How to build an $n$-dimensional object?

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The 24-cell
a “simple” 4D object
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a “simple” 4D object

24 0D vertices
96 1D edges
96 2D faces
24 3D volumes
1 4D hypervolume
objects in more than 3D are complex!
Some background
0D: a vertex
0D: a vertex

\[ (x_0, x_1, \ldots) \]
1D: an edge
1D: an edge
1D: an edge

\[(x_0, x_1, \ldots)\]

\[(x_0, x_1, \ldots)\]
1D: an edge

A 1D object can be described by its 0D boundaries.

$$(x_0, x_1, \ldots)$$

$$(x_0, x_1, \ldots)$$
2D: a face
2D: a face
2D: a face

A 2D object can be described by its 1D boundary.
2D: a face
2D: a face

which can be described by its 0D boundaries
2D: a face

\((x_0, x_1, \ldots)\)
3D: a volume
3D: a volume

A 3D object can be described by its 2D boundary.
3D: a volume
3D: a volume

which can be described by its 1D boundaries
3D: a volume
3D: a volume which can be described by their 0D boundaries
4D, 5D, ... = 8 cubes
however, there is a problem in practice...
2D: a face
2D: a face
2D: a face

A 2D object is described by a set of 1D objects.
2D: a face

a “soup” of line segments
2D: a face
2D: a face

$(x_0, x_1, \ldots)$

same coordinates
2D: a face
2D: a face

build an object!
2D: a face
3D: a volume
3D: a volume

a “soup” of faces
3D: a volume
3D: a volume

build a volume from a set of faces
The solution: incremental construction

- Start from a set of 0D vertices
- Connect them to form 1D edges
- Connect these to form 2D faces
- Connect these to form 3D volumes
- …
Building two tetrahedra
Building two tetrahedra

(a)
Building two tetrahedra
Building two tetrahedra
Building two tetrahedra

(d)

1 3

2 4

5

(d)
Building two tetrahedra
Building two tetrahedra
Building two tetrahedra
Building two tetrahedra
Building two tetrahedra
Building two tetrahedra

(b)

done!
Build a tesseract
Build a tesseract

build each cube separately
Build a tesseract

build each cube separately, then join them
Build a tesseract

done!
Methodology

- Analyse the problem
- Split it into small, manageable subproblems
- Try sketches/ideas on paper
- When mature enough, build a program to test these
- Start with simple shapes, move towards more complex ones
Thank you!

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Images from:

- http://commons.wikimedia.org/wiki/File:Stereographic_polytope_24cell_faces.png
- http://blogs.lt.vt.edu/foundationdesignlab/category/materials/
- http://commons.wikimedia.org/wiki/File:Schlegel_wireframe_8-cell.png