Assignment 3: Linear Algebra

This assignment aims to practice the basic linear algebra skills needed for your future Geomatics courses and to apply some of them to solve practical problems.

- This assignment consists of three parts corresponding to the three lectures on linear algebra respectively. It is highly recommended to **work on and finish each part after the corresponding lecture**.
- You're allowed for this assignment to work in a **group of 2** (and thus submit only one solution for both of you). You are free to form a group yourself. It's also fine if you prefer to work alone.
- You are free to use any text editor (e.g., Word, LaTex) for writing your answers. **An answer without an explanation is considered invalid**! For each question, your answer should include a final result and an explanation of how it is computed. A good explanation can be an equation, intermediate results, texts describing the idea, or any of their combination. We highly encourage expressing ideas with equations.
- **What and where to submit?** Write your answers and necessary explanation or important intermediate steps **in PDF format with a file name "**studentID1_familyName1_studentID2_FamilyName2.pdf**". Click here to submit**: https://surfdrive.surf.nl/files/index.php/s/gEDC2fyZAJ7GsLB
- **The deadline is 27 October 2020 at 22:59.**
- For late submission, 10% will be removed for each day that you are late.

Part 1 (After lecture C1): Vectors, matrices, vector/matrix arithmetic

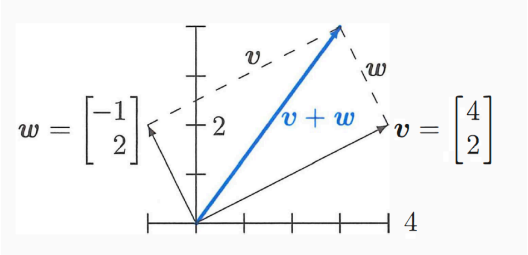
1. Given two vectors $\boldsymbol{u} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $\boldsymbol{v} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$,

   (a) which vector has a longer length? (2%)

   (b) what is the dot product of them? (3%)

   (c) are the two vectors perpendicular? If not, what is the angle between them (an equation expression is sufficient)? (3%)

2. In the following figure, the parallelogram has a diagonal $\boldsymbol{v} + \boldsymbol{w}$. What is the other diagonal? (8%)



3. $c\boldsymbol{u} + d\boldsymbol{v}$ is a linear combination of the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$. For $\boldsymbol{u} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $\boldsymbol{v} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$, what is $3\boldsymbol{u} + 2\boldsymbol{v}$? (8%)

4. Given two vectors $\boldsymbol{u} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $\boldsymbol{v} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$, the linear combinations $c\boldsymbol{u} + d\boldsymbol{v}$ fill a plane (in the 3D space) that passes through the origin (i.e., 0, 0, 0). What is the normal vector of this plane? (8%)

Part 2 (After lecture C2): Linear systems, least squares regression

5. Given the following system of linear equations

$$\begin{cases} 2x_1 + x_2 - 3x_3 = 4 \\ x_1 + 5x_2 + 4x_3 = 1 \\ 3x_1 + 6x_2 + x_3 = 3 \end{cases}$$

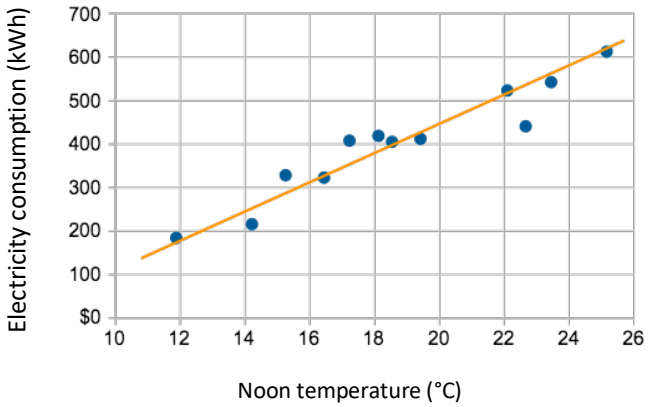
(a) write the above system equations in matrix form (i.e., matrix-vector multiplication form). (8%)

(b) does the system have a single solution, no solution, or an infinite number of solutions? How do you know? (10%)

6. A factory keeps track of electricity consumption versus the average noon temperature on a day. Here is its data for the last 12 days:

x - Noon temperature (°C)	y - Electricity consumption (kWh)
14.2	215
16.4	325
11.9	185
15.2	332
18.5	406
22.1	522
19.4	412
25.1	614
23.4	544
18.1	421
22.6	445
17.2	408

And the following figure shows the same data as a scatter plot (i.e., points that show the relationship between temperature and electricity consumption). From the scatter plot, it is easy to see that there is a linear relationship between electricity consumption and temperature (An extra yellow line on the scatter plot is drawn to illustrate the best fitting line).



(a) Use any programming language (e.g., Python, C/C++, Matlab) or any solver to figure out the empirical relationship between x and y. (15%)

(b) How much electricity will be consumed on a day with a noon temperature at 21 °C? (5%)

Part 3 (After lecture C3): Eigen values and eigen vectors, singular value decomposition

7. Given a matrix $A = \begin{bmatrix} 8 & 3 \\ 2 & 7 \end{bmatrix}$, compute the eigen values and eigen vectors of the following matrices

   - A (5%)

   - A^2 (5%)

   - A^n, where n is a positive integer larger than 1. (5%)

8. Decomposition transformation $M = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$ into $M = RS$, where R and S represent a rotation transformation and a stretch transformation respectively. You can use any programming language (e.g., Python, C/C++, Matlab) or any solver/library to solve it. (Hint: this is an application of the SVD decomposition, but it not a direct application and you will have to do some transformation). (15%)