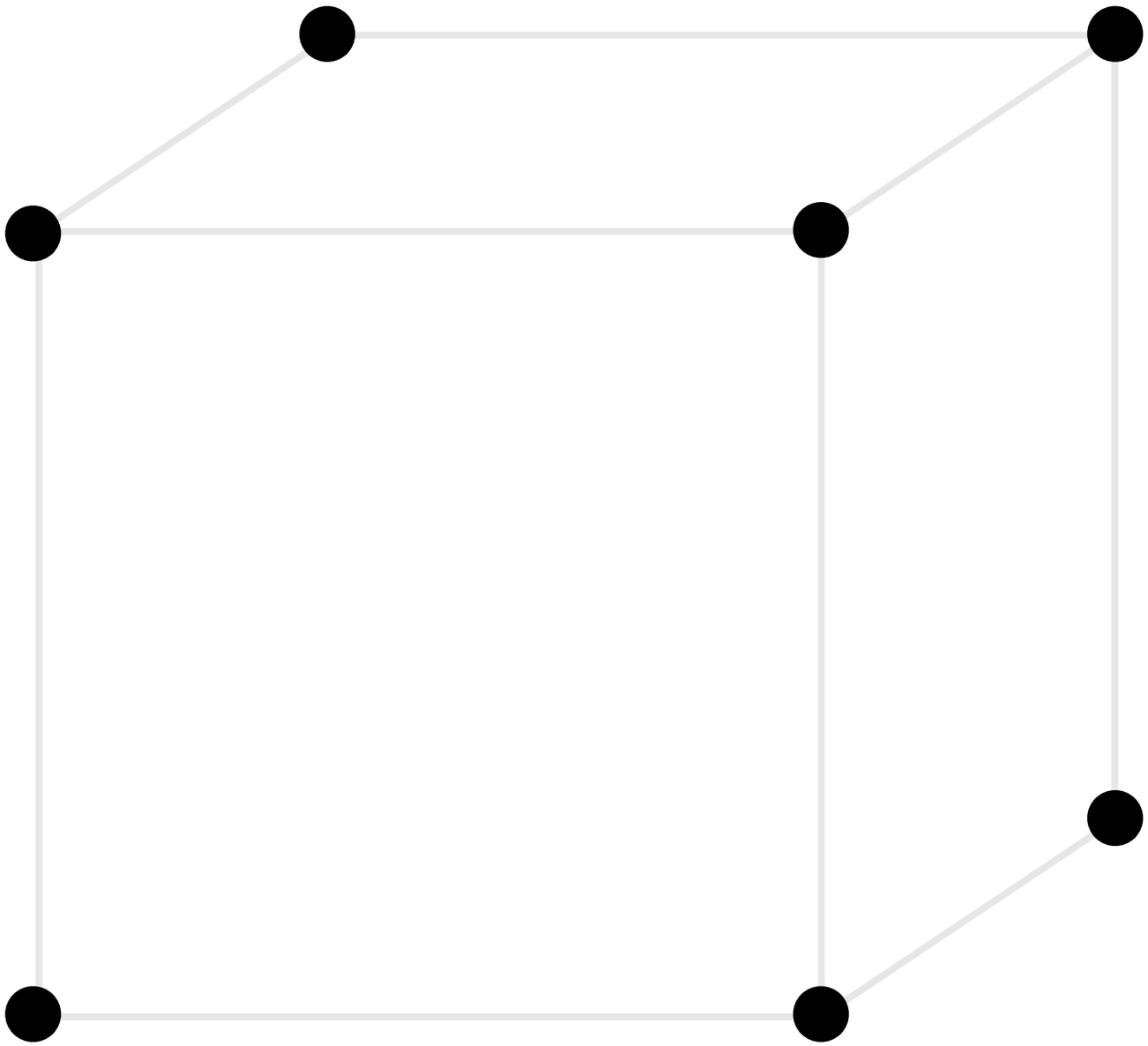


which can be described by its 1D boundaries

3D: a volume

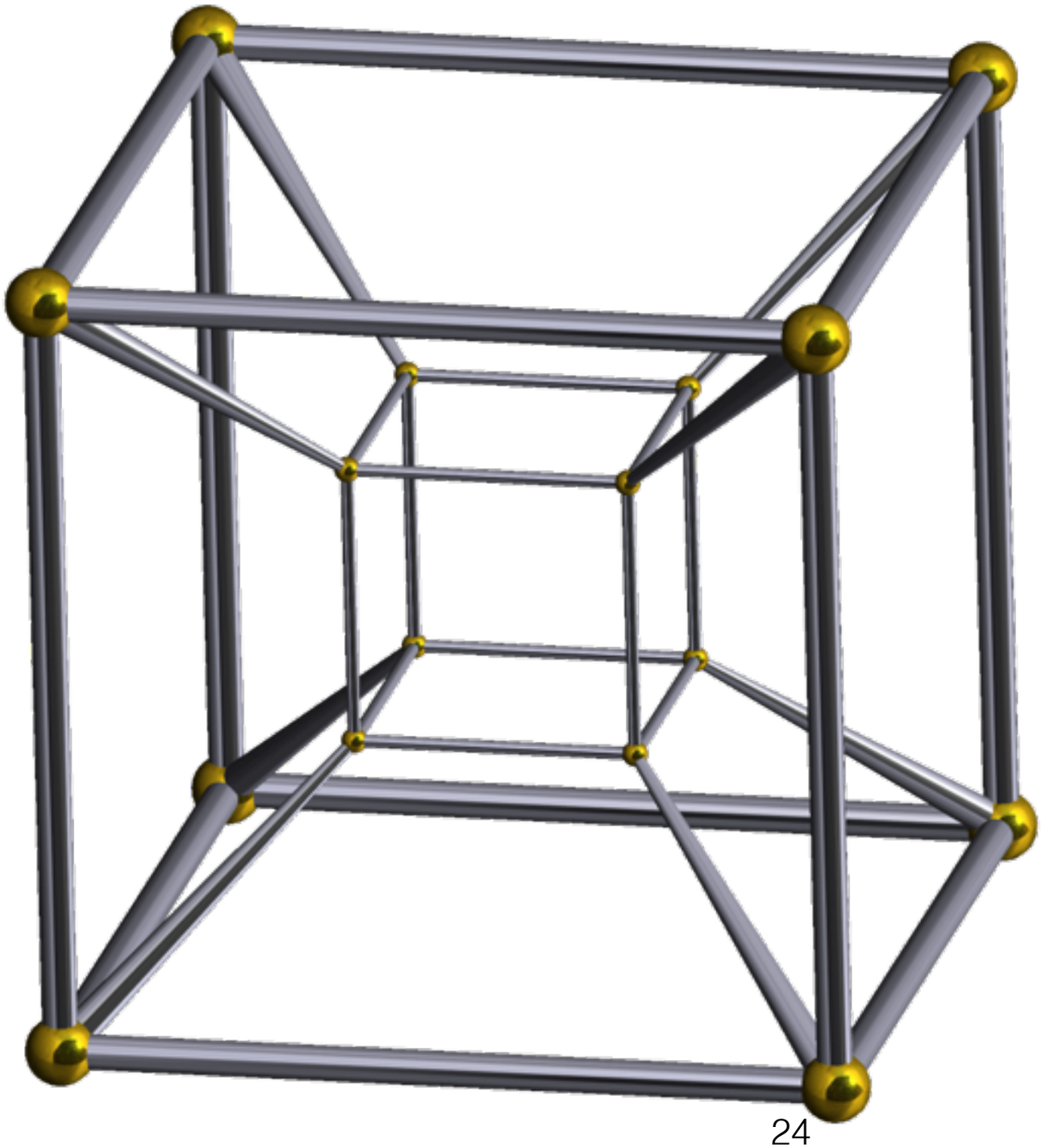


3D: a volume



which can be described by their 0D boundaries

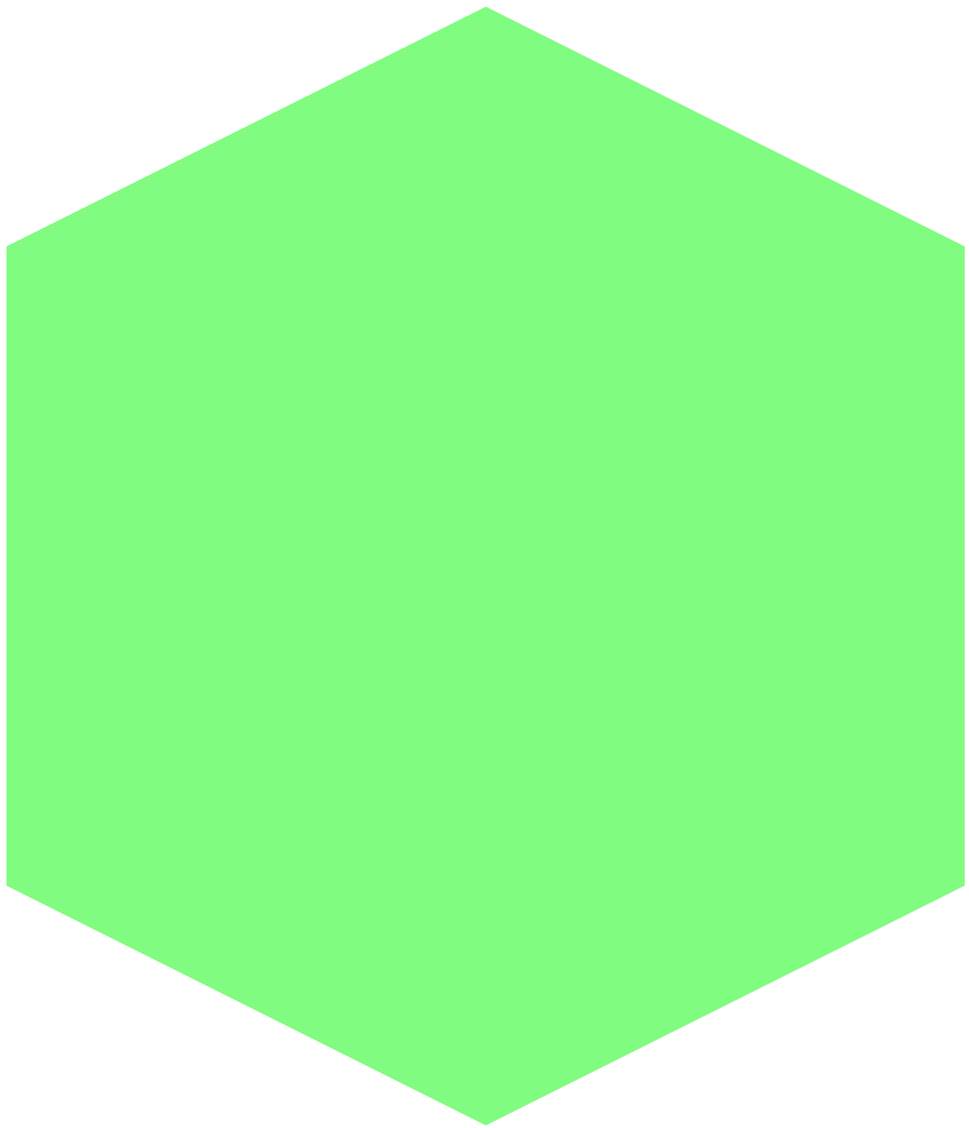
4D, 5D, . . .



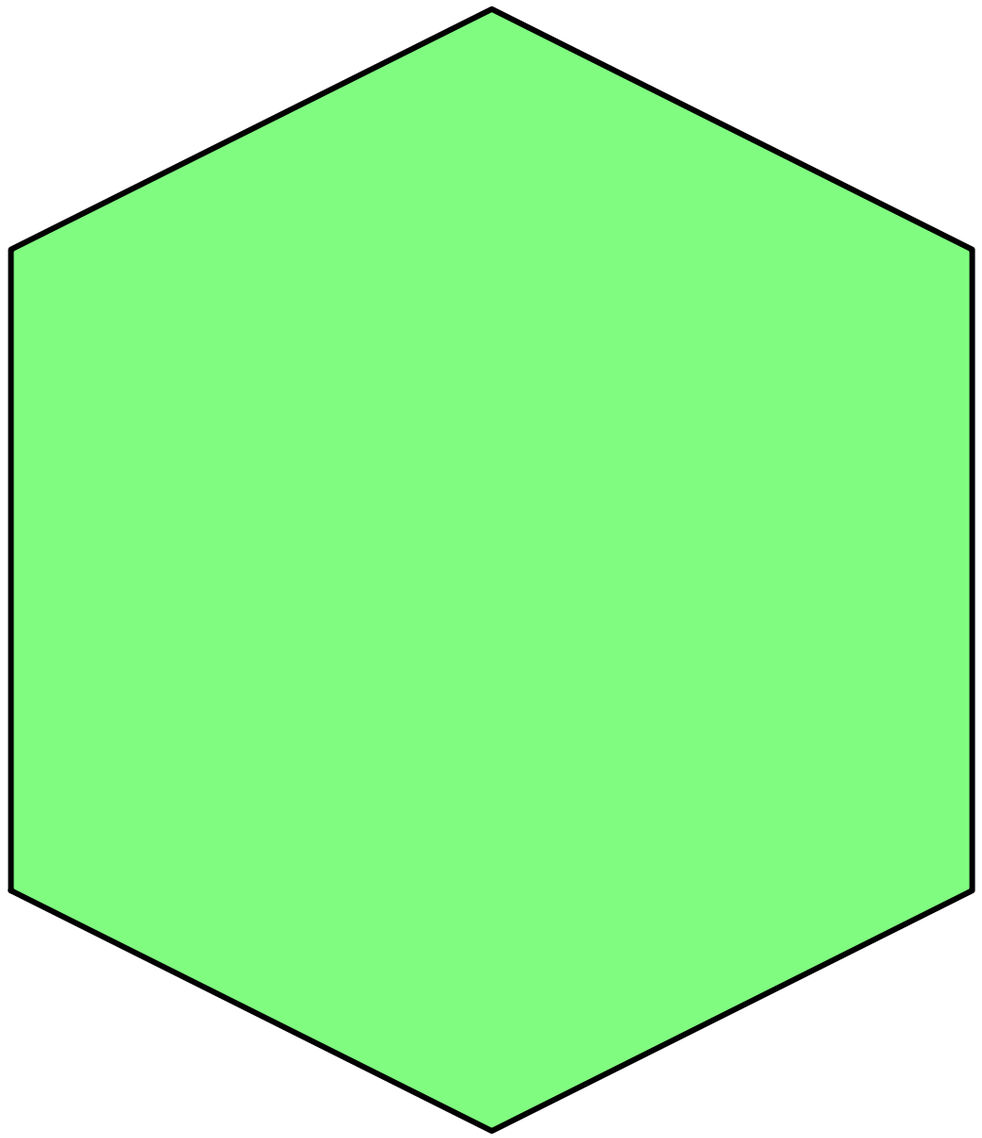
= 8 cubes

however, there is a problem in practice…

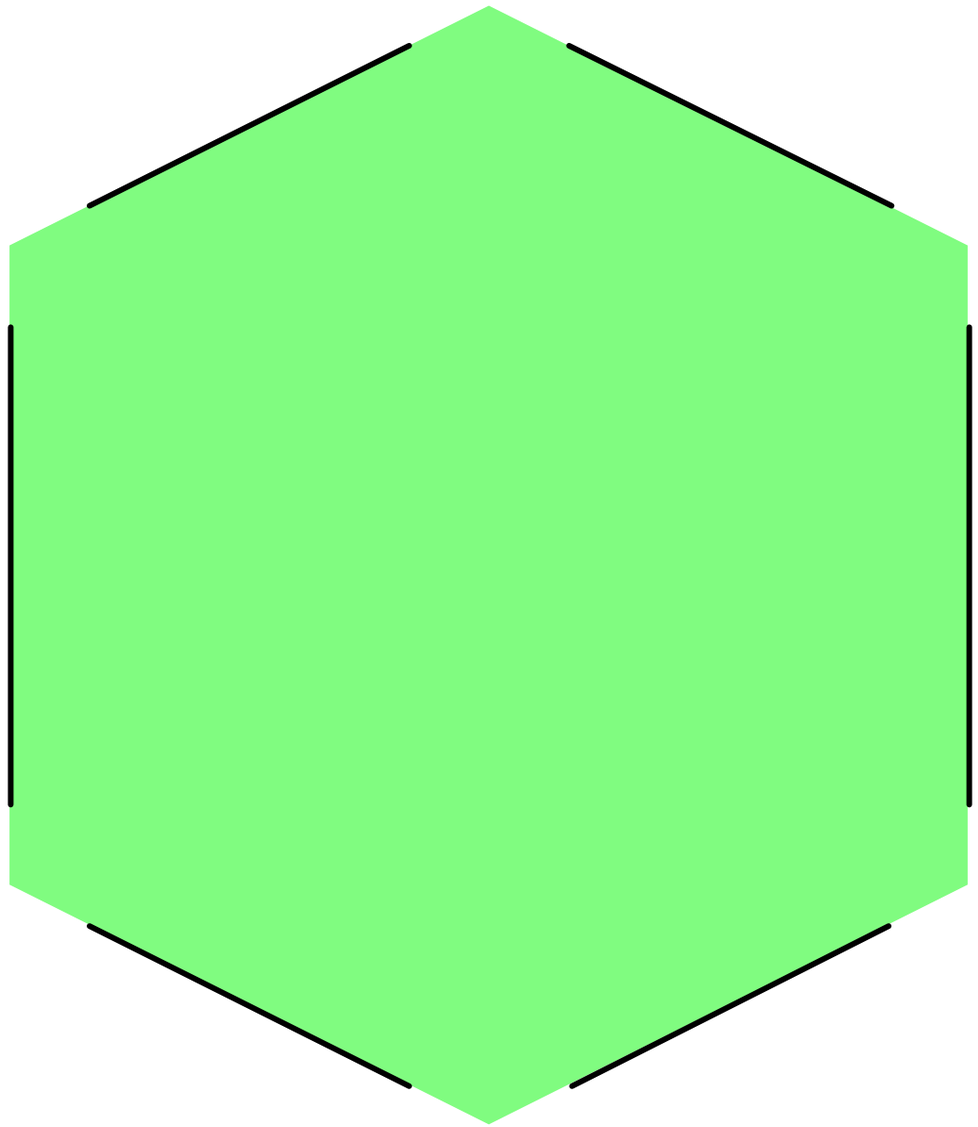
2D: a face



2D: a face

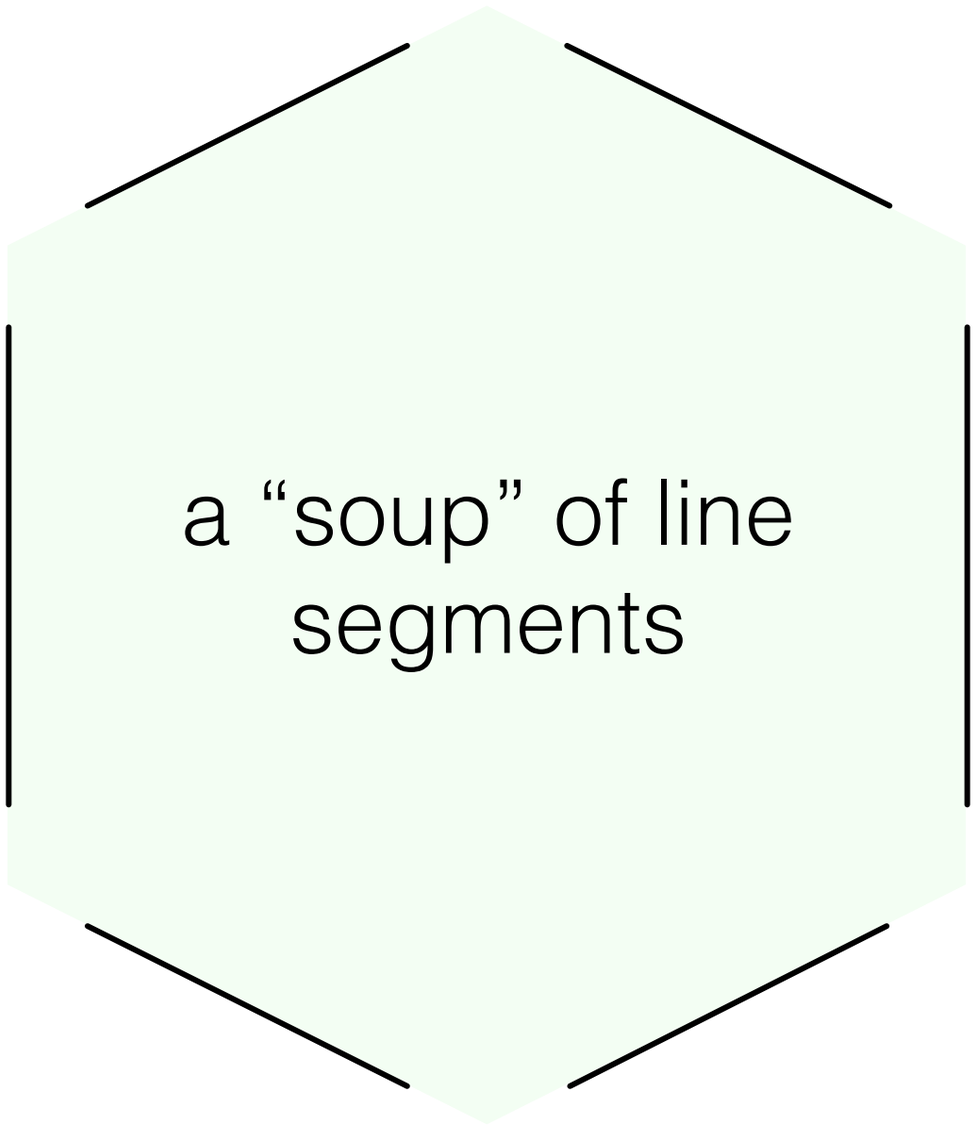


2D: a face

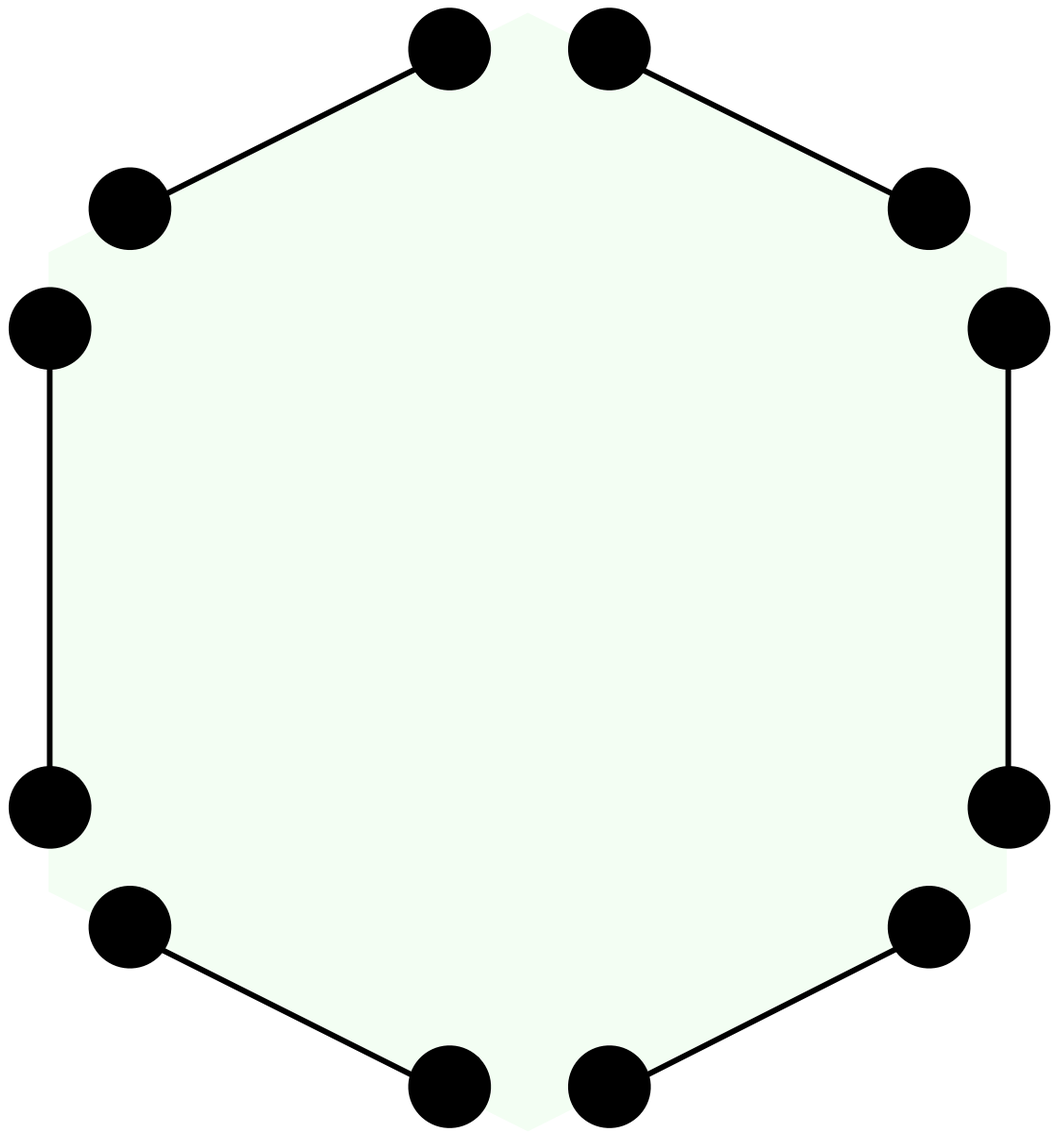


a 2D object is described by **a set** of 1D objects

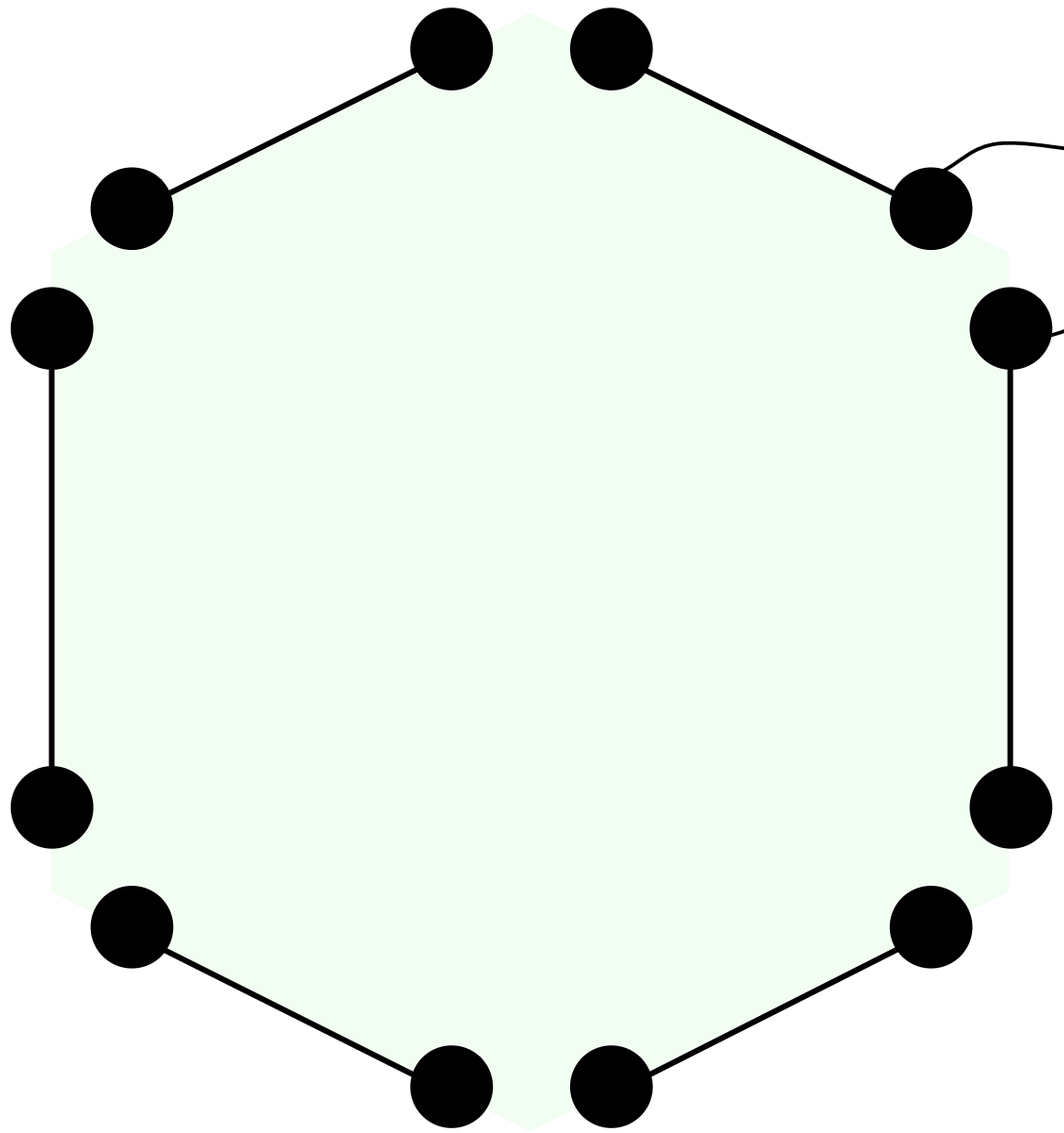
2D: a face



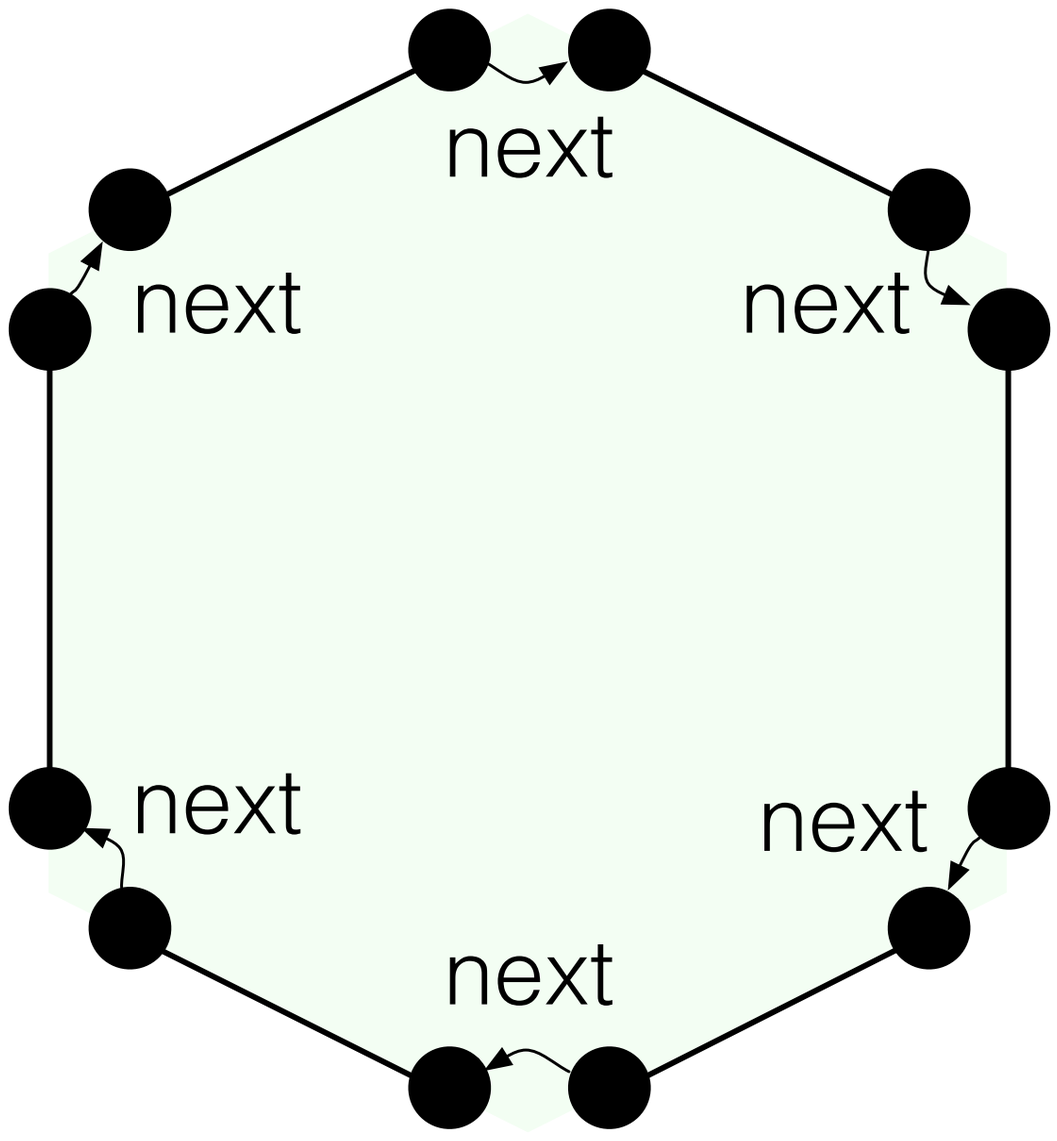
2D: a face



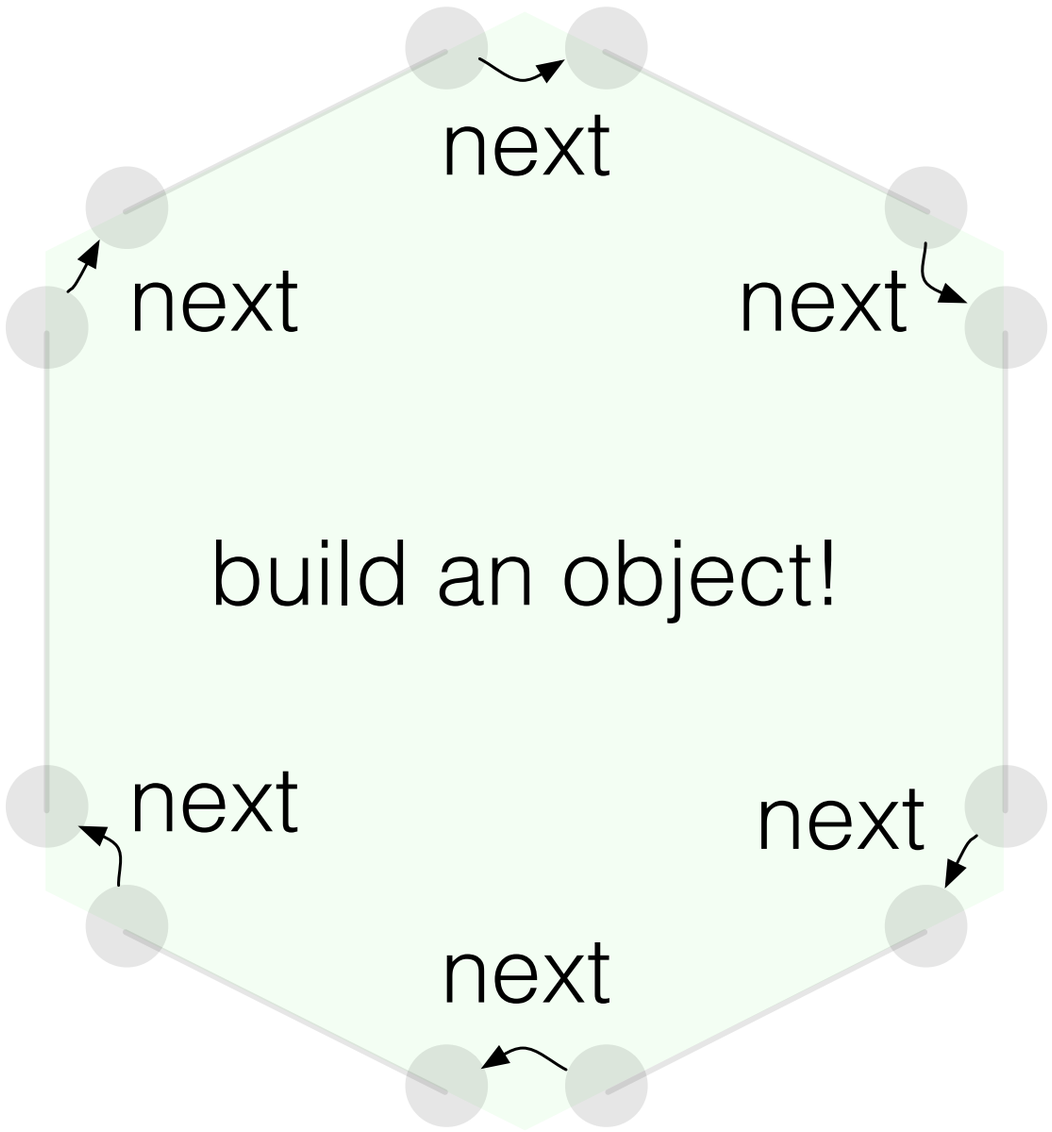
2D: a face



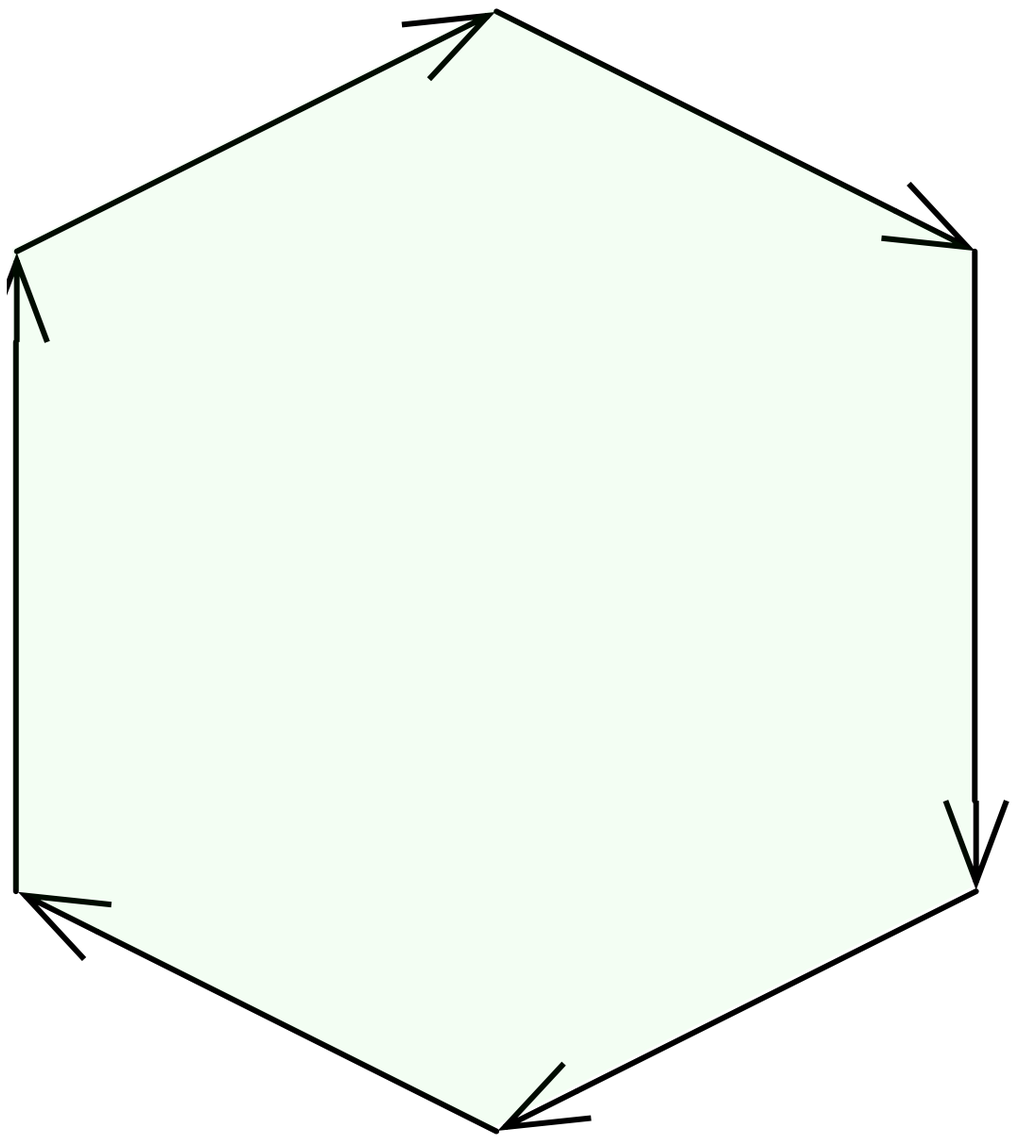
2D: a face



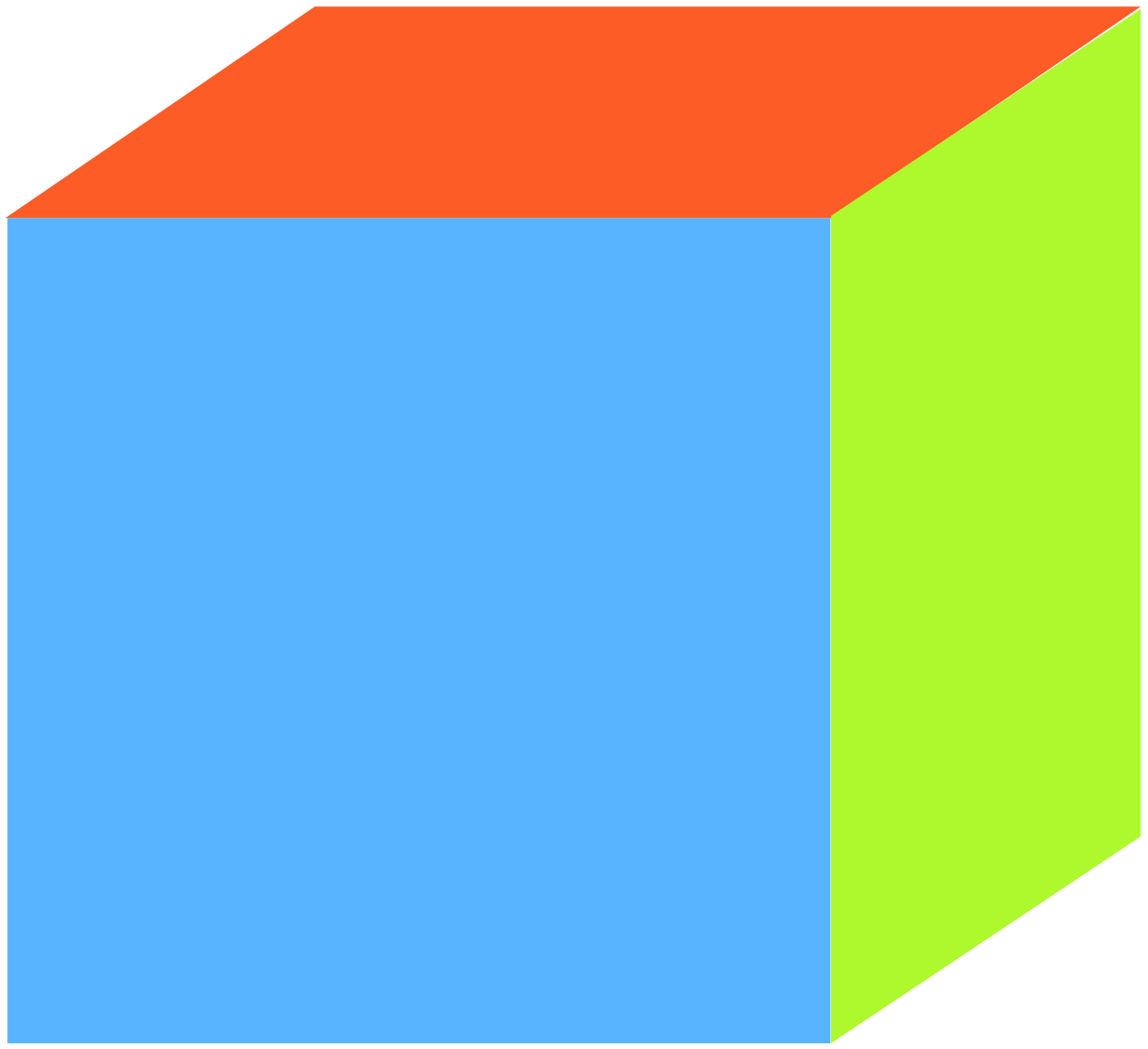
2D: a face



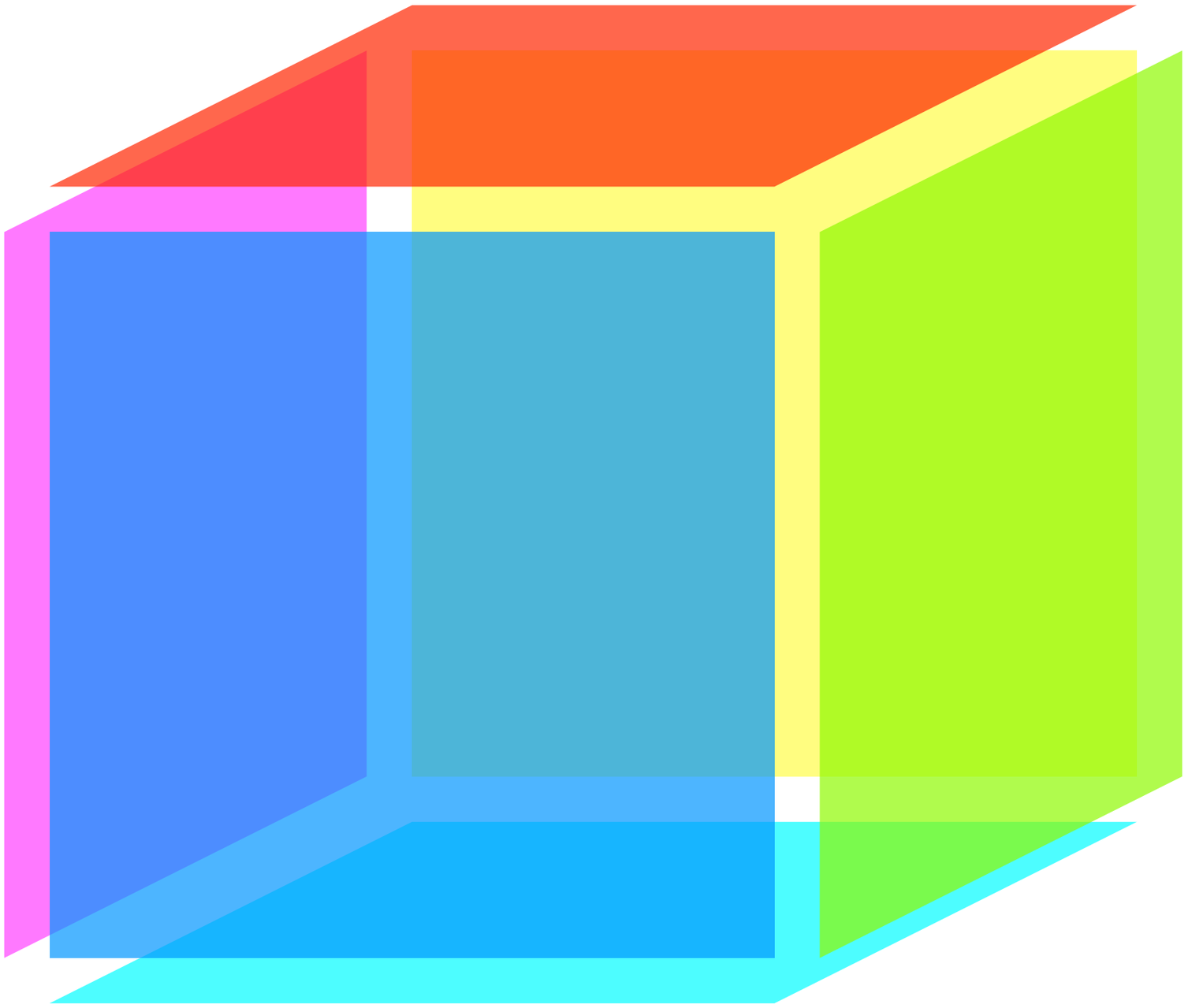
2D: a face



3D: a volume

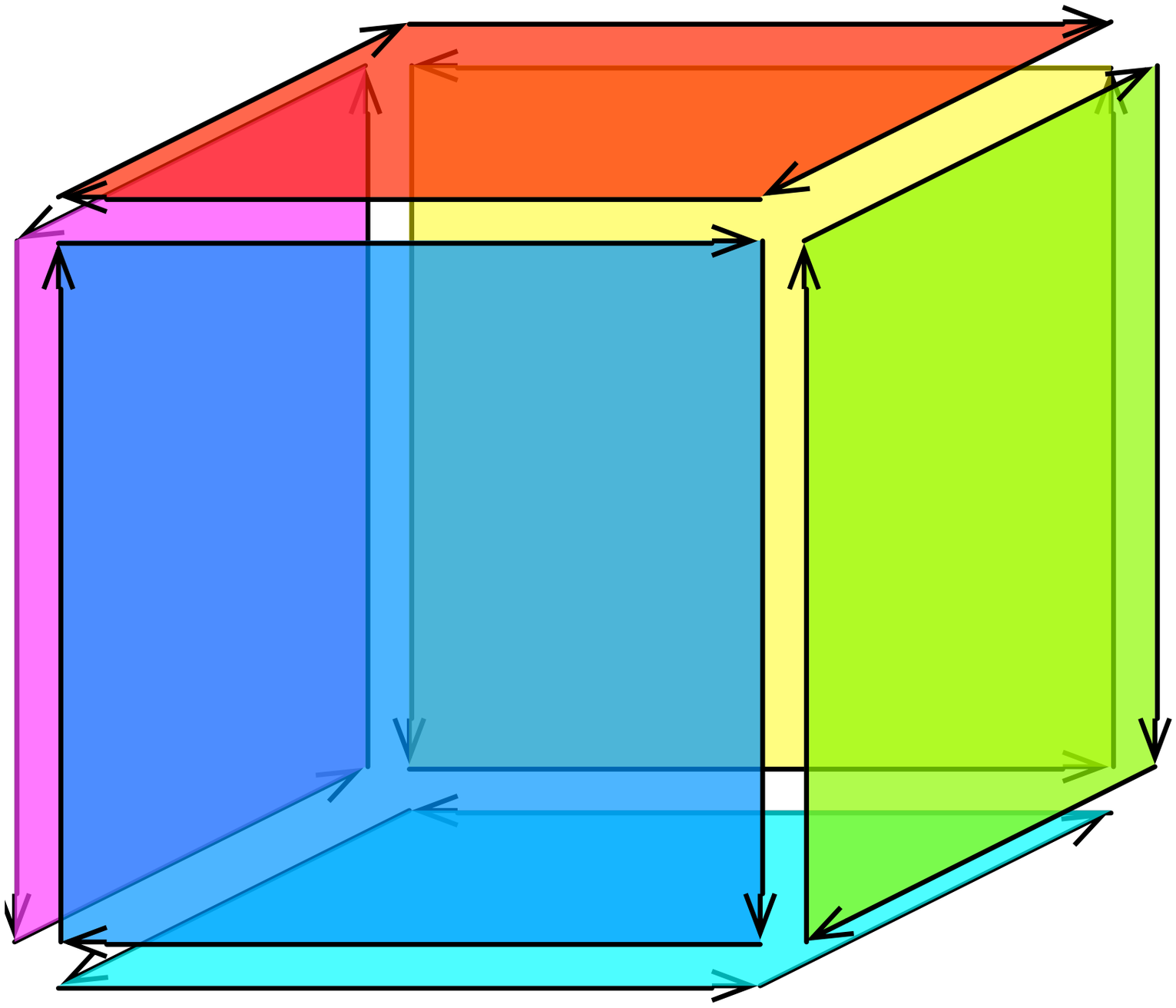


3D: a volume

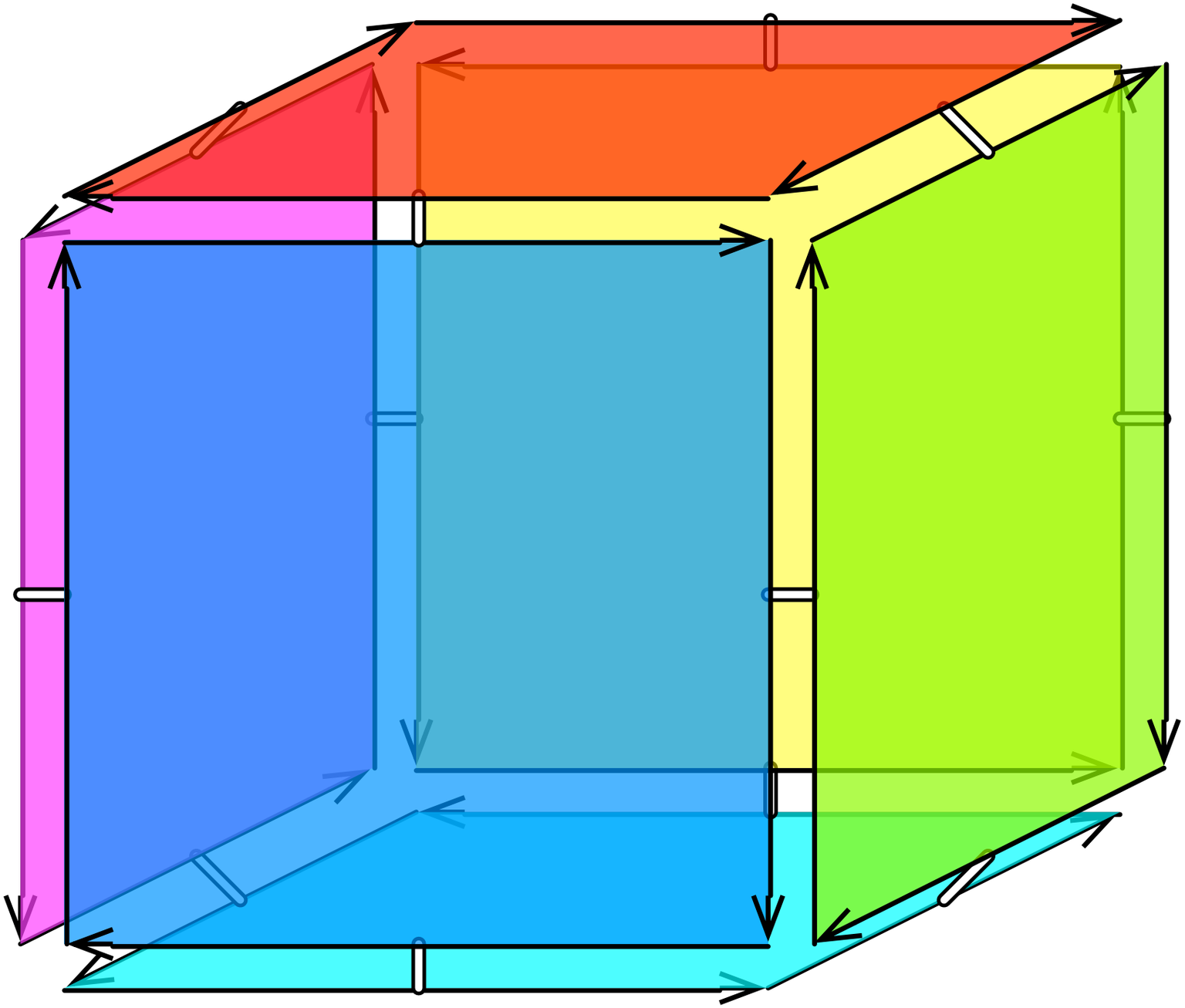


a “soup”
of faces

3D: a volume



3D: a volume

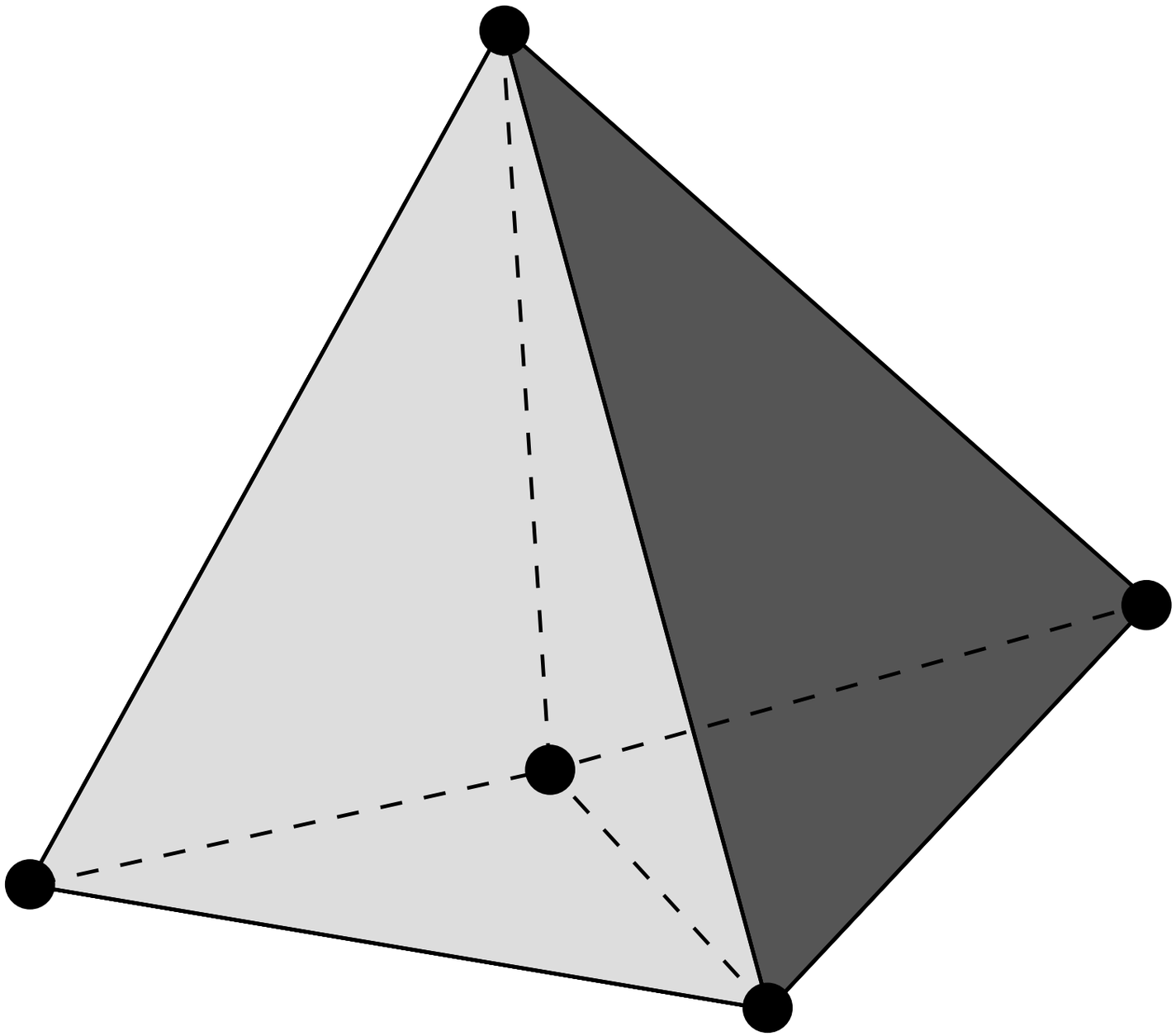


build a volume from a set of faces

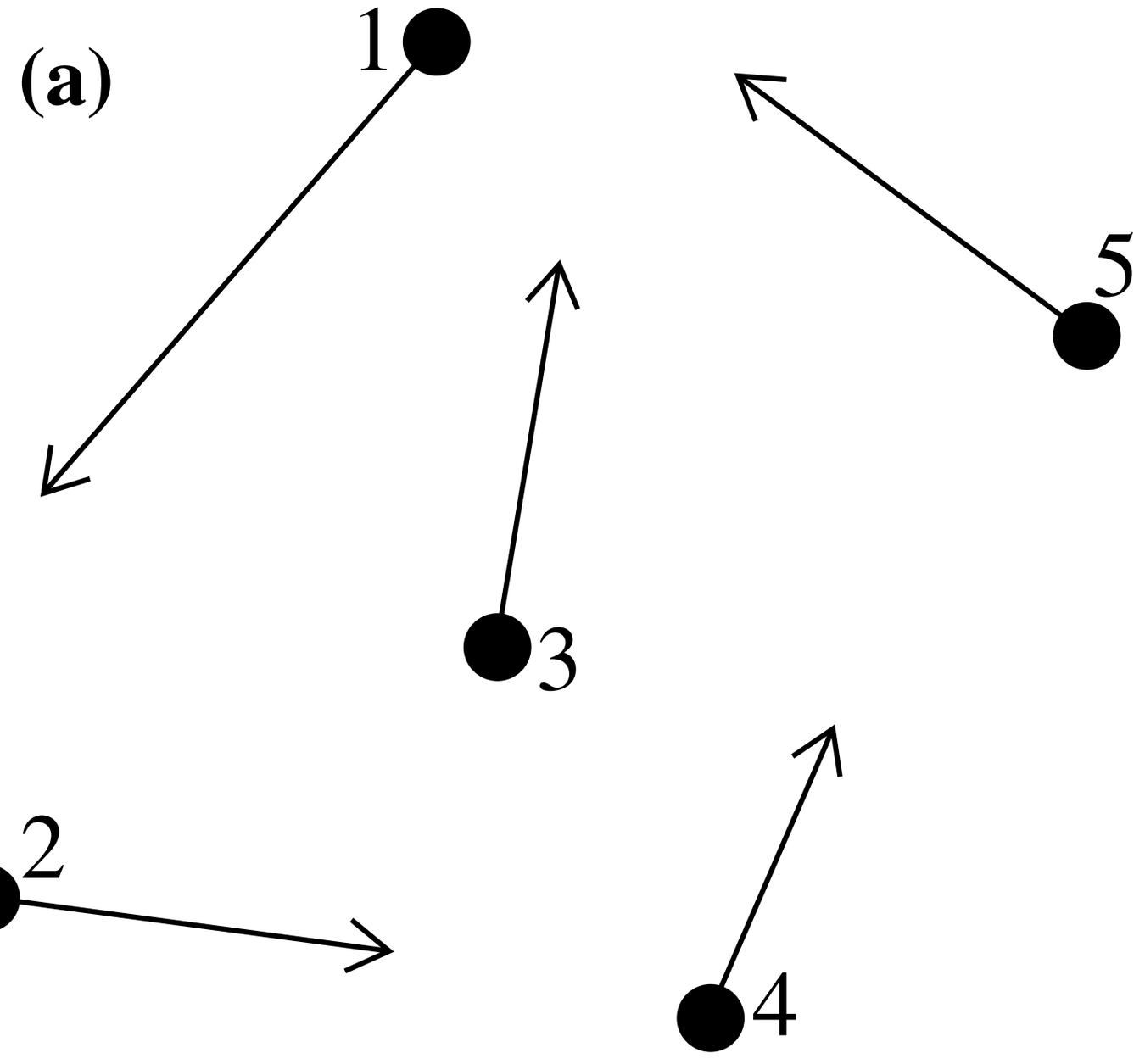
The solution: incremental construction

- Start from a set of 0D vertices
- Connect them to form 1D edges
- Connect these to form 2D faces
- Connect these to form 3D volumes
- …

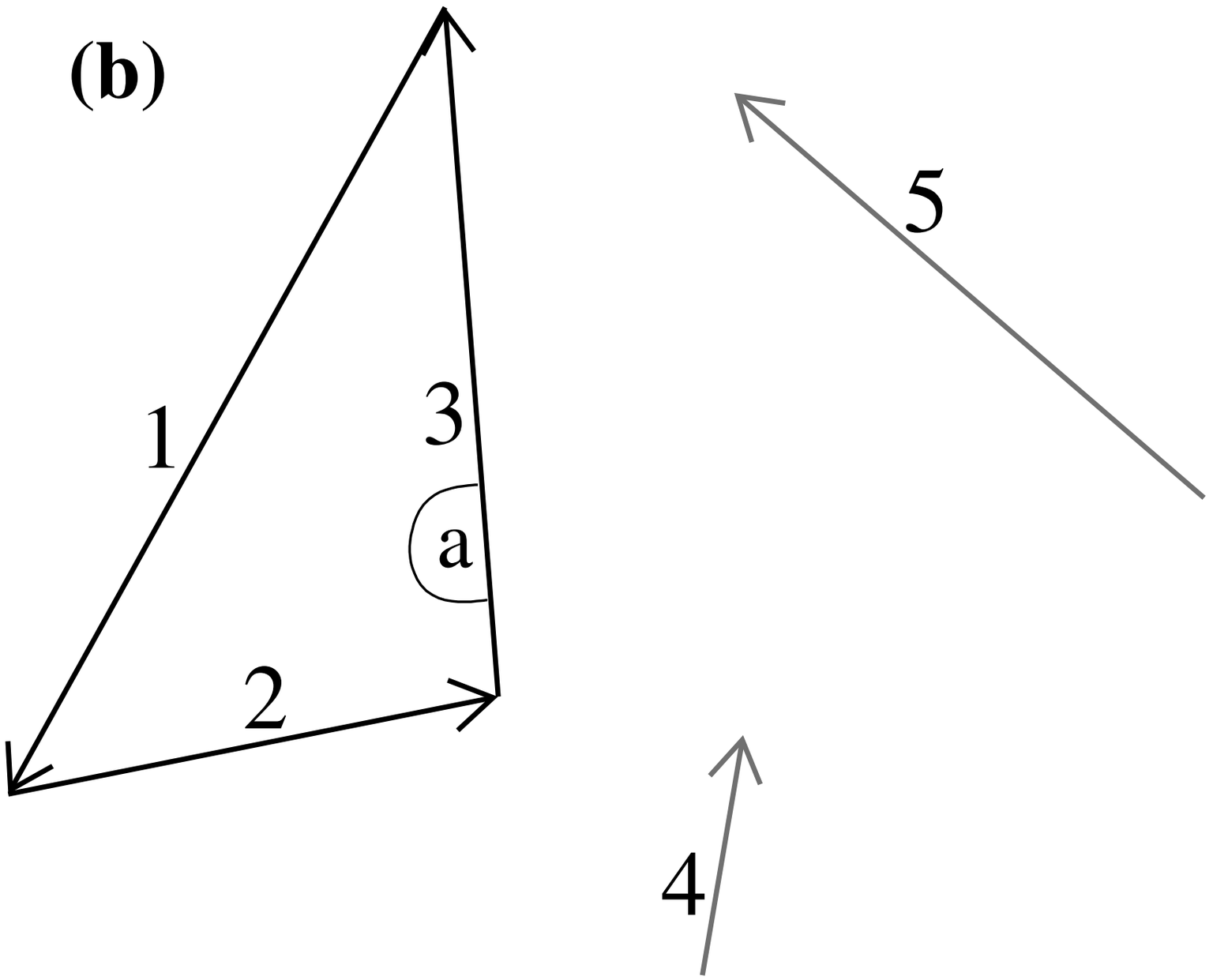
Building two tetrahedra



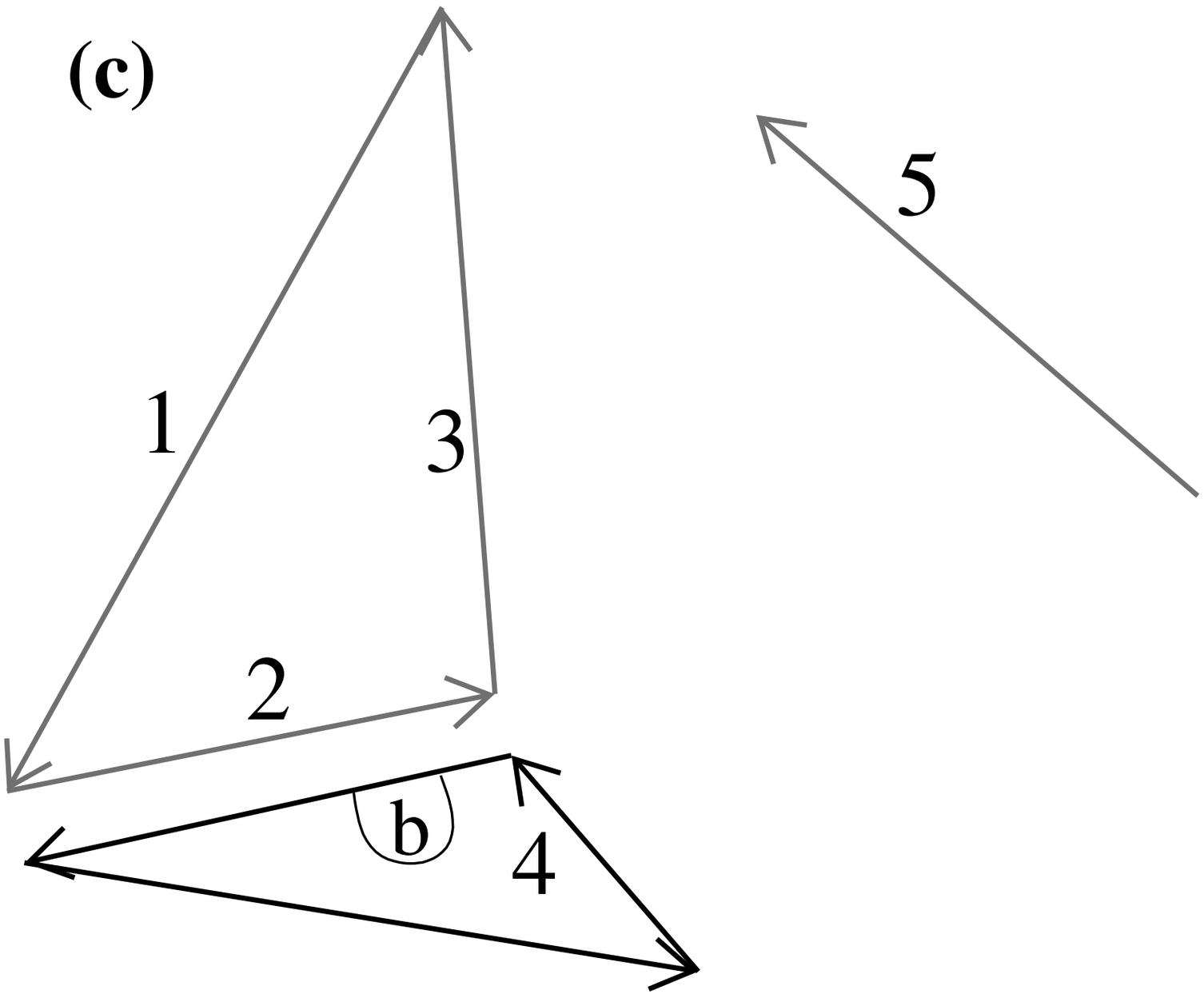
Building two tetrahedra



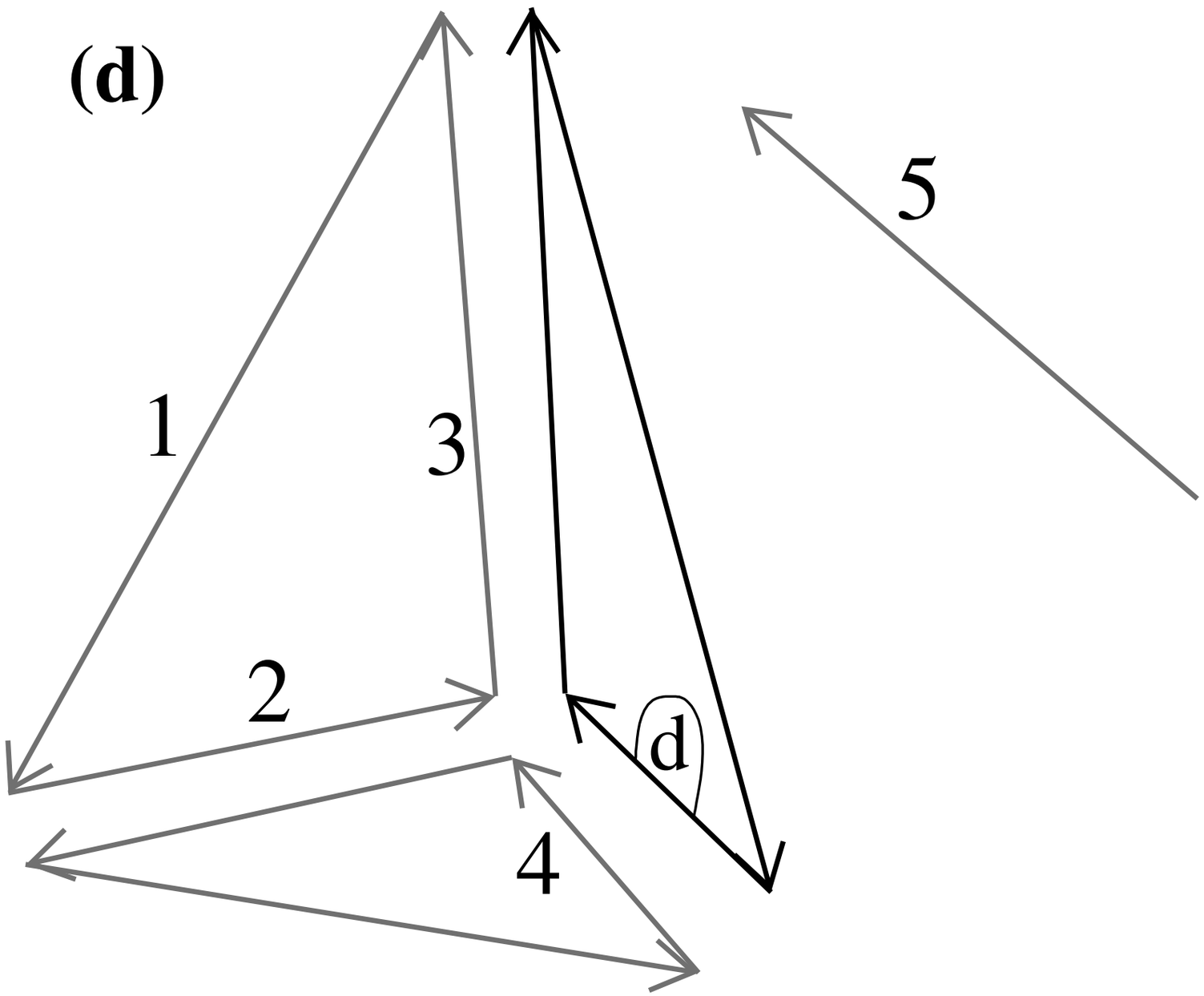
Building two tetrahedra



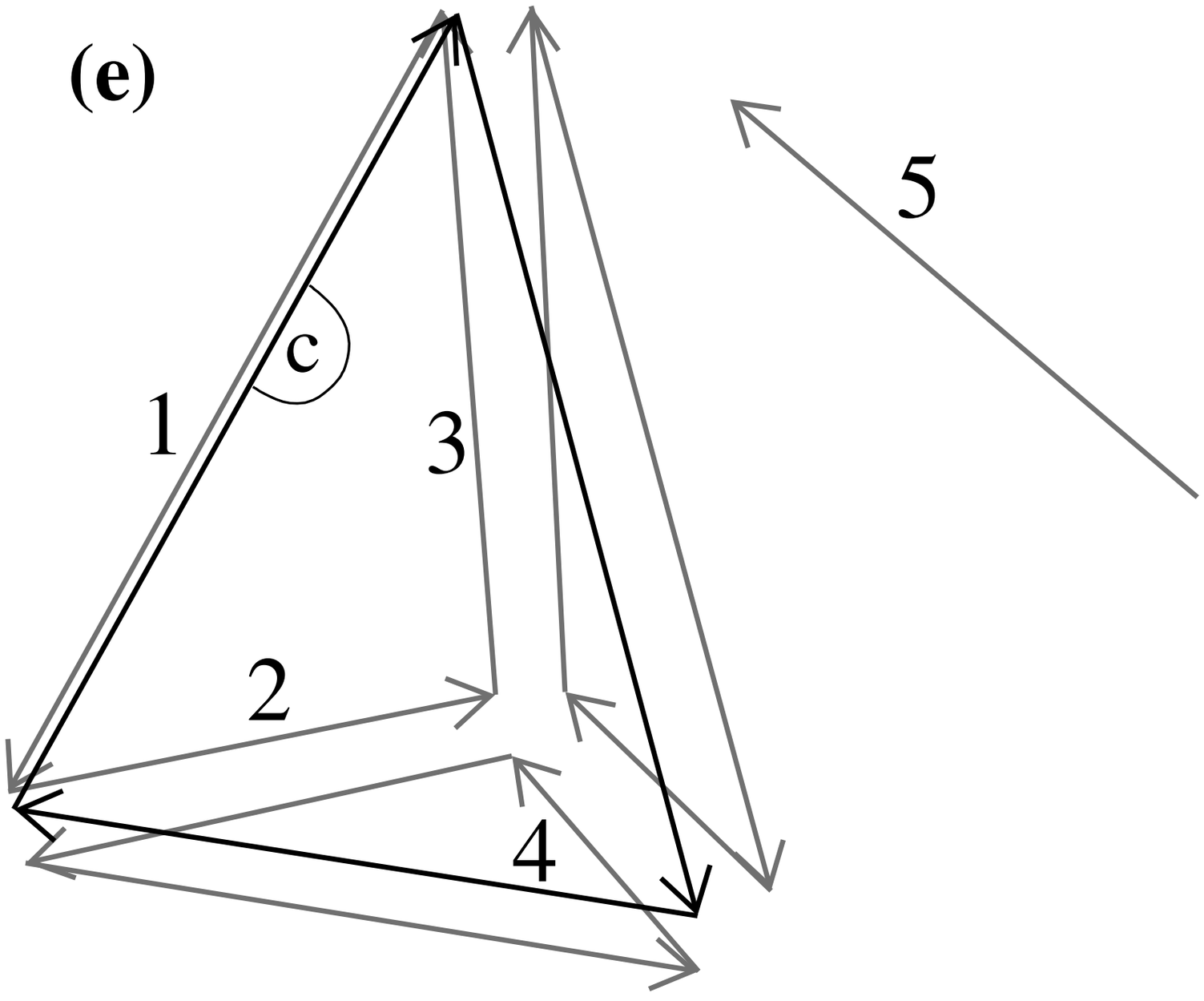
Building two tetrahedra



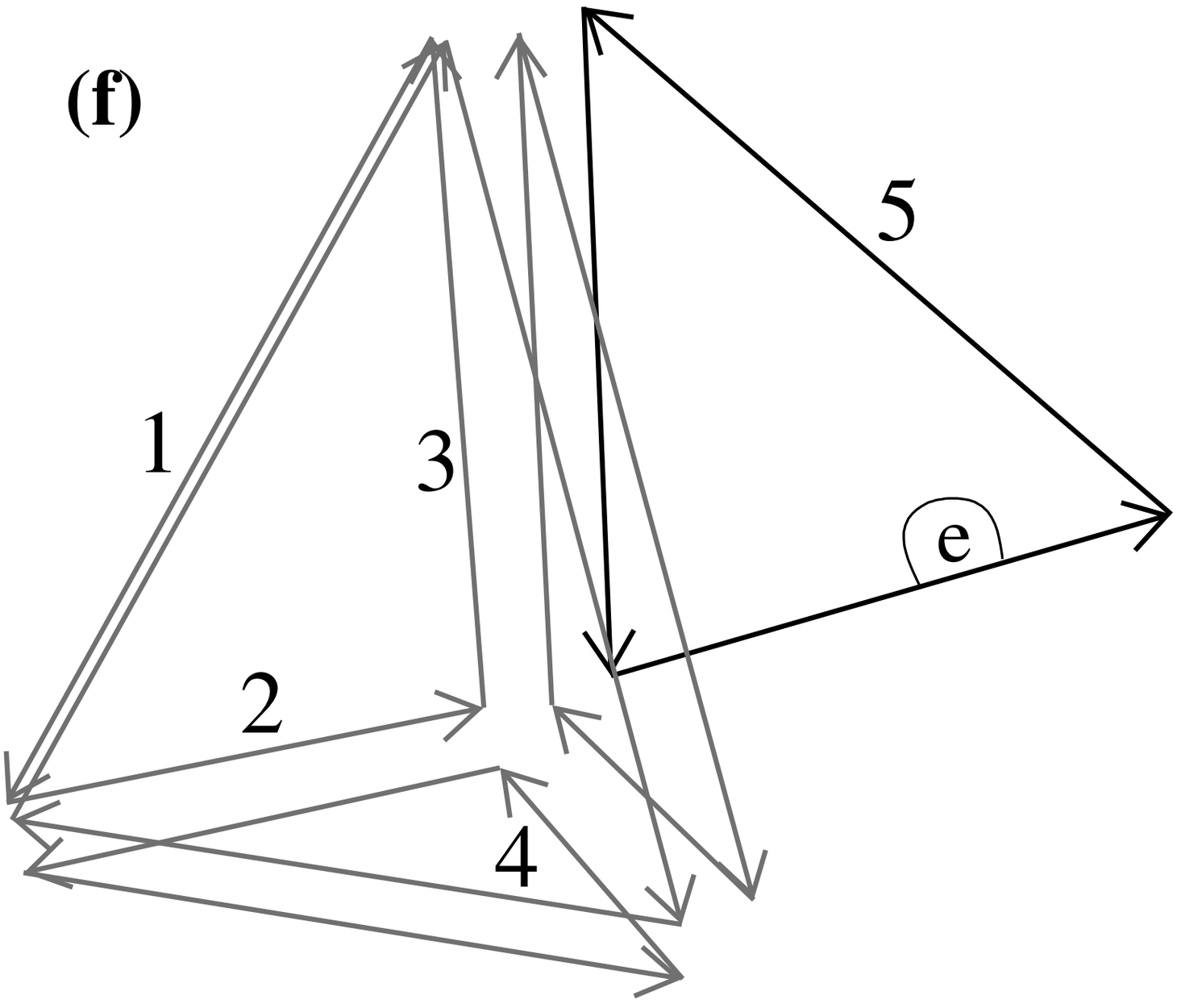
Building two tetrahedra



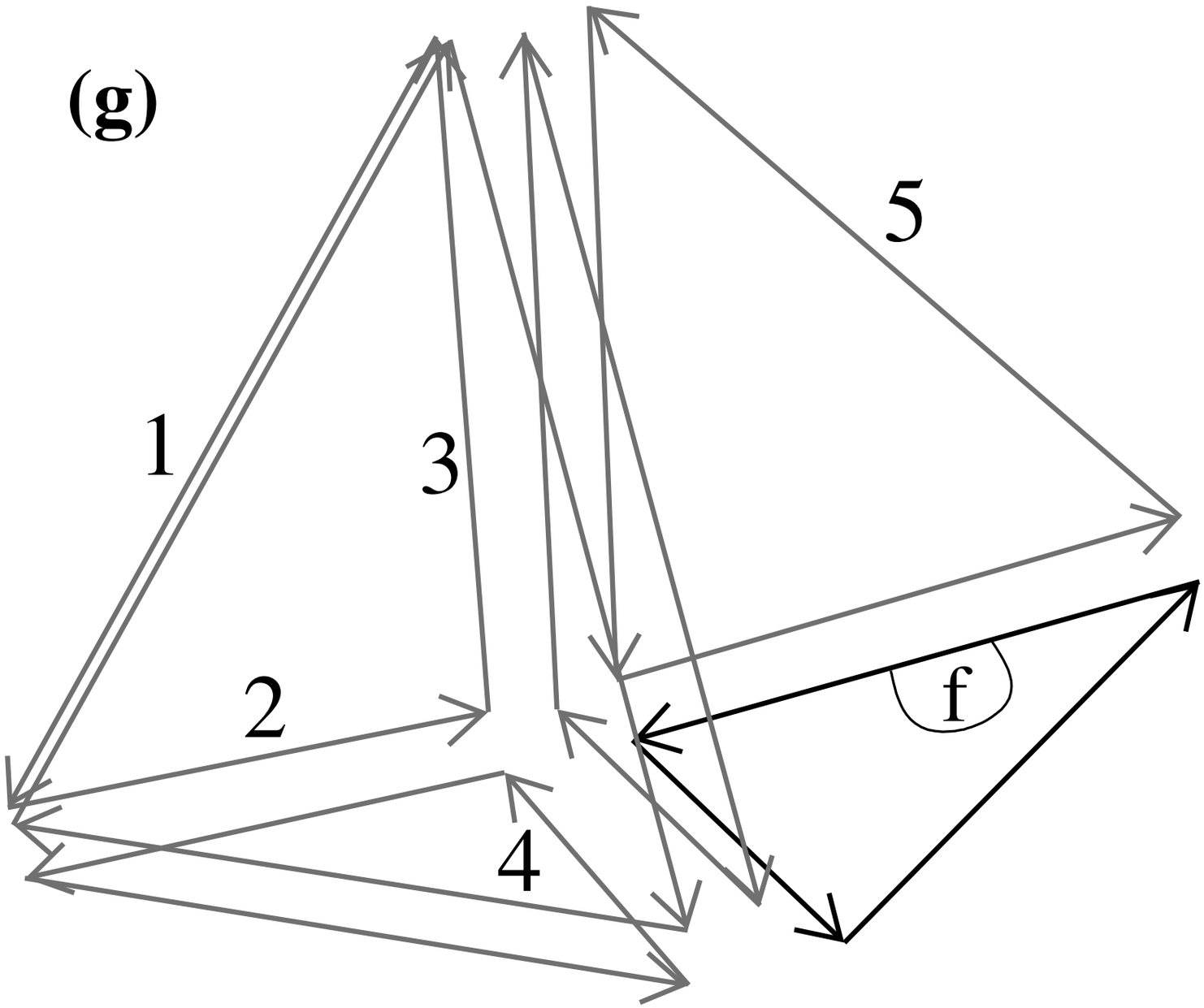
Building two tetrahedra



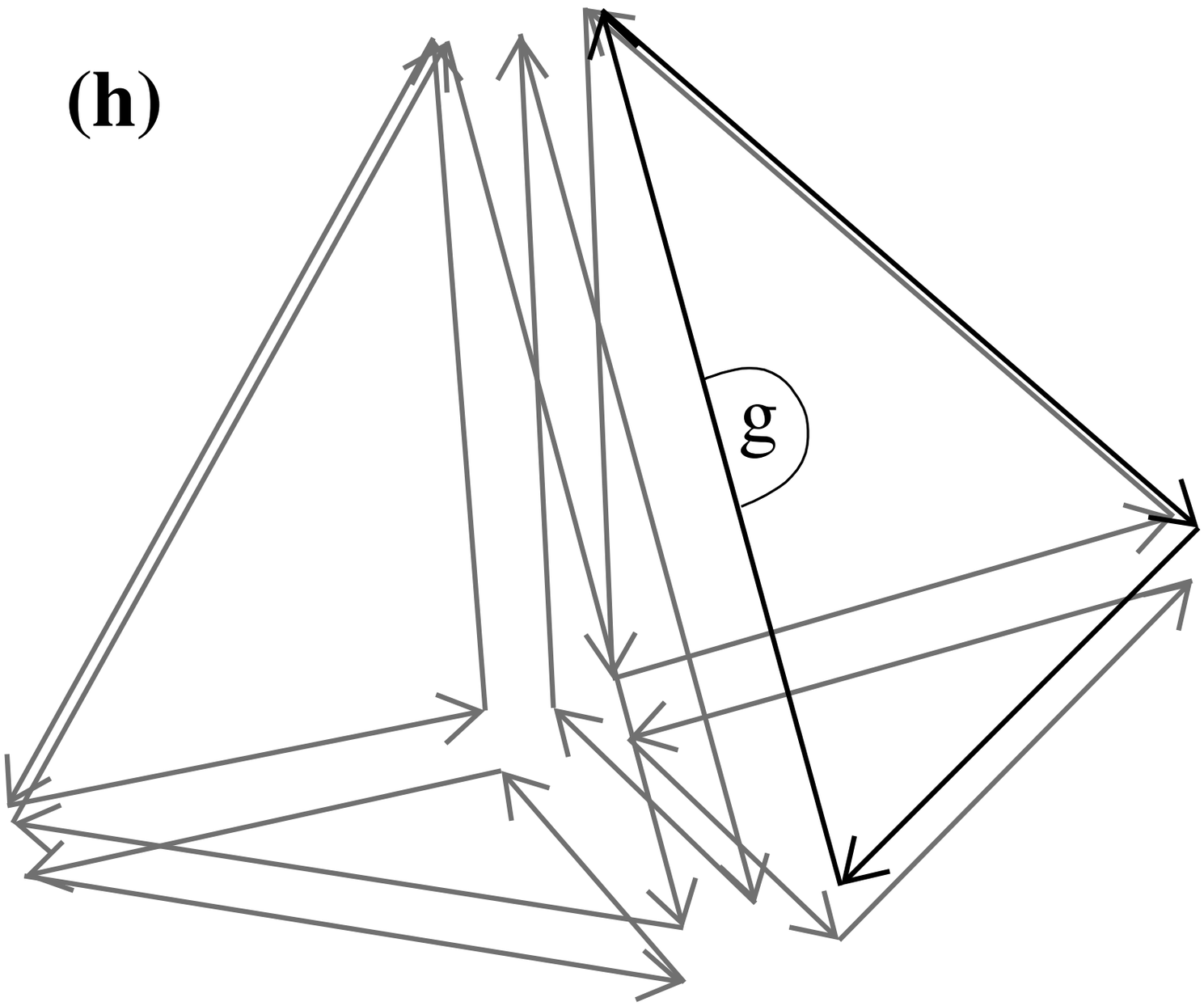
Building two tetrahedra



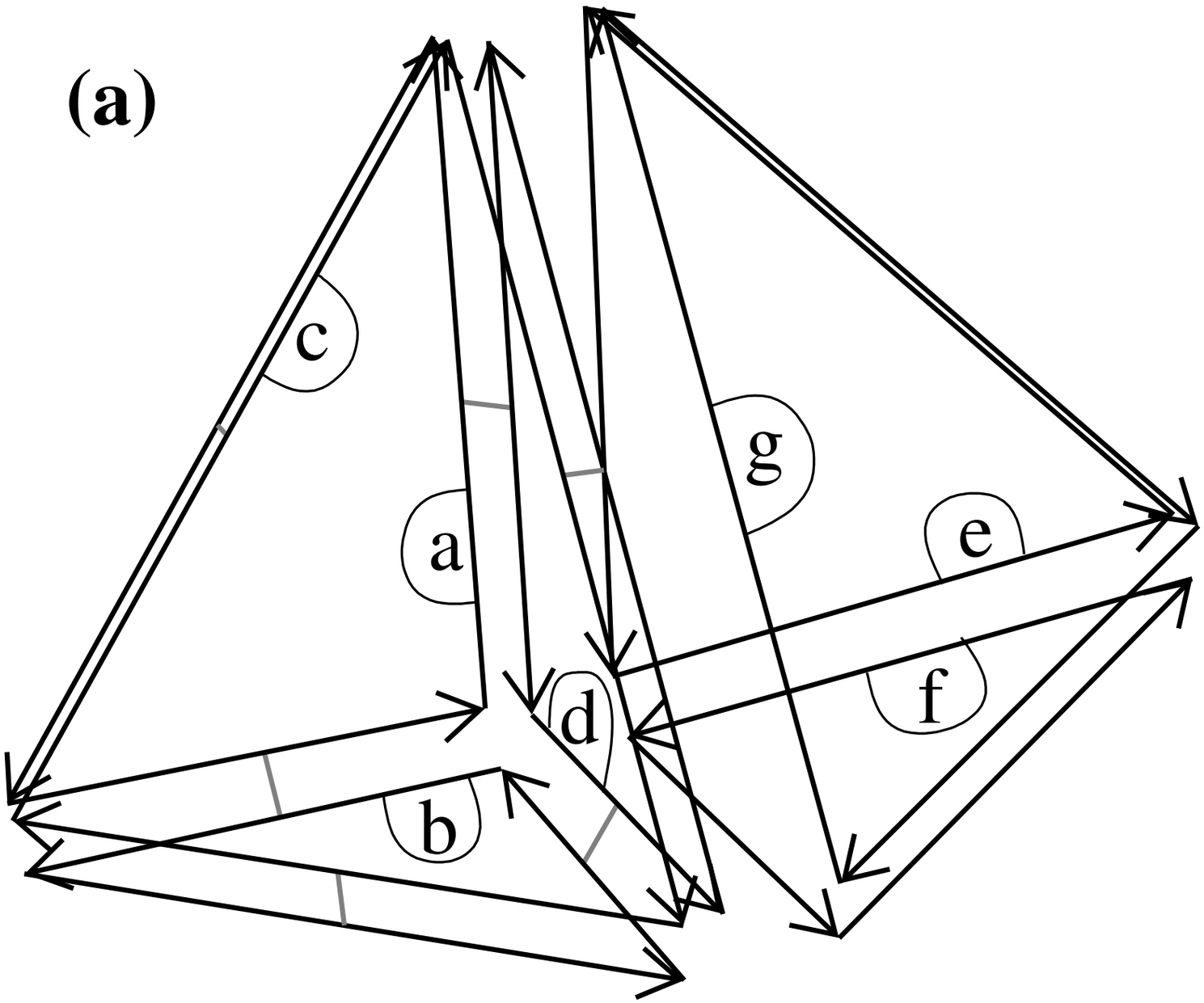
Building two tetrahedra



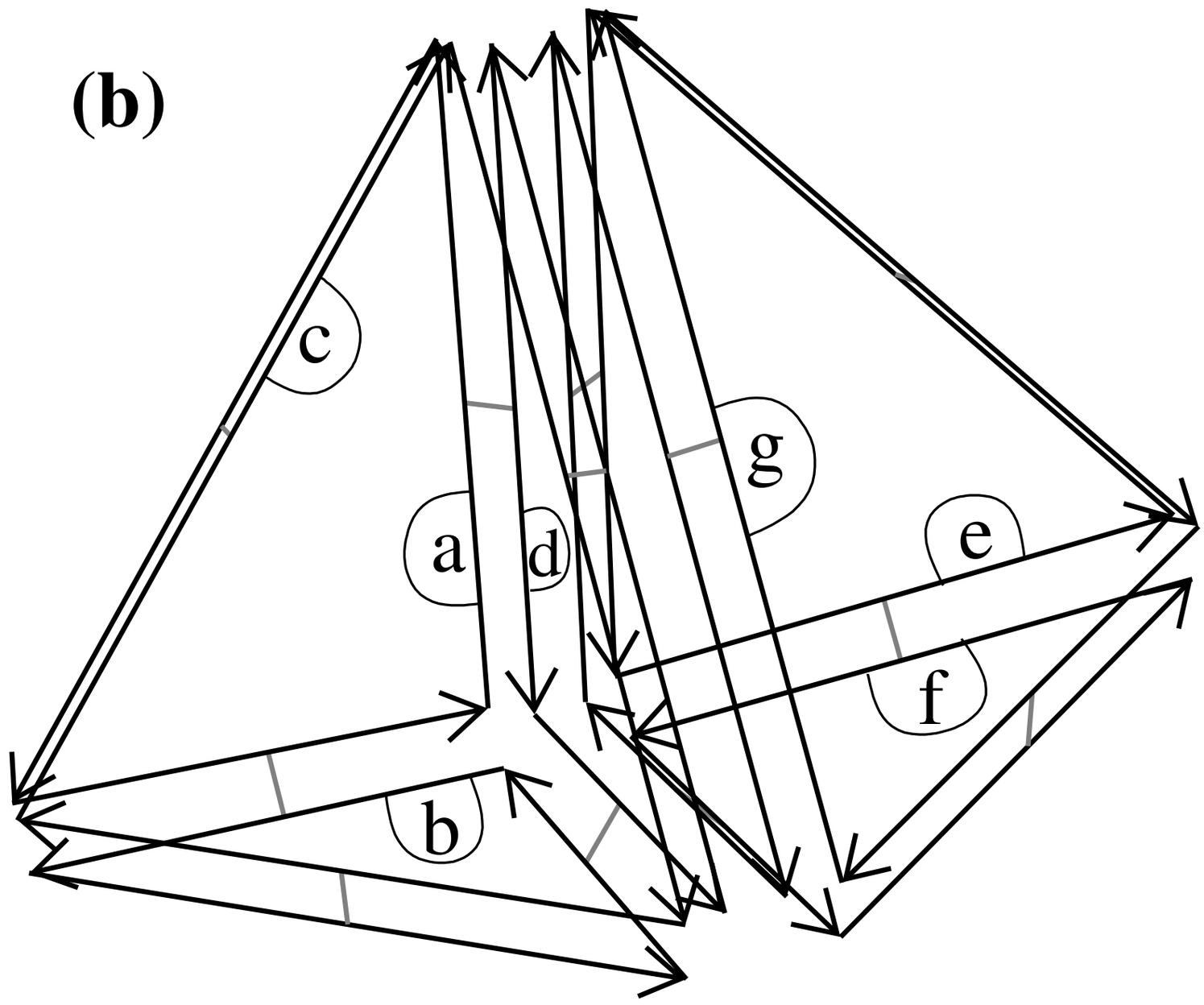
Building two tetrahedra



Building two tetrahedra

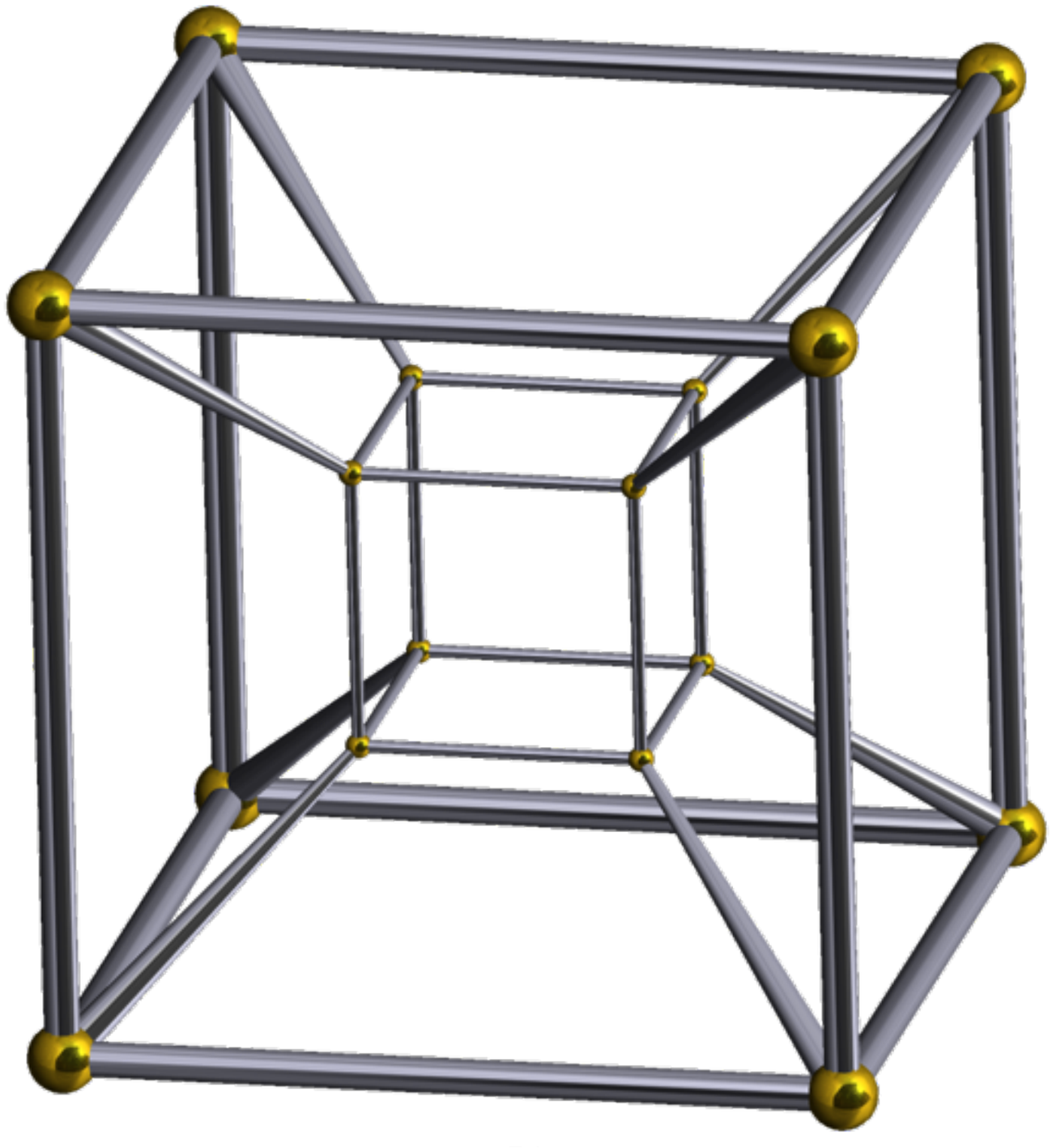


Building two tetrahedra

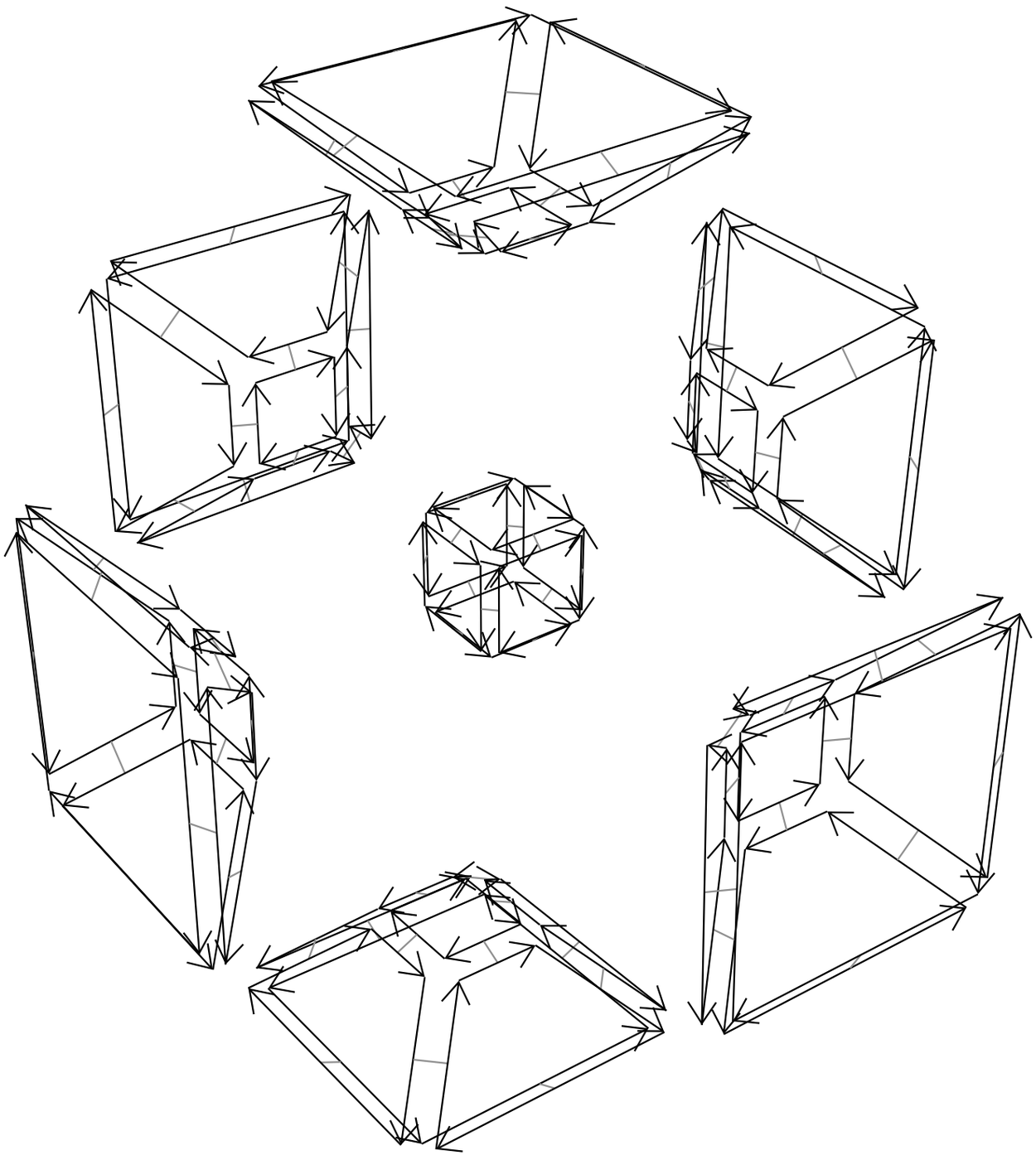


done!

Build a tesseract

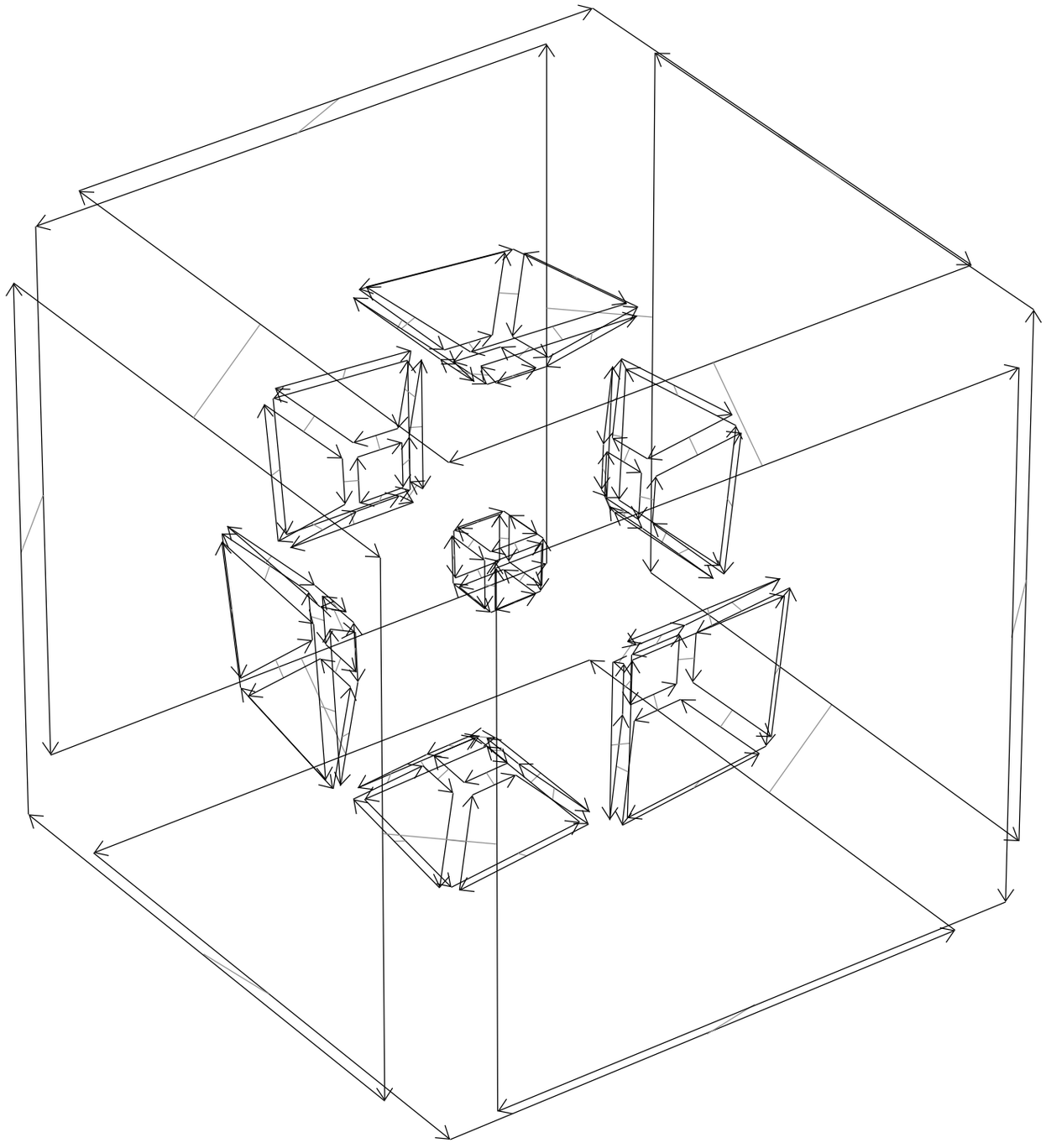


Build a tesseract



build each cube separately

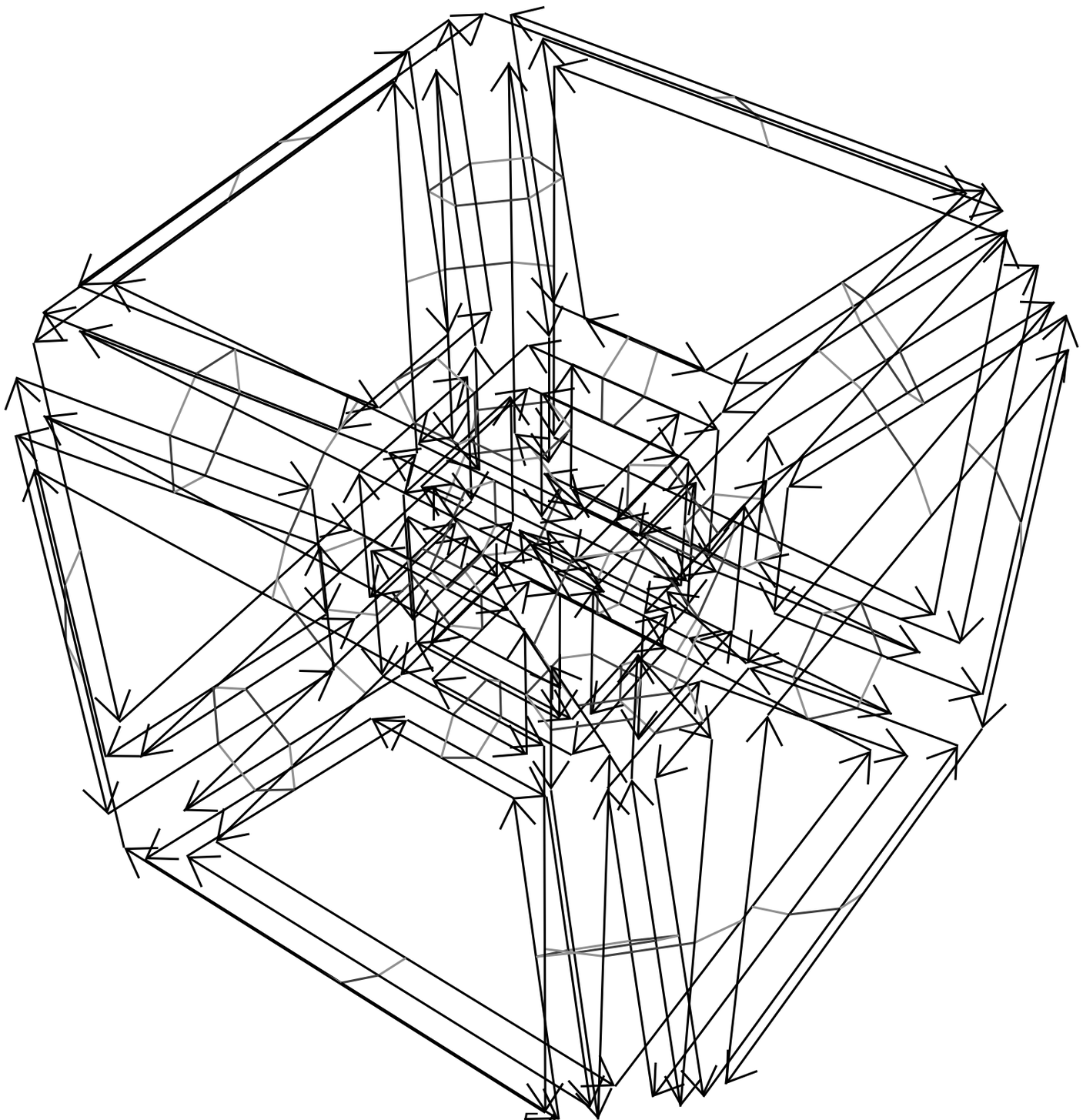
Build a tesseract



build each cube separately,

then join them

Build a tesseract



done!

Methodology

- Analyse the problem
- Split it into small, manageable subproblems
- Try sketches/ideas on paper
- When mature enough, build a program to test these
- Start with simple shapes, move towards more complex ones

Thank you!

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Images from:

- http://commons.wikimedia.org/wiki/File:Stereographic_polytope_24cell_faces.png

- http://blogs.lt.vt.edu/foundationdesignlab/category/materials/

- http://commons.wikimedia.org/wiki/File:Schlegel_wireframe_8-cell.png

More info

Ken Arroyo Ohori, Guillaume Damiand and Hugo Ledoux. **Constructing an n-dimensional cell complex from a soup of (n-1)-dimensional faces**. In Prosenjit Gupta and Christos Zaroliagis (eds.), *Applied Algorithms*, Volume 8321 of Lecture Notes in Computer Science, Springer International Publishing Switzerland, January 2014, pp. 37–48.