Using extrusion to generate higher-dimensional GIS datasets

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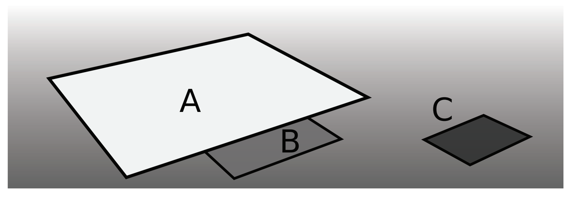
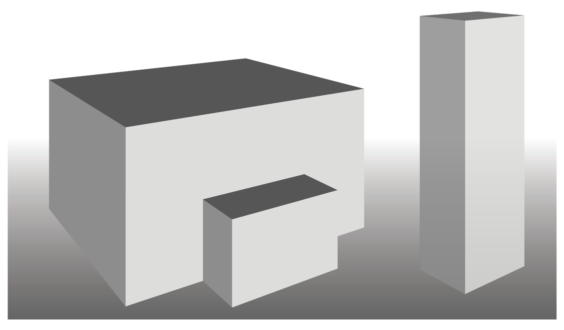
1. Introduction





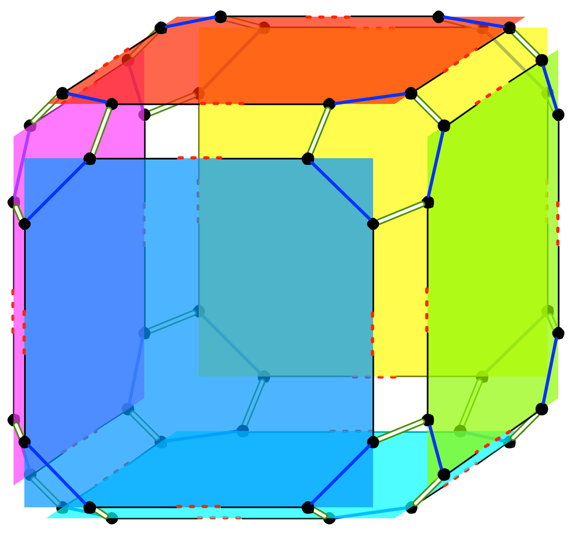
There is a growing interest in the use of higher-dimensional (>4D) digital objects, but their use is hampered by a lack of methods and algorithms.

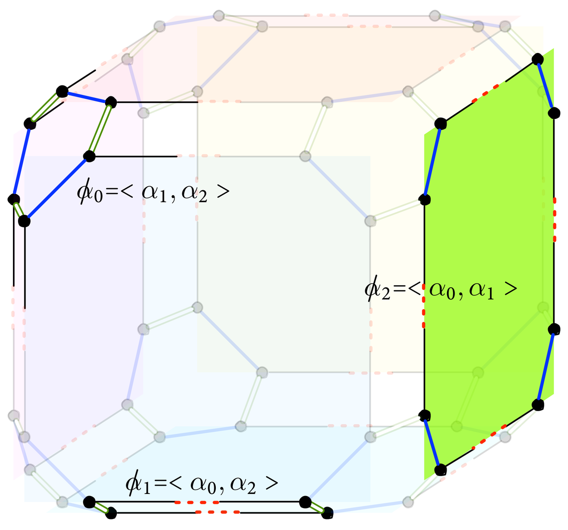
Akin to 2D to 3D extrusion (left), *n*D extrusion allows us to create an (*n*+1)-dimensional model from an n-dimensional one by assigning it a range along the (*n*+1)-th dimension.

We present here a dimension-independent extrusion algorithm for linear geometries using generalised maps. It is optimal and straightforward to implement.

We have implemented it in Python and made experiments in which 2D real-world GIS datasets were extruded to 3D and 4D.

2. Generalised maps





Generalised maps (G-maps) are a topological model in arbitrary dimensions developed by Lienhardt (1994). It is composed of darts and involutions (left). A dart is a combination of a cell of every dimension. Involutions connect darts related along a certain dimension.

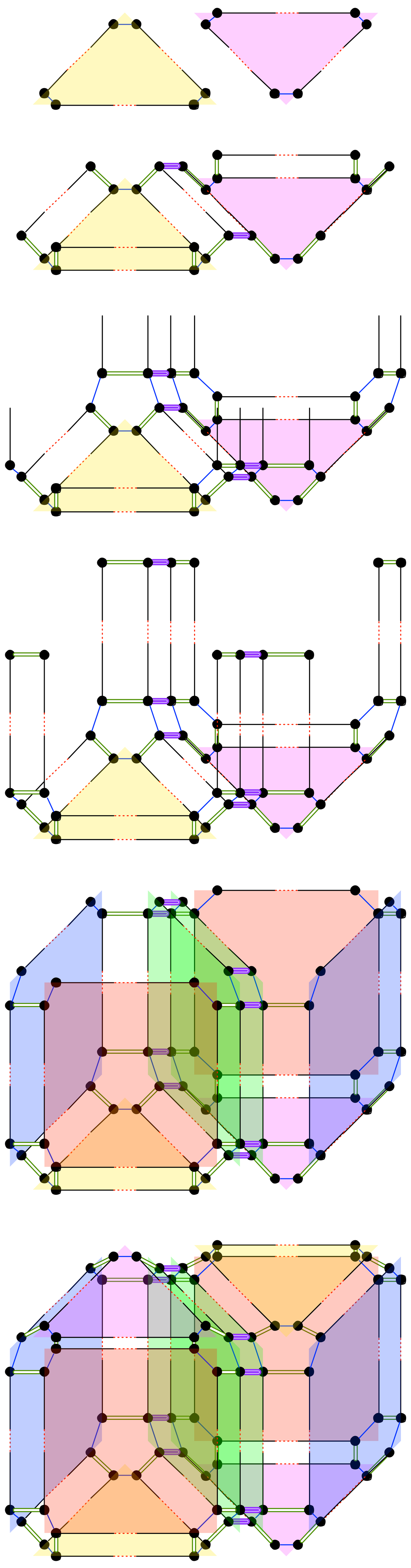
A simple implementation (right) is based on structures per dart and per geometric embedding.

```
struct Dart {
    Dart *involutions[n+1];
    Embeddings *embeddings[n+1];
};

struct Embedding {
    Dart *referenceDart;
    Embedding *holes[];
    int dimension;
    ...
    float red, green, blue;
};

struct PointEmbedding : Embedding {
    float x, y, z;
};
```

3. Extrusion algorithm



Our extrusion algorithm takes two input arguments: an (*n*-1)-G-map, and a given range [$r_{\min}$, $r_{\max}$] where these will exist along an *n*-th dimension. Its result is an *n*-G-map representing a set of prismatic *n*-polytopes.

Algorithm 1: EMBEDDINGSEXTRUSION

Input : E, $[r_{\min}, r_{\max}]$
Output: E', $base$, top, ex
foreach $e \in E$ **do**
  $base(e), top(e), ex(e) \leftarrow e$
  if $e.dimension = 0$ **then**
    Append $r_{\min}$ to $base(e)$'s coordinates
    Append $r_{\max}$ to $top(e)$'s coordinates
  $ex(e).dimension \leftarrow ex(e).dimension + 1$
  Put $base(e), top(e)$ and $ex(e)$ in E'

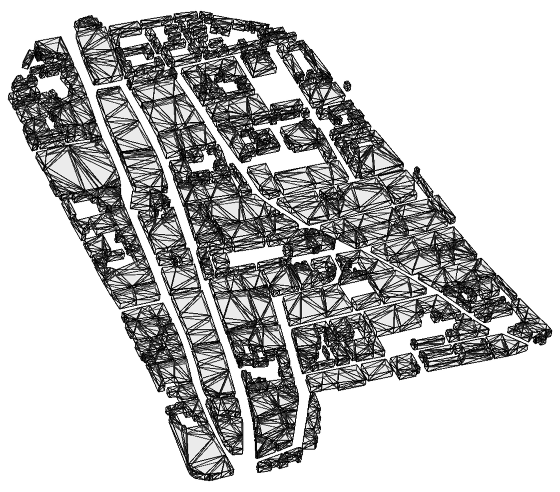
Algorithm 2: GMAPEXTRUSION

Input : $G = (D, \alpha_0, \alpha_1, \dots, \alpha_{n-1})$, E, E', $base$, top, ex, e, e'
Output: $G' = (D', \alpha'_0, \alpha'_1, \dots, \alpha'_n)$
for $dim \leftarrow n$ **to** 0 **do**
  GMAPLAYER$(G, G', dim, 1, last, E, E', e, e', base, ex)$
  $last \leftarrow cur$
for $dim \leftarrow 0$ **to** n **do**
  GMAPLAYER$(G, G', dim, 0, last, E, E', e, e', top, ex)$
  $last \leftarrow cur$

Algorithm 3: GMAPLAYER

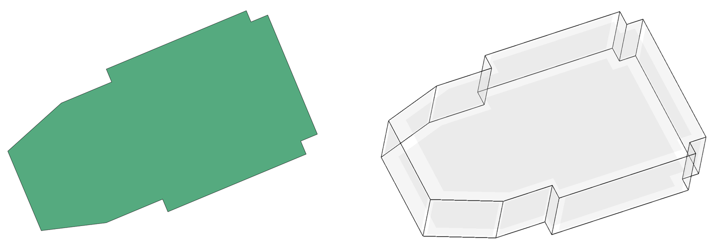
Input : $G = (D, \alpha_0, \alpha_1, \dots, \alpha_{n-1})$, $G' = (D', \alpha'_0, \alpha'_1, \dots, \alpha'_n)$, dim, $offset$, $last$, E, E', e, e', el, ex
Output: $G' = (D', \alpha'_0, \alpha'_1, \dots, \alpha'_n)$, $last$, cur, e'
foreach $d \in D$ **do**
  $cur(d) \leftarrow$ new dart
  Put $cur(d)$ in D'
foreach $d \in D$ **do**
  for $inv \leftarrow 0$ **to** $dim - 1$ **do**
    $\alpha'_{inv}(cur(d)) \leftarrow cur(\alpha_{inv}(d))$
  $\alpha'_{dim+offset}(cur(d)) \leftarrow last(d)$
  $\alpha'_{dim+offset}(last(d)) \leftarrow cur(d)$
  for $inv \leftarrow dim + 2$ **to** n **do**
    $\alpha'_{inv}(cur(d)) \leftarrow cur(\alpha_{inv-1}(d))$
  for $emb \leftarrow 0$ **to** dim **do**
    $e'_{emb}(cur(d)) \leftarrow el(e_{emb}(d))$
  for $emb \leftarrow dim + 1$ **to** n **do**
    $e'_{emb}(cur(d)) \leftarrow ex(e_{emb-1}(d))$

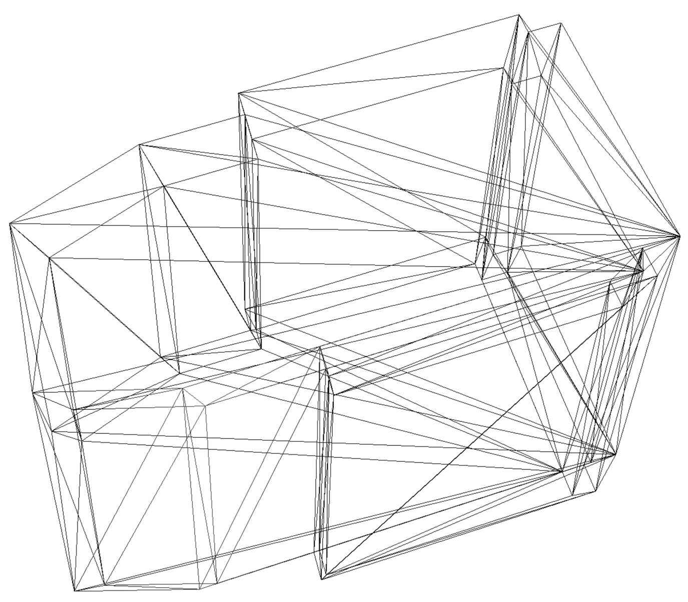
4. Experiments



We have implemented the algorithm in Python and tested it with a few datasets in the area of Delft.

For the first test (left), an area of the TOP10NL dataset was extruded along the range [0,15] m. We verified the results using the procedure described in Ledoux and Meijers (2011), based on a constrained tetrahedralisation of the dataset.





For the second test (right), we used a building footprint from the GBKN dataset. It was extruded along the range [0,25] m, and extruded again along the range [1960,2060] yr for a 4D (3D+time) model.

5. Discussion

We have shown that it is possible to use our algorithm to extrude (*n*-1)-dimensional cell complexes represented as G-maps into *n*-dimensional ones. Our algorithm is optimal in time, easy to implement and addresses both the geometry and the topology of the objects.

References

Pascal Lienhardt. N-dimensional generalized combinatorial maps and cellular quasi-manifolds. *International Journal of Computational Geometry and Applications*, 4(3):275-324, 1994.

Hugo Ledoux and Martijn Meijers. Topologically consistent 3D city models obtained by extrusion. *International Journal of Geographical Information Science*, 25(4):557-574, 2011.

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