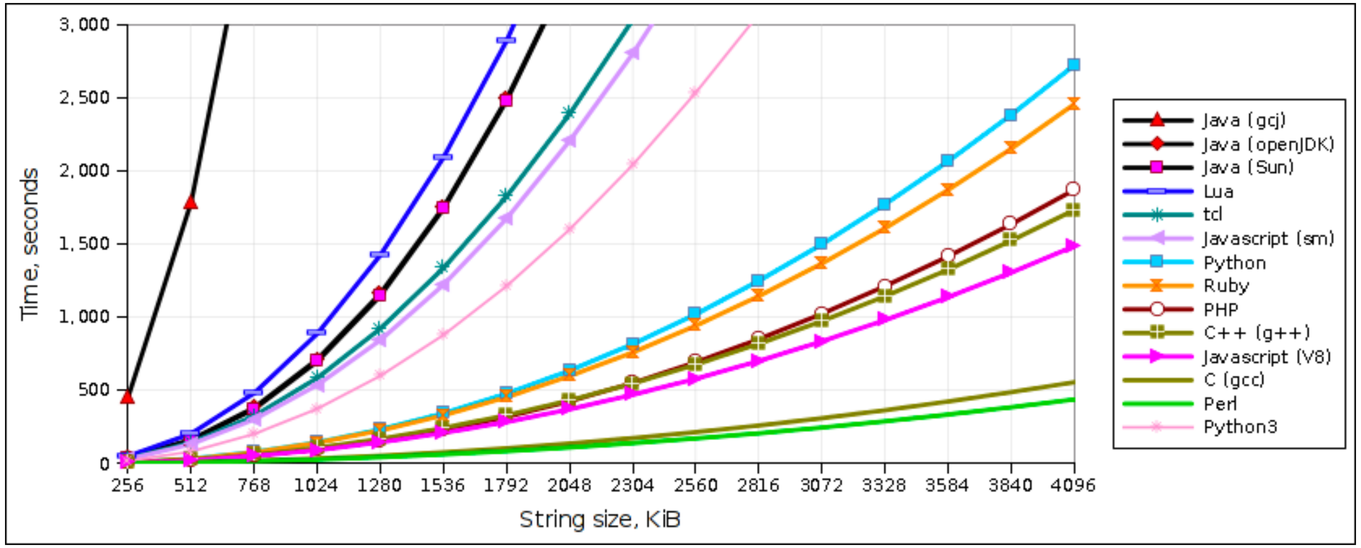
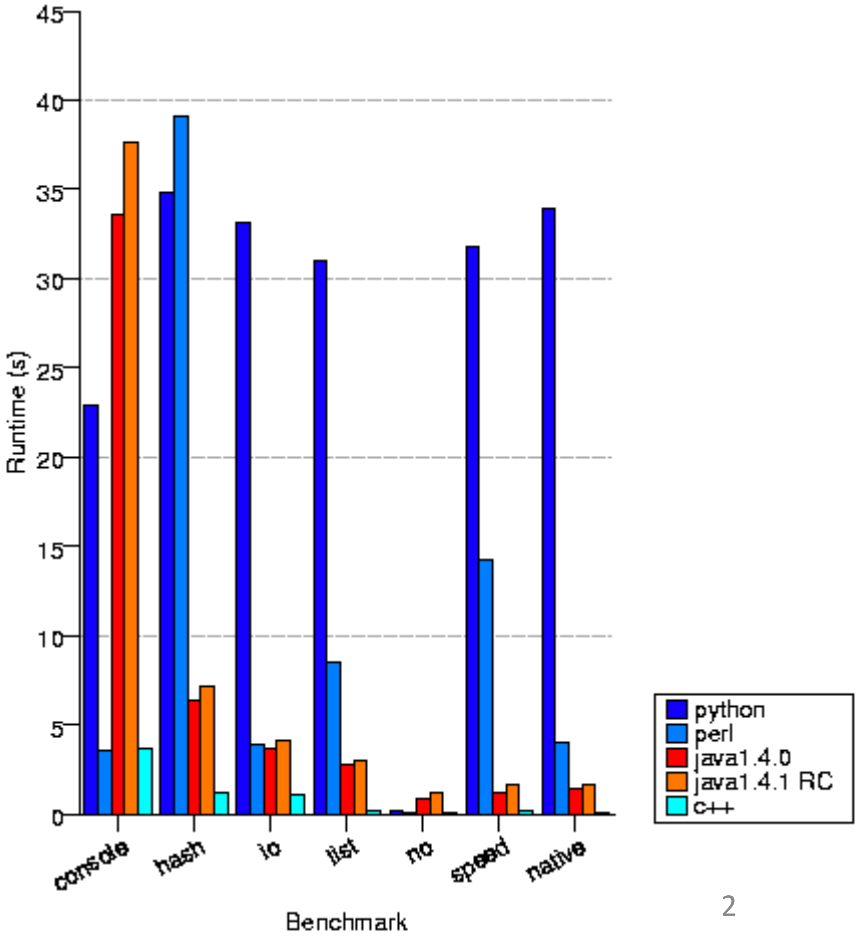
Feedback and Discussion

- Why C++?
- What is “K * [R, **t**]”?
- A1: Calibration
  - What is the main source of the error in camera calibration?
  - Why cannot I crop the image (but snapshots of the full viewer)?
  - How to determine the sign of rho?
- A2: Triangulation
  - Determine the correct R-t pair
  - The effect of errors in K on reconstruction
  - What will be the ideal evaluation method?
  - Interpretation of “reprojection error”
  - The accuracy of reconstruction from two similar views

Why C++?

- Performance is critical
  - Large images, point clouds, matrices
  - SfM, MVS, numerical algorithms …
- Deployment
- “bilingual” ☺





What is “K * [R, **t**]”?

- It is not K * R * **t**
- [R, **t**] denotes the concatenation of R and **t**
  - Appending **t** to R so **t** becomes its last column
- Notation: T or **t?**
  - T is a matrix
  - T and **t** represent the same translation transformation
  - K * [R, **t**] = K * T * R
    - Be careful: it is not K * R * T

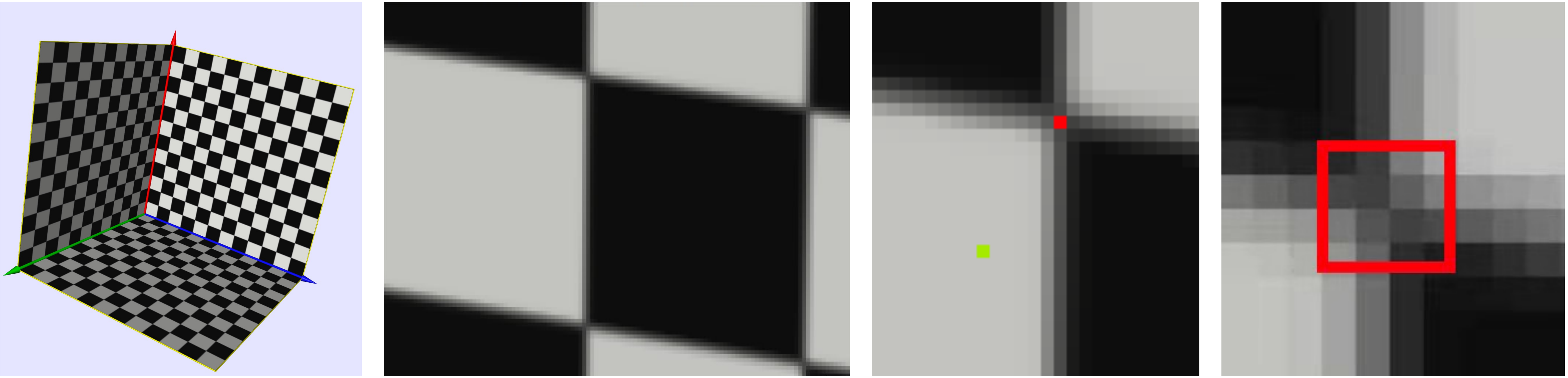
$$\mathbf{P'} = \mathbf{T} \cdot R \cdot \mathbf{P} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

A1: Calibration

- What is the main source the error in camera calibration?

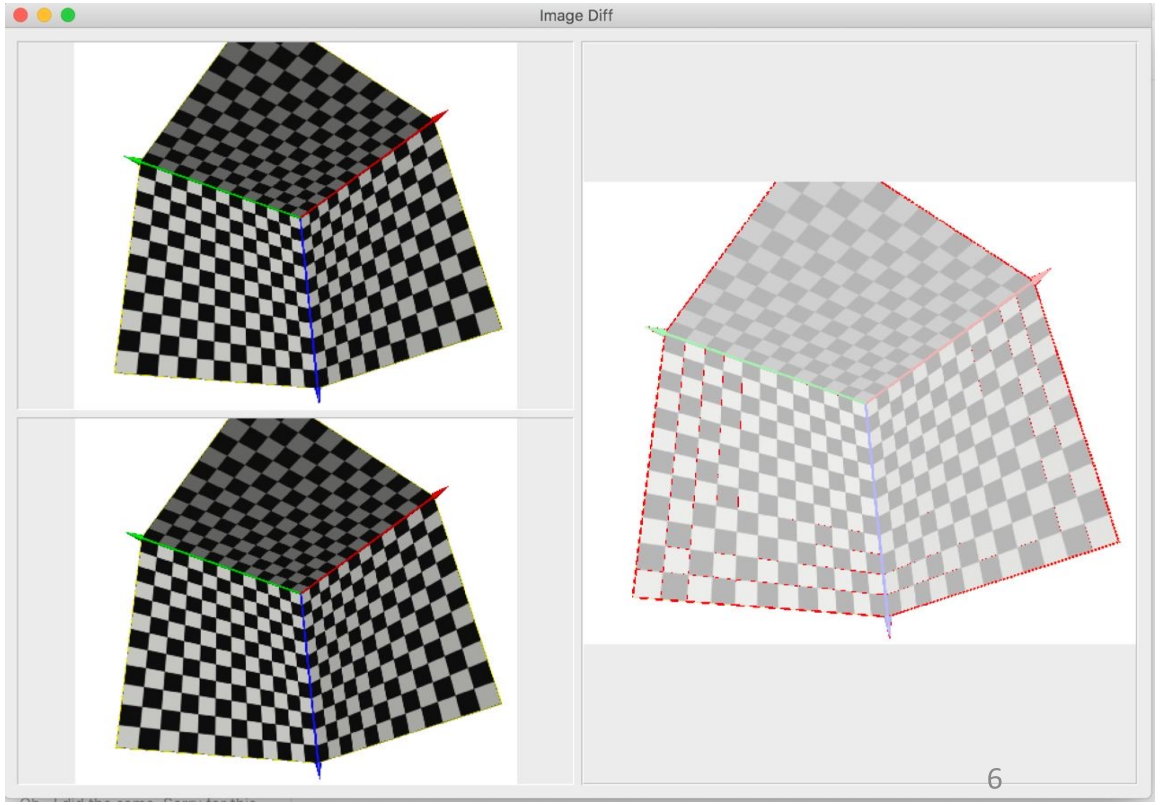
A1: Calibration

- What is the main source of the error in camera calibration?
  - Inaccuracy in pixel locations: manually picking pixels



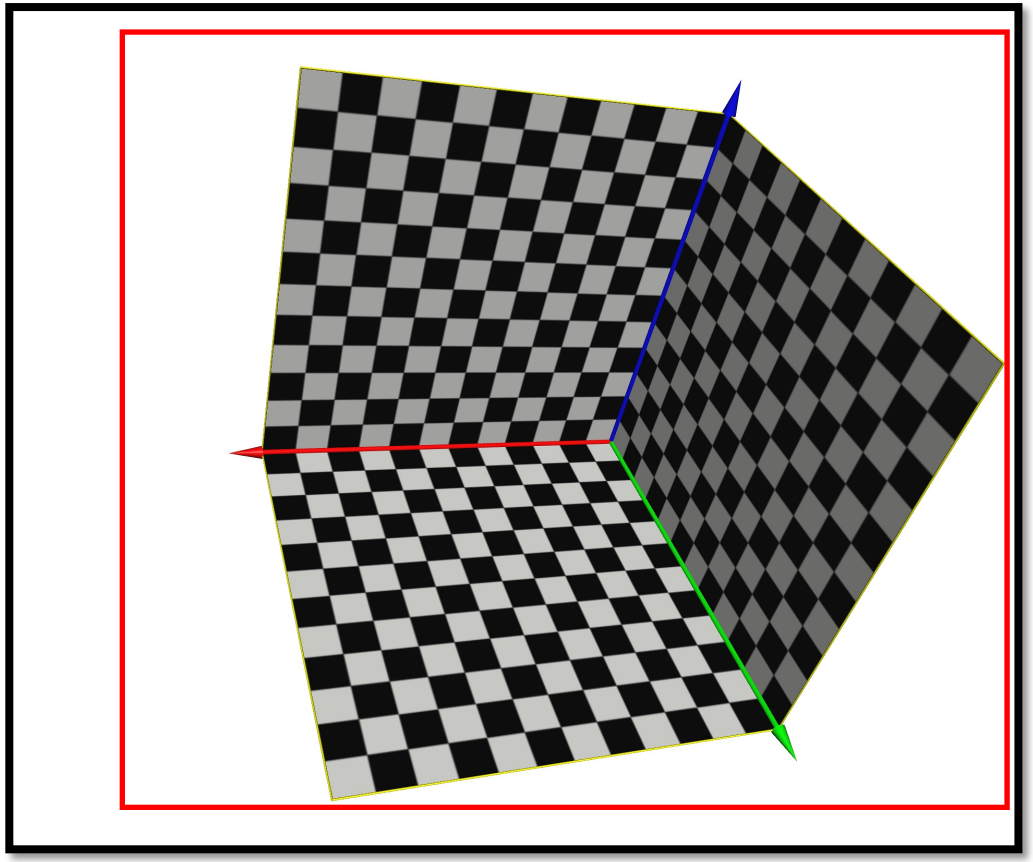
A1: Calibration

- What is the main source of the error in camera calibration?
  - Inaccuracy in pixel locations: manually picking pixels



A1: Calibration

- Why cannot I crop the image (but using the snapshot of the full viewer)?



$$P\mathbf{m} = 0 \;\Rightarrow\; \begin{array}{ll} \underset{\mathbf{m}}{\text{minimize}} & \|P\mathbf{m}\|^2 \\ \text{subject to} & \|\mathbf{m}\|^2 = 1 \end{array}$$

$$M = K\begin{bmatrix} R & \mathbf{t} \end{bmatrix}$$

$$K = \begin{bmatrix} f_x & s & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_0 \\ 0 & \frac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix}, \; \mathbf{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$M = \begin{bmatrix} \alpha\mathbf{r}_1^T - \alpha\cot\theta\mathbf{r}_2^T + u_0\mathbf{r}_3^T & \alpha t_x - \alpha\cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta}\mathbf{r}_2^T + v_0\mathbf{r}_3^T & \frac{\beta}{\sin\theta}t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{bmatrix}$$

SVD-solved projection matrix

SVD-solved projection matrix is known up to scale, i.e., $\rho\mathcal{M} = M$

The true values of project matrix

$$\mathcal{M} = \frac{1}{\rho}M = \frac{1}{\rho}\begin{bmatrix} \alpha\mathbf{r}_1^T - \alpha\cot\theta\mathbf{r}_2^T + u_0\mathbf{r}_3^T & \alpha t_x - \alpha\cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta}\mathbf{r}_2^T + v_0\mathbf{r}_3^T & \frac{\beta}{\sin\theta}t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{bmatrix}$$

A1: Calibration

- How to determine the sign of rho?
  - Size of rho: the scale factor between the SVD-solved projection matrix and the actual projection matrix
  - Sign of rho

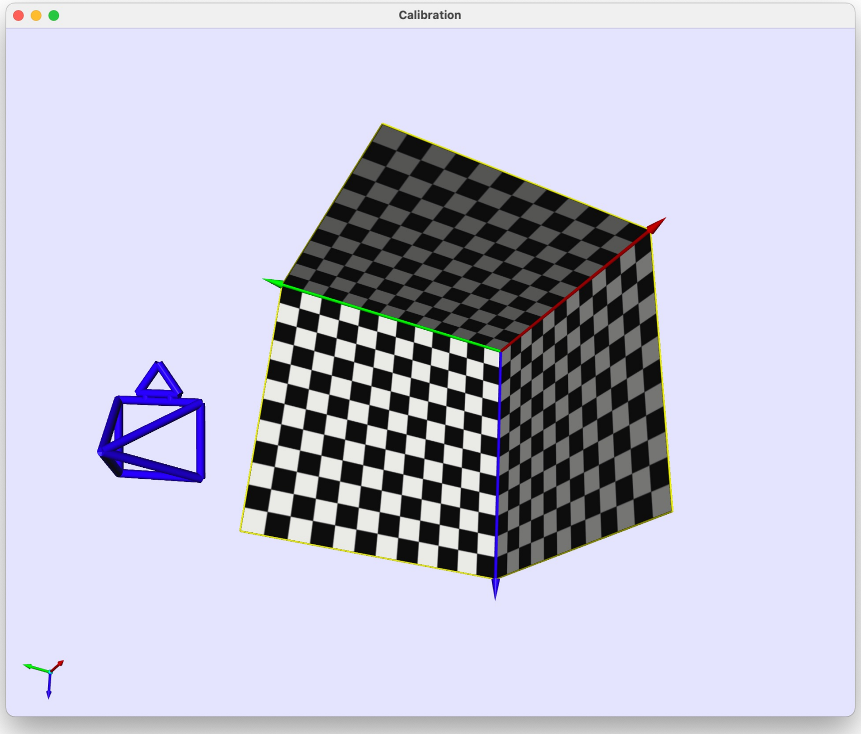
Extrinsic parameters:

$$\mathbf{r_1} = \frac{\mathbf{a_2} \times \mathbf{a_3}}{\|\mathbf{a_2} \times \mathbf{a_3}\|}$$

$$\mathbf{r_2} = \mathbf{r_3} \times \mathbf{r_1}$$

$$\mathbf{r_3} = \rho \mathbf{a_3}$$

$$\boxed{\mathbf{t} = \rho K^{-1} \mathbf{b}}$$



A1: Calibration

- How to determine the sign of rho?
  - Size of rho: the scale factor between the SVD-solved projection matrix and the actual projection matrix
  - Sign of rho
  - What about testing

    reprojection_error_with_positive_sign < reprojection_error_with_positive_sign

A2: Triangulation

- Determine the correct R-t pair
- The effect of errors in K on reconstruction
- What will be the ideal evaluation method?
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A2: Triangulation

- Determine the correct R-**t** pair
  - Determine if a 3D point lies in front of both cameras

M_0 and M_1 are the projection matrices of the two cameras

```
    int front_of_camera_0 = 0;
    int front_of_camera_1 = 0;
    for (const Vector3D& P : points3D) {
        // Point in the first camera coordinate system
        Vector3D P_cam0 = M_0 * P.homogeneous();
        if (P_cam0.z() > 0) {
            ++front_of_camera_0;
        }

        // Point in the second camera coordinate system
        Vector3D P_cam1 = M_1 * P.homogeneous();
        if (P_cam1.z() > 0) {
            ++front_of_camera_1;
        }
    }
```

A2: Triangulation

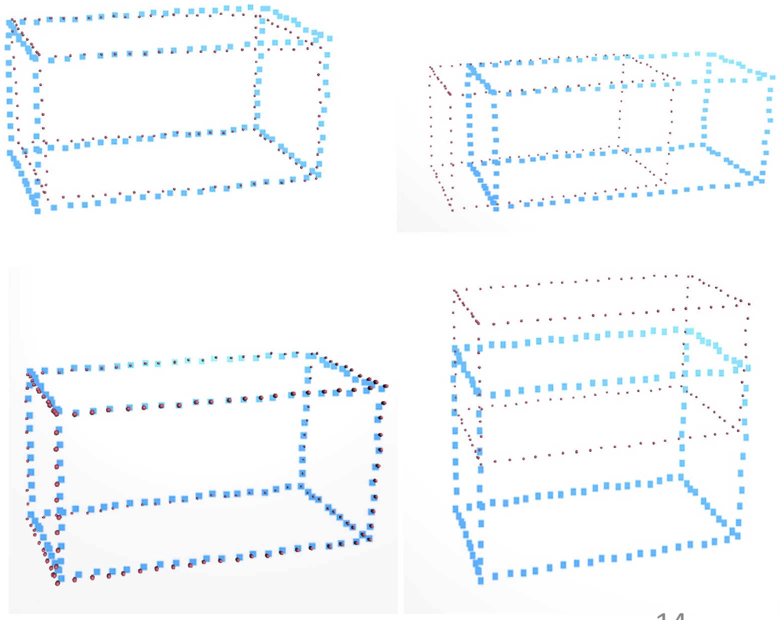
- Determine the correct R-**t** pair
  - The one having the largest number of 3D points in front of **BOTH** cameras

```
const vec3 p3d = triangulate(M, M_prime, p1, p2);
const vec4 p3d_h = vec4{ p3d.x, p3d.y, p3d.z, w_in: 1.0 };

// First camera check
if (p3d.z > 0) found++;
// Second camera check
if ((Rt * p3d_h).z > 0) found++;
```

```
const vec3 p3d = triangulate(M, M_prime, p1, p2);
const vec4 p3d_h = vec4{ p3d.x, p3d.y, p3d.z, w_in: 1.0 };

if (p3d.z > 0 && (Rt * p3d_h).z > 0)
    found++;
```

A2: Triangulation

- The effect of errors in K on reconstruction
  - How K can be obtained in practice?
  - Does K have to be very accurate?



Accurate Camera Calibration from Multi-View Stereo and Bundle Adjustment
Furukawa and Ponce. CVPR 2008

A2: Triangulation

- What will be the ideal evaluation method?
  - Reprojection error?

A2: Triangulation

- Interpretation of “reprojection error”
  - MSE (Mean Squared Error)

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

  - RMSE (Root Mean Squared Error)

$$RMSE = \sqrt{MSE}$$

  - Average distance
  - Reprojection error

$$\sum_i \|M\hat{\mathbf{P}}_i - \mathbf{p}_i\|^2$$

A2: Triangulation

- The accuracy of reconstruction from two views
  - Why?
  - How to improve?