Lesson dtvd3d: some extras

GEO1004:
3D modelling of the built environment

https://3d.bk.tudelft.nl/courses/geo1004

3D geoinformation
Department of Urbanism
Faculty of Architecture and the Built Environment
Delft University of Technology

2 Voronoi cells

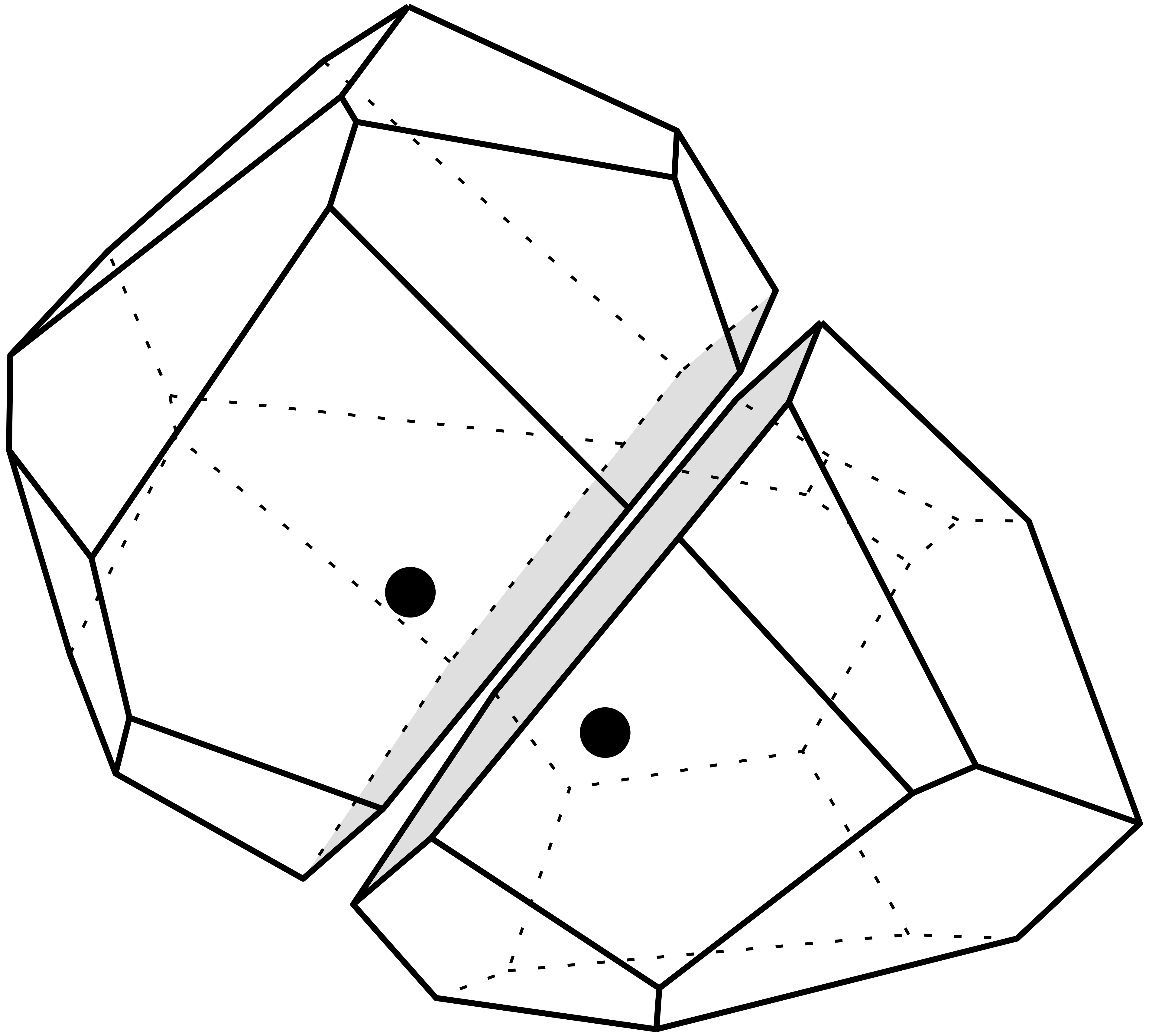
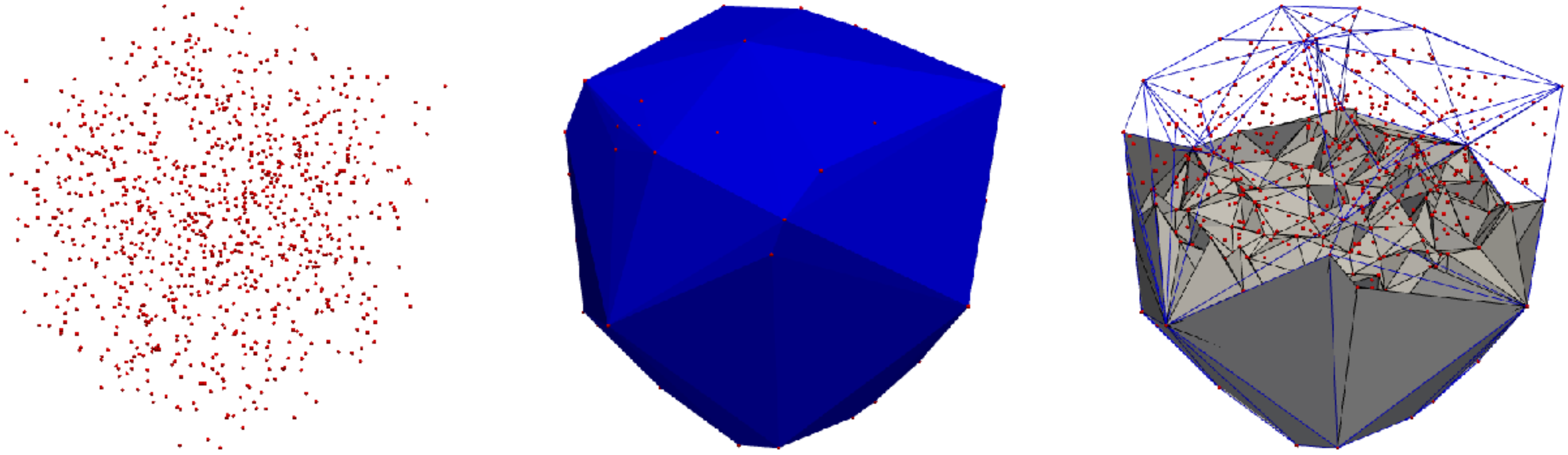


Figure 3.3: Two Voronoi cells adjacent to each other in $\mathbb{R}^3$, they share the grey face.

Delaunay tetrahedralisation



(a) (b) (c)

Figure 3.8: (a) A set of 1000 points randomly distributed in a cube. **(b)** Its convex hull. **(c)** The Delaunay tetrahedralisation of the points, 'sliced' in the middle and the upper tetrahedra removed (to be able to visualise the interior).

Incremental construction

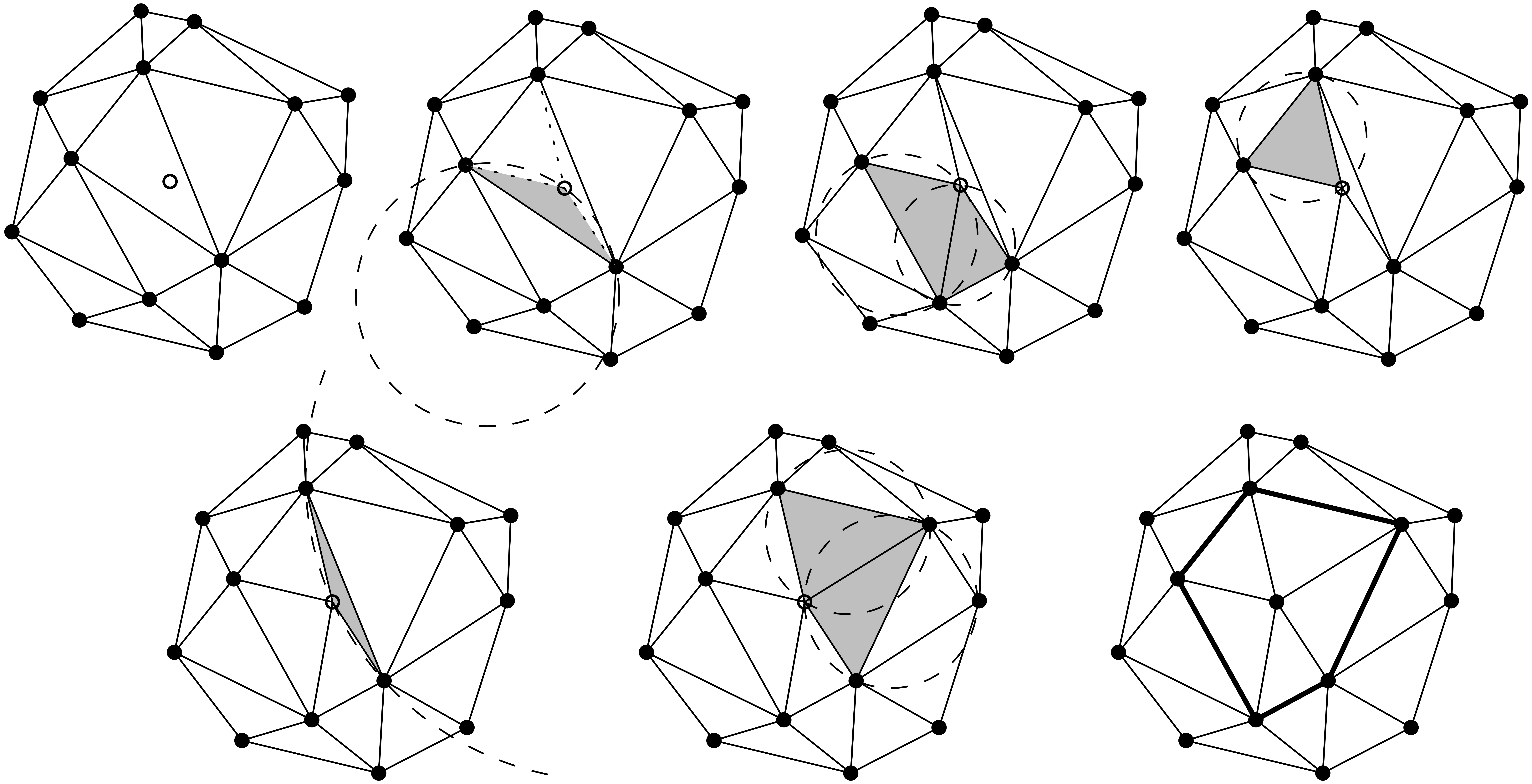


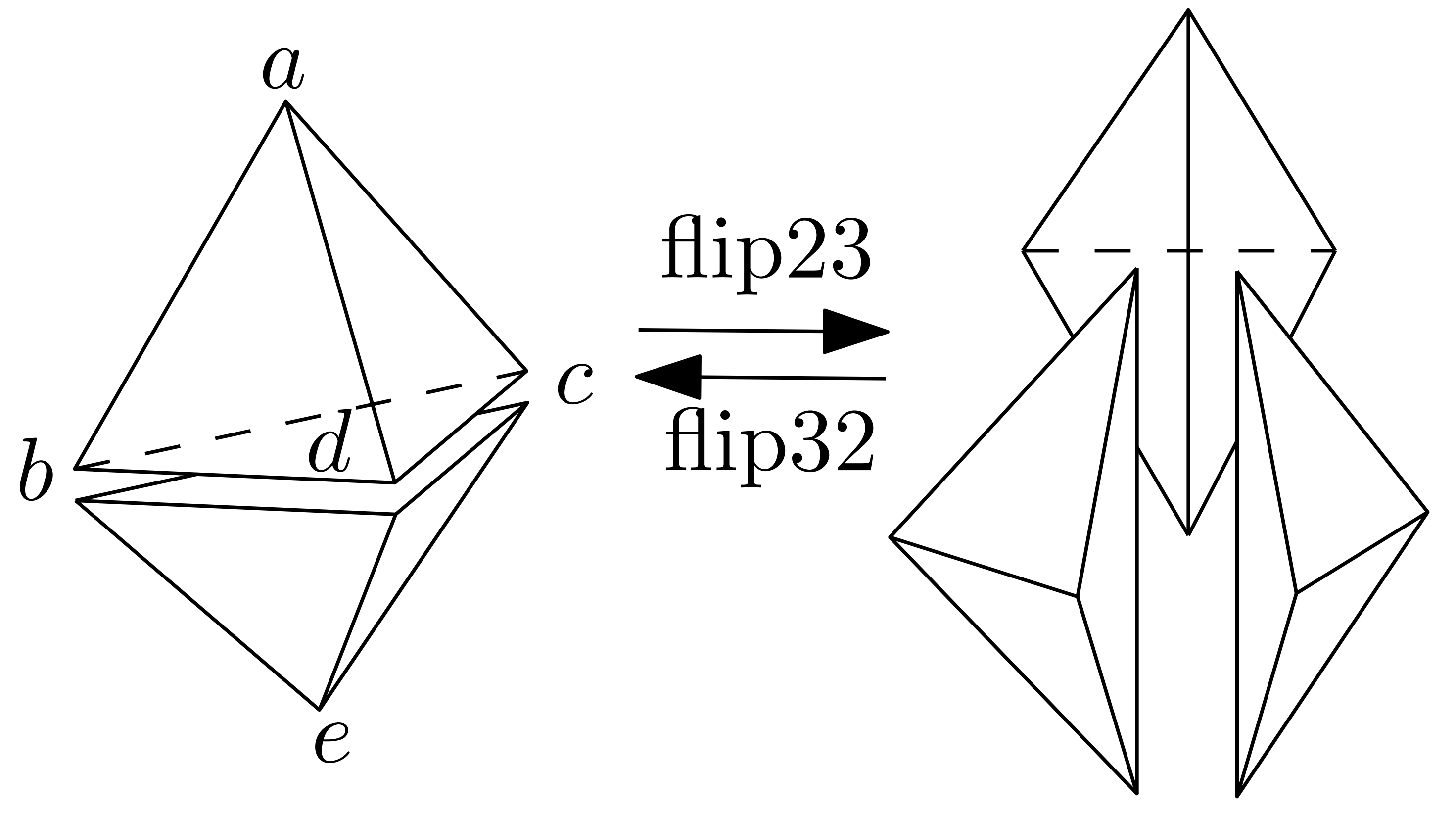
Figure 7: Step-by-step insertion, with flips, of a single point in a DT in two dimensions.

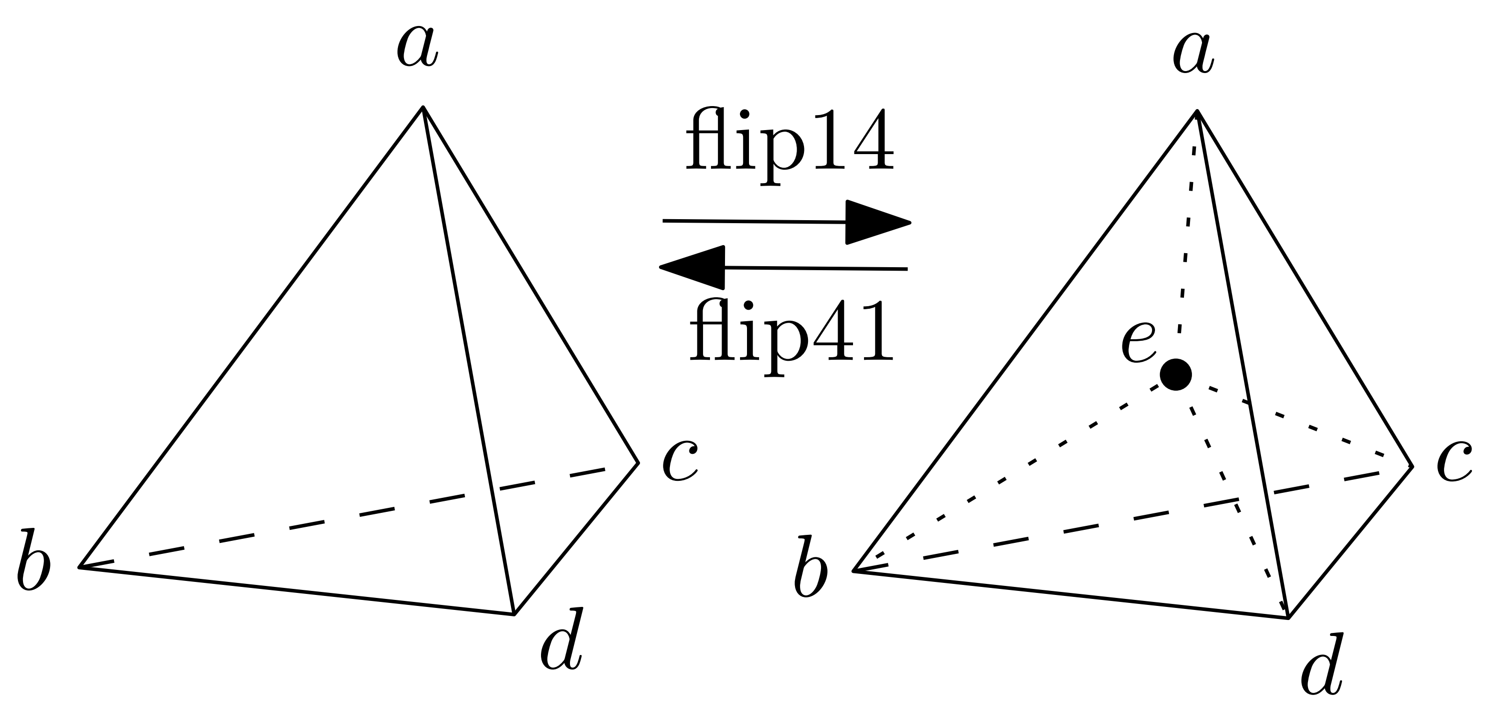
Incrementation construction

Algorithm 1: Algorithm to insert one point in a DT

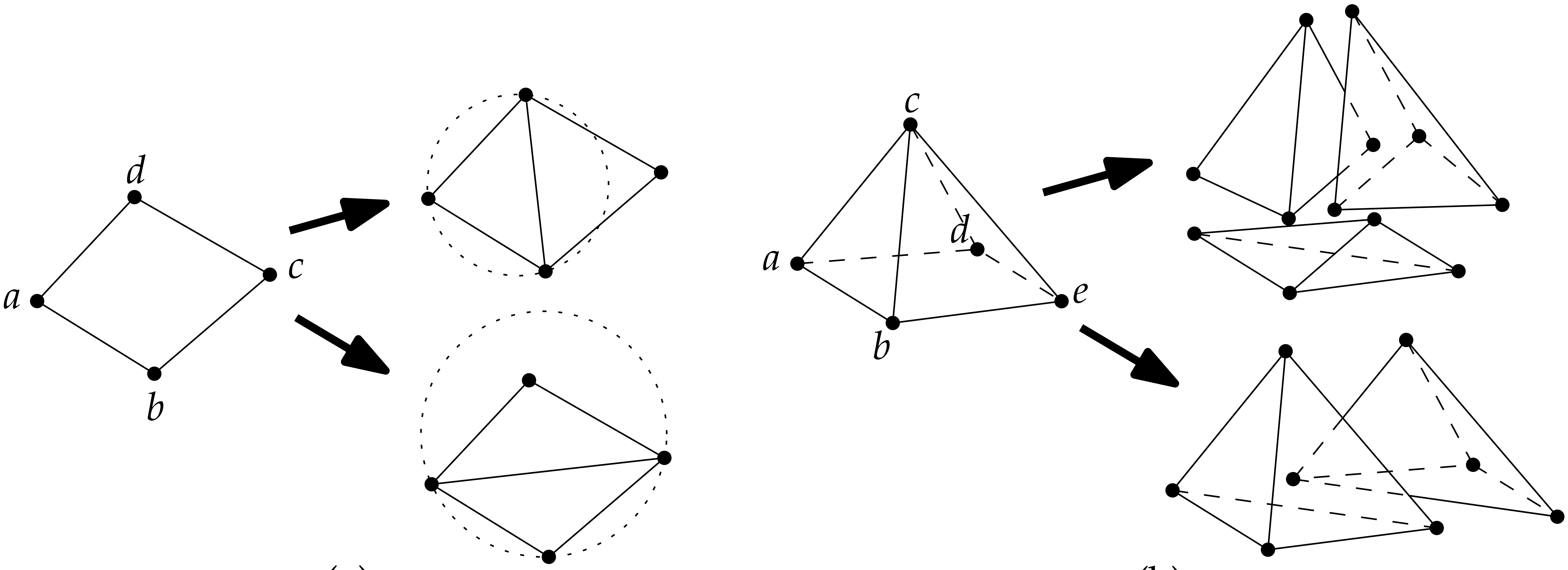
Input: A DT(S) $\mathcal{T}$ in $\mathbb{R}^3$, and a new point p to insert
Output: $\mathcal{T}^p = \mathcal{T} \cup \{p\}$

find tetrahedron τ containing p
insert p in τ by splitting it in to 4 new tetrahedra (flip14)
push 4 new tetrahedra on a stack
while *stack is non-empty* **do**
  $\tau = \{p, a, b, c\} \leftarrow$ pop from stack
  $\tau_a = \{a, b, c, d\} \leftarrow$ get adjacent tetrahedron of τ having the edge *abc* as a face
  if *d is inside circumsphere of τ* **then**
    if *configuration of τ and τ_a allows it* **then**
      flip the tetrahedra τ and τ_a (flip23 or flip32)
      push 2 or 3 new tetrahedra on stack
    else
      Do nothing





Slivers in DT



(a) (b)

3D DT == 4D convex hull

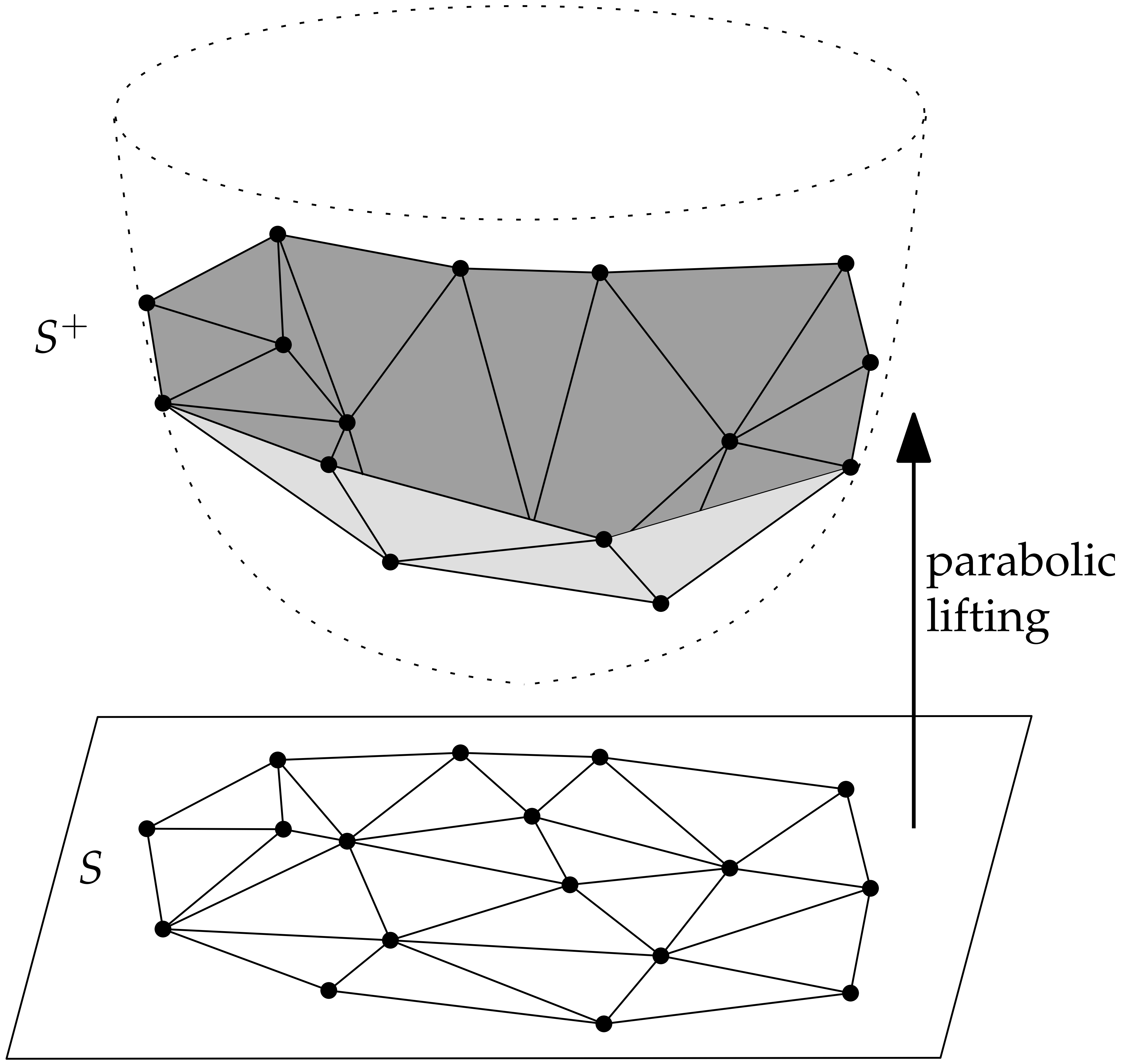


Figure 6: The parabolic lifting map for a set S of points $\mathbb{R}^2$.

3D == no “fixed” orientation

3.3.2 Predicates

The ‘orientation’ of points in three dimensions is somewhat tricky because, unlike in two dimensions, we can not simply rely on the counter-clockwise orientation. In three dimensions, the orientation is always relative to another point of reference, ie given three points we cannot say if a fourth one is left of right, this depends on the orientation of the three points.

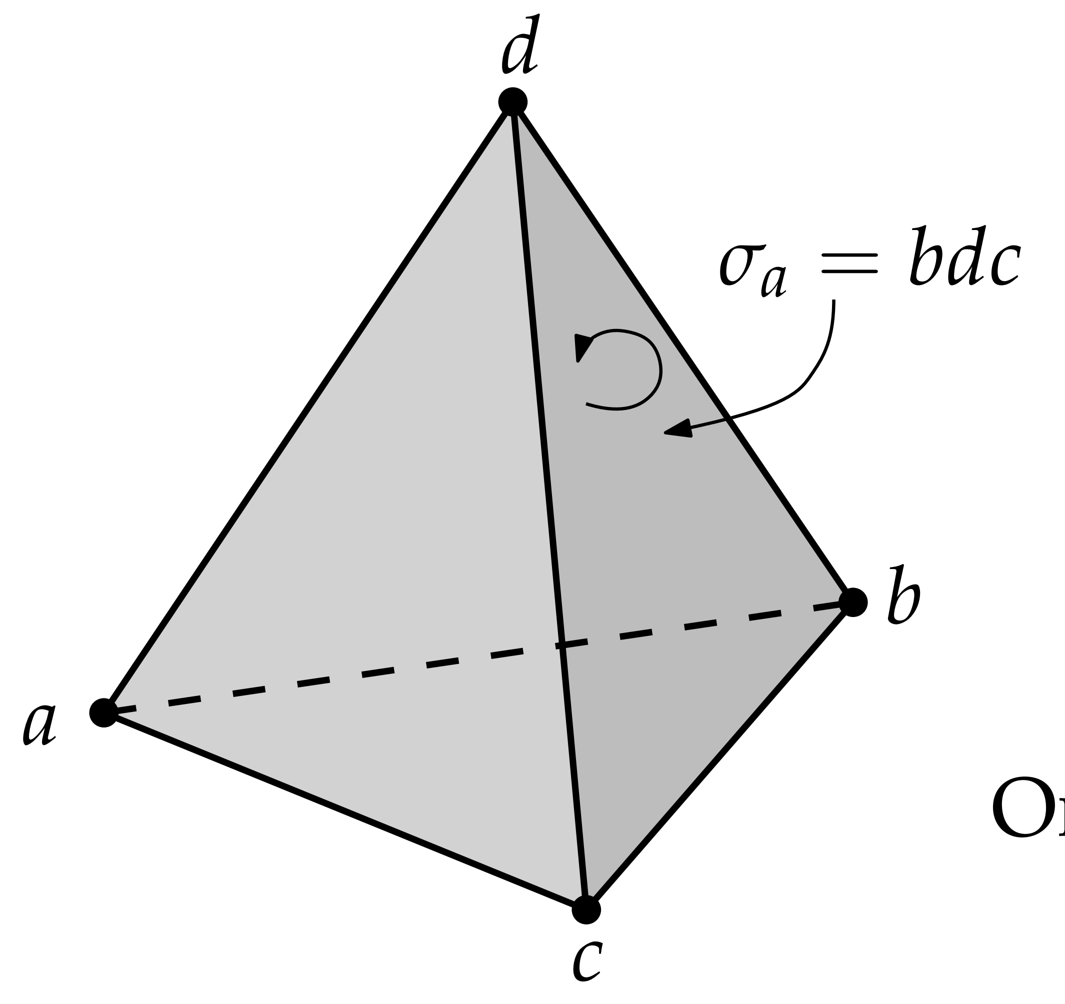
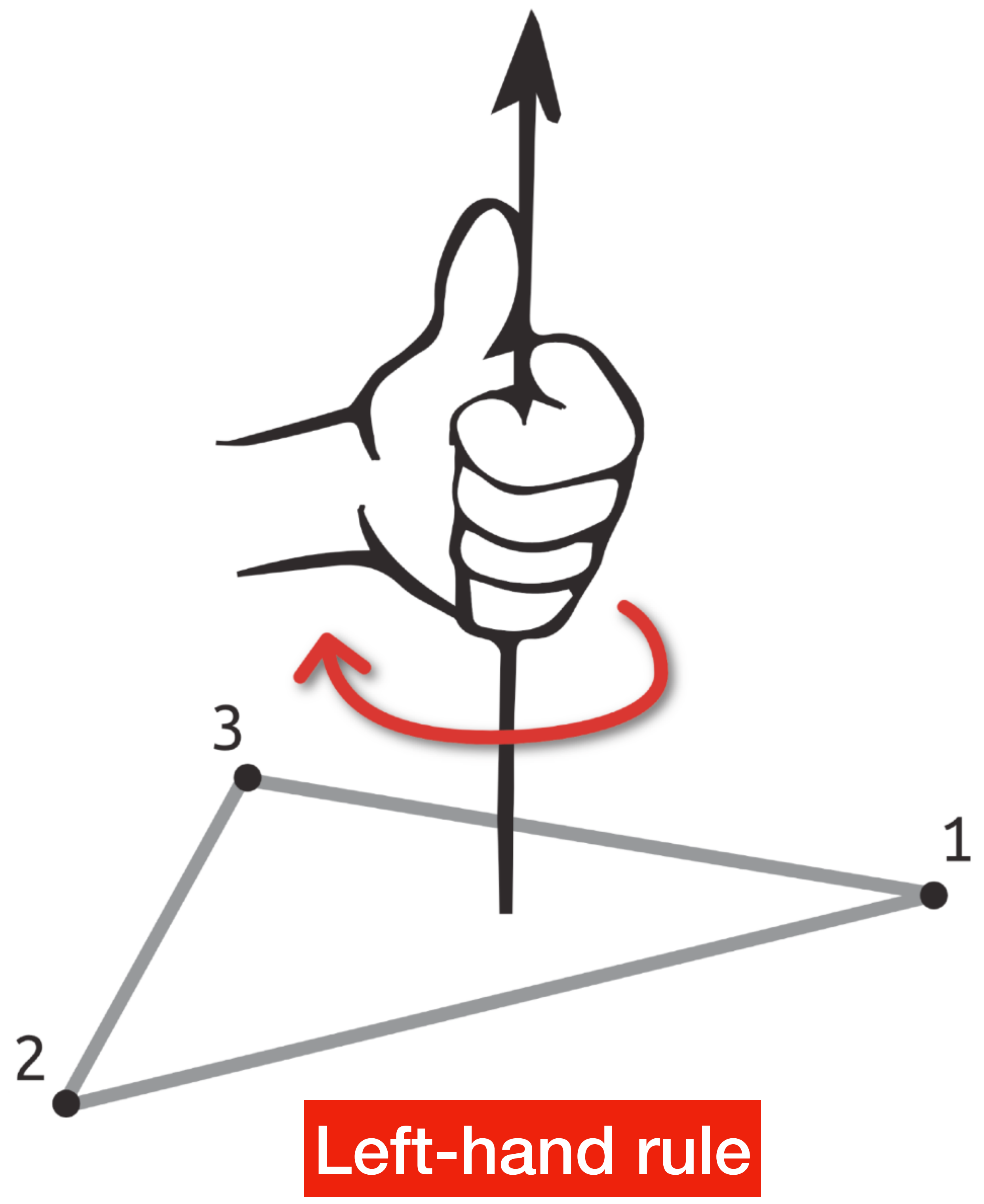


Figure 3.13: The tetrahedron $abcd$ is correctly oriented since Orient (a, b, c, d) returns a positive result. The arrow indicates the correct orientation for the face σ_a, so that Orient (σ_a, a) returns a positive result.



Orient can be implemented as the determinant of a matrix:

$$\text{Orient}(a, b, c, p) = \begin{vmatrix} a_x & a_y & a_z & 1 \\ b_x & b_y & b_z & 1 \\ c_x & c_y & c_z & 1 \\ p_x & p_y & p_z & 1 \end{vmatrix} \tag{3.2}$$

Duality VD <-> DT in 3D

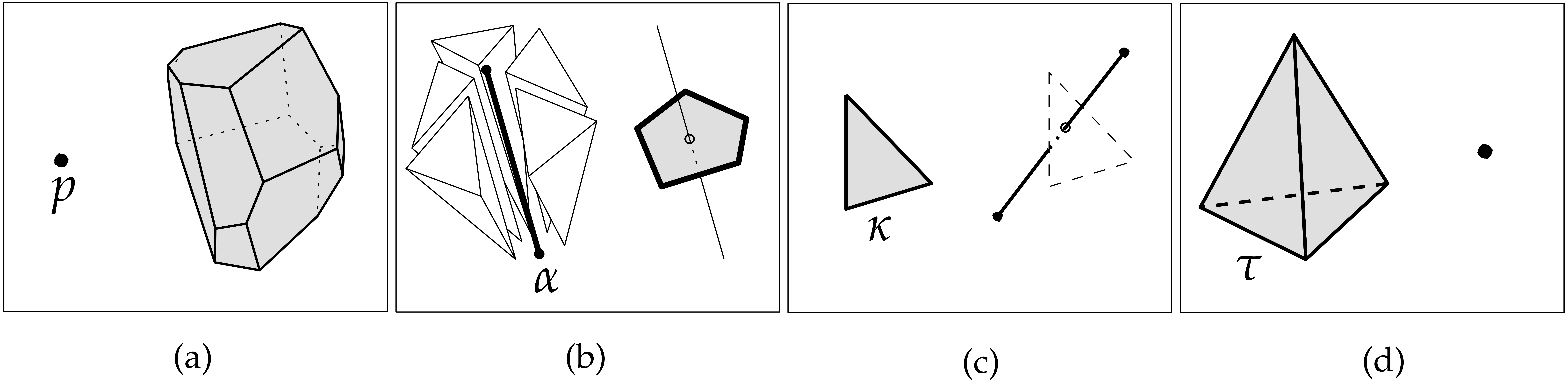
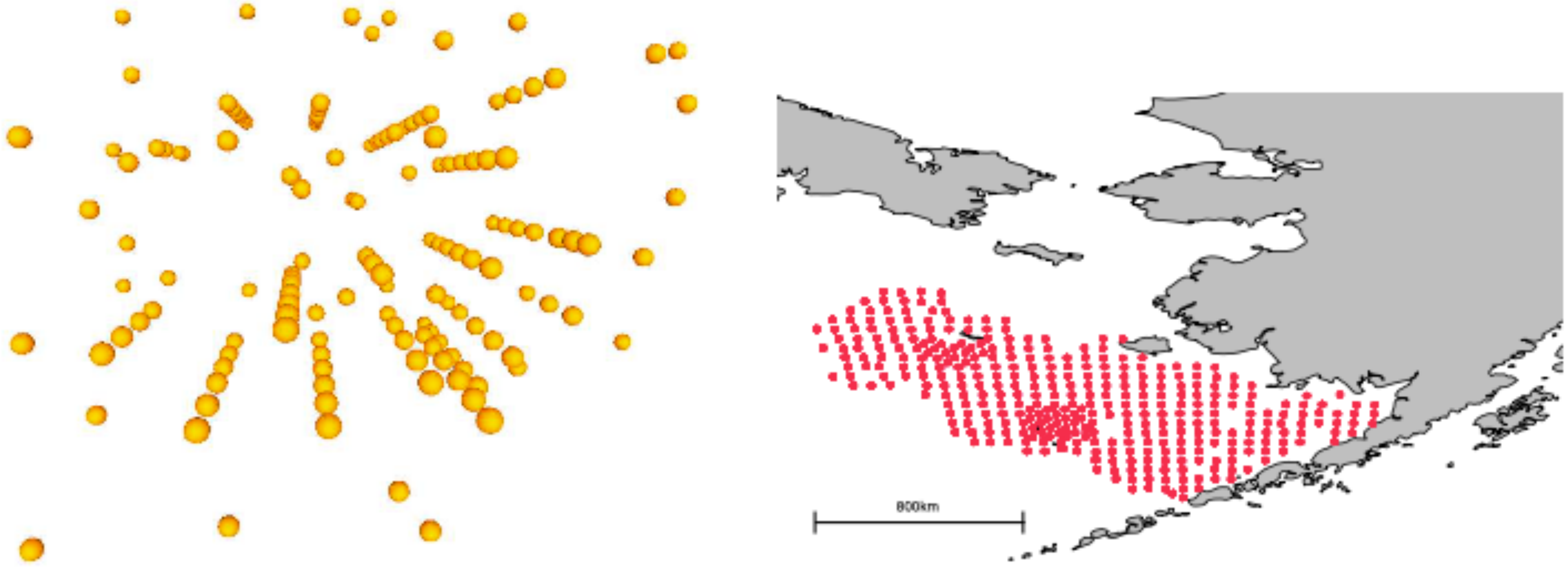


Figure 3: Duality in $\mathbb{R}^3$ between the elements of the VD and the DT.

App#1: 3DVD as alternative to voxels



(a) (b)

Figure 10: **(a)** Example of a dataset in geology, where samples were collected by drilling a hole in the ground. Each sample has a location in 3D space ($x - y - z$ coordinates) and one or more attributes attached to it. **(b)** An oceanographic dataset in the Bering Sea in which samples are distributed along water columns. Each red point represents a (vertical) water column, where samples are collected every 2m, but water columns are about 35km from each other.

App#2: spatial interpolation

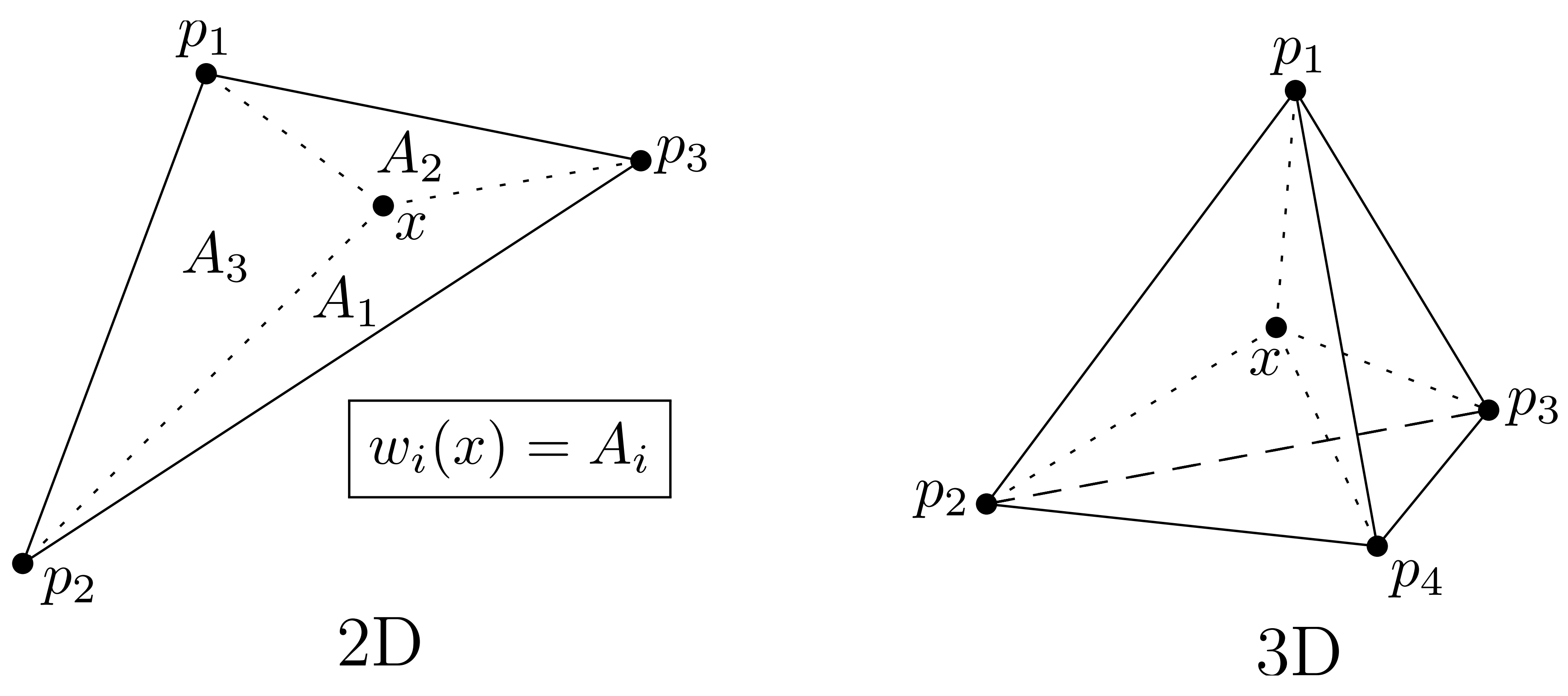
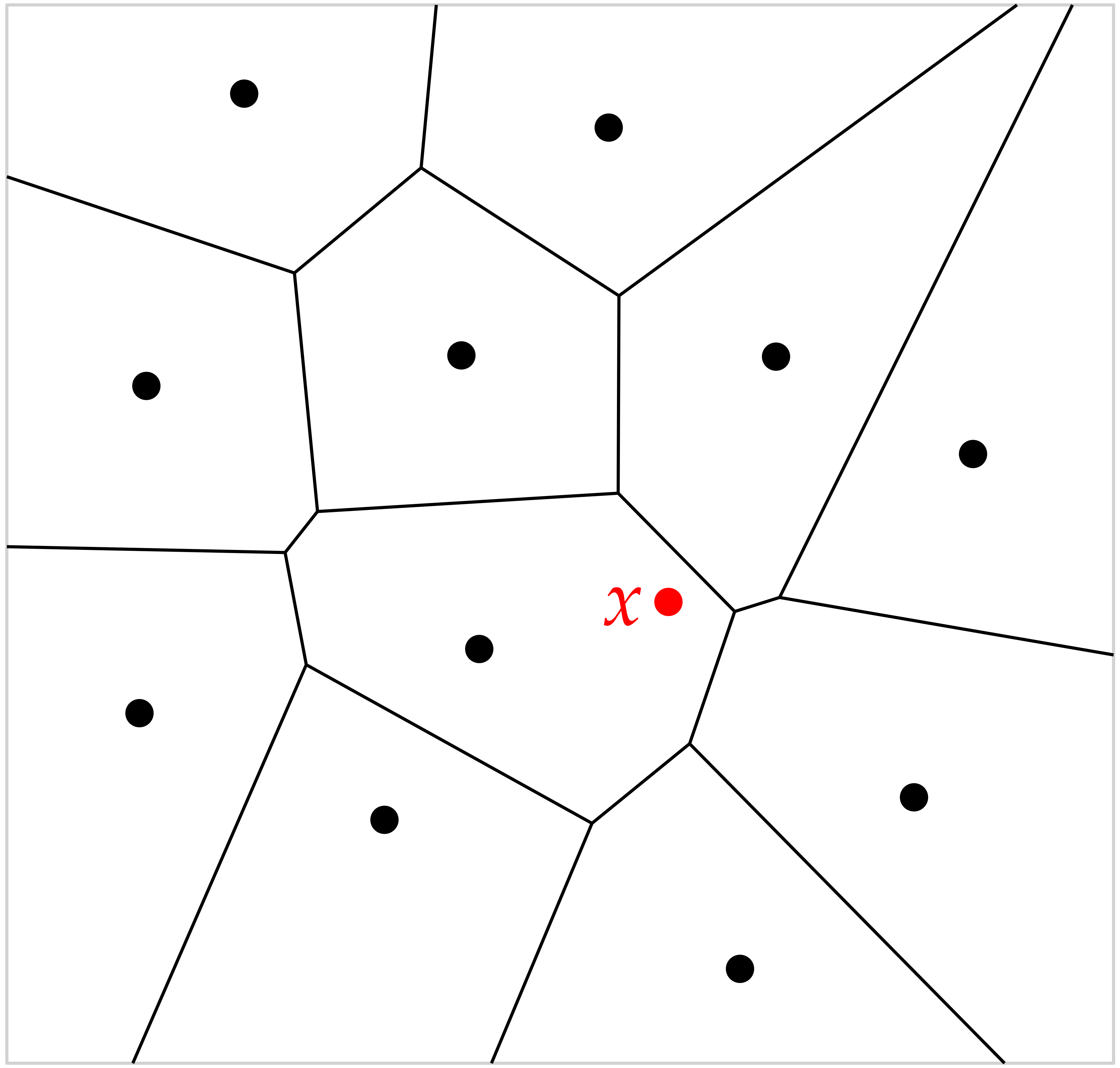
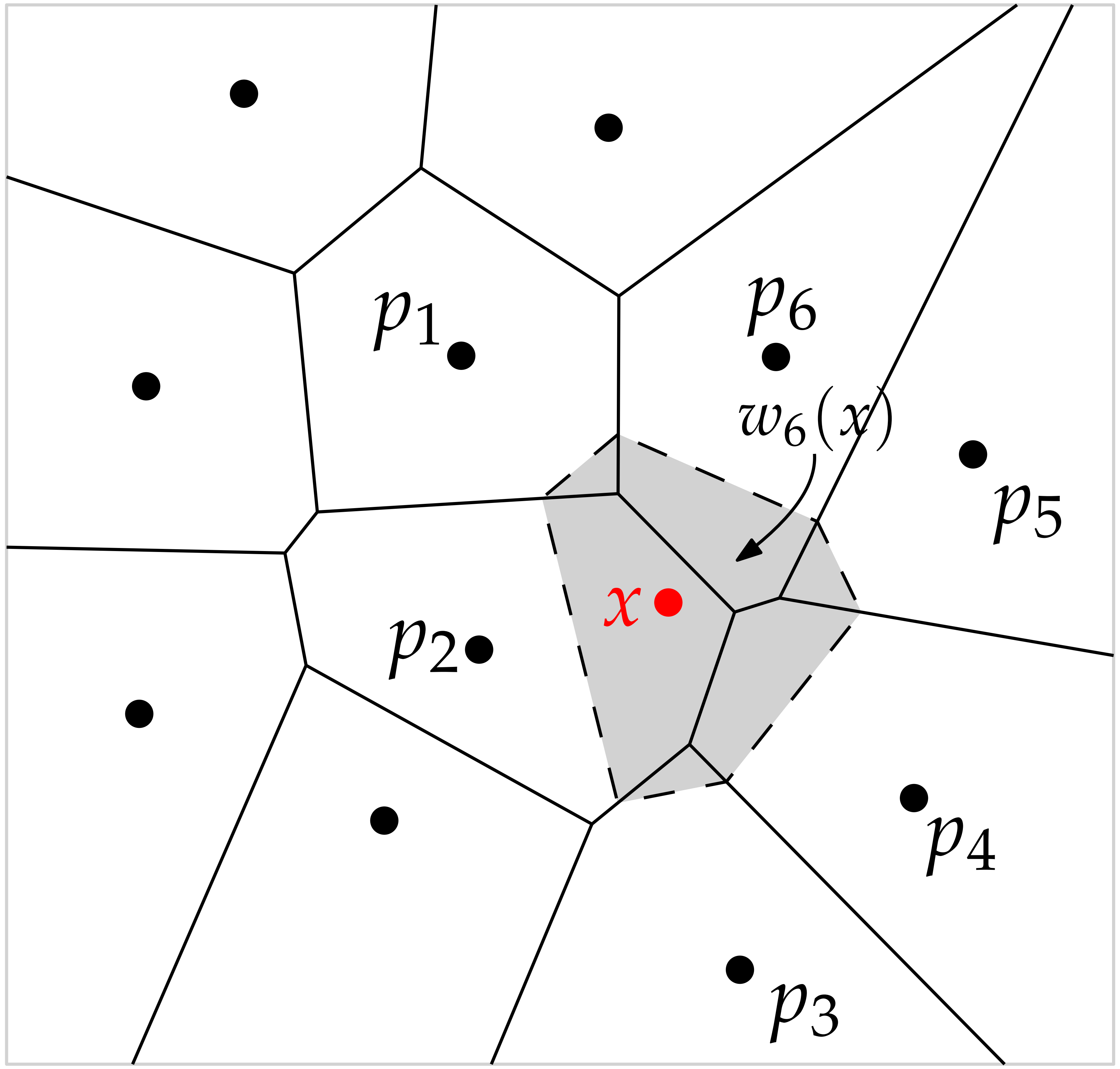


Figure 11: Barycentric coordinates in two and three dimensions. A_i represents the area of the triangle formed by x and one edge.

$$vol(\sigma) = \frac{1}{d!} \left| det \begin{pmatrix} v^0 & \dots & v^d \\ 1 & \dots & 1 \end{pmatrix} \right|$$

App#2: spatial interpolation





$$w_i(x) = stolenarea$$

App#3: visualisation with iso-surfaces

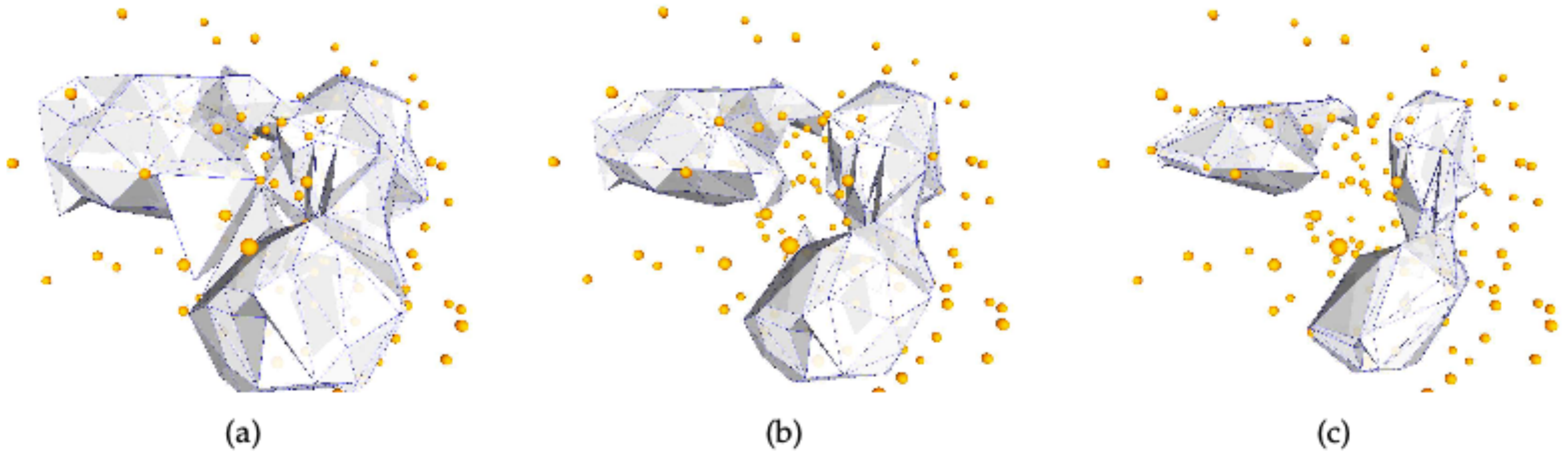
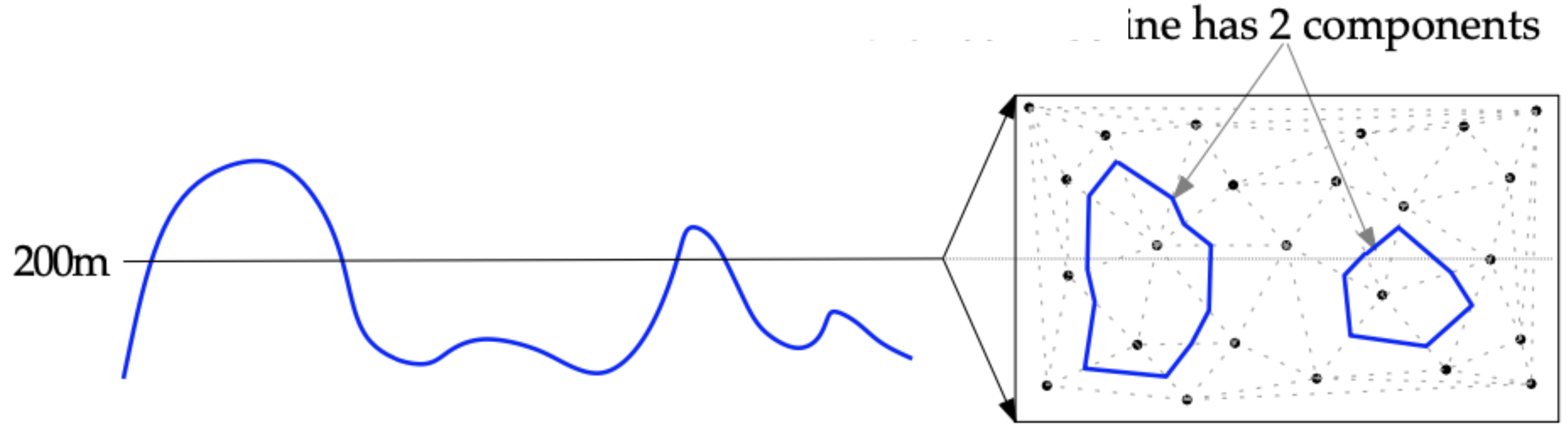


Figure 12: An example of an oceanographic dataset where each point has the temperature of the water, and three isosurface extracted (for a value of respectively 2.0, 2.5 and 3.5) from this dataset.



App#3: visualisation with iso-surfaces

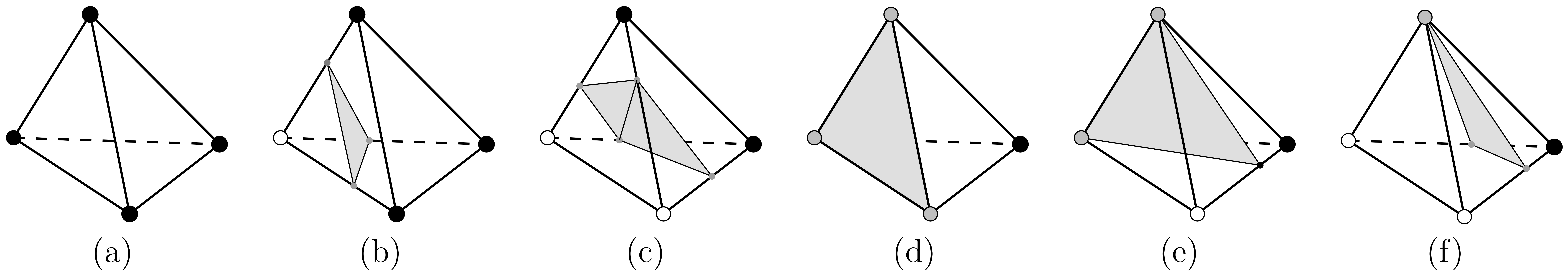
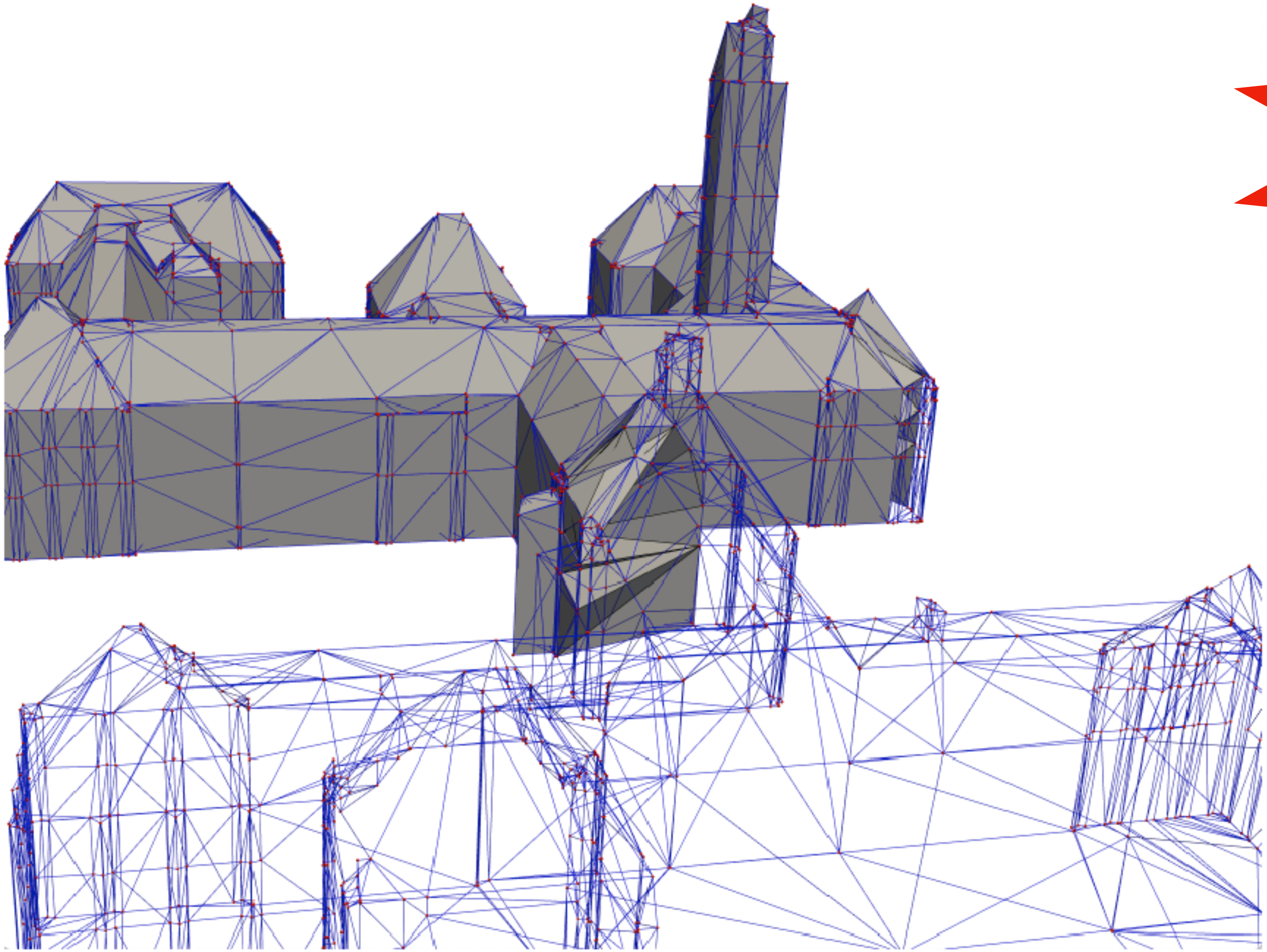


Figure 3.15: Potential isosurface (for an attribute value v) extracted for one tetrahedron. Black vertex means that the attribute of this vertex is below v; white vertex means it is above; and grey that it is equal.

Triangulating a building (or any 3D model)



Two very different cases

demo with one building

- MeshLab: https://www.meshlab.net
- TetGen: http://tetgen.org
- ParaView: https://www.paraview.org
- Mapple: https://github.com/LiangliangNan/Easy3D

https://3d.bk.tudelft.nl/courses/geo1004

3D geoinformation
Department of Urbanism
Faculty of Architecture and the Built Environment
Delft University of Technology