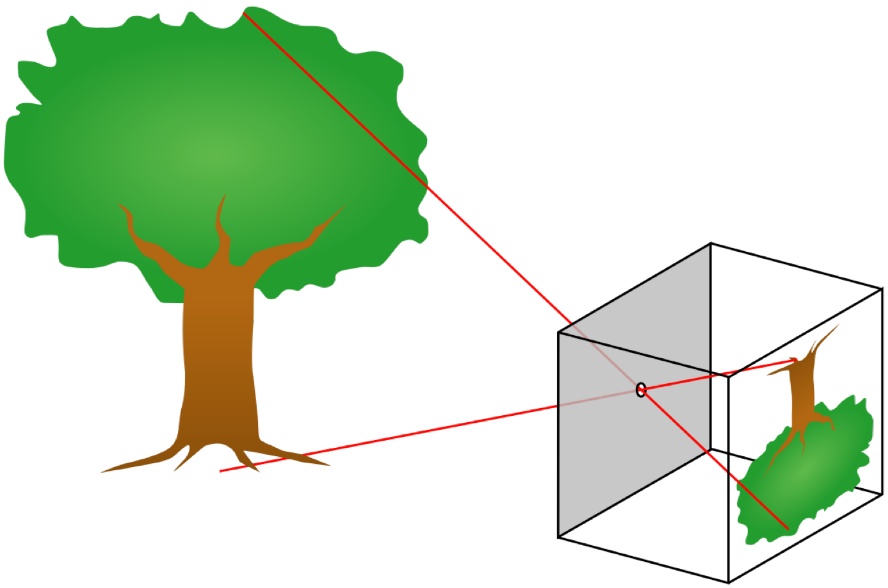
Linear Algebra

Liangliang Nan

What is linear algebra?

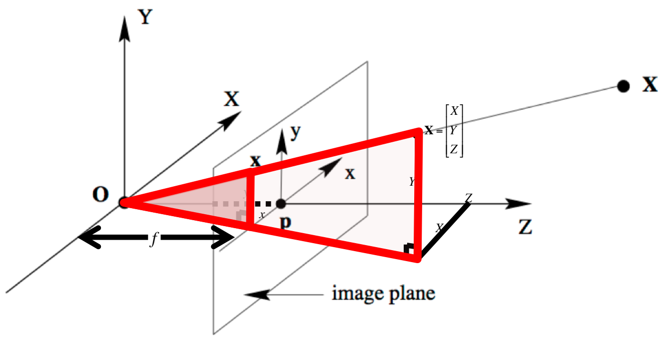
Problem → Conceptual Solution → Math Solution



What is linear algebra?

Problem → Conceptual Solution → Math Solution

How to get the pixel location on an image of a 3D point?



$$\mathbf{x} = \mathbf{K}\left[\mathbf{R} \middle| \mathbf{t}\right]\mathbf{X}$$

What is linear algebra?

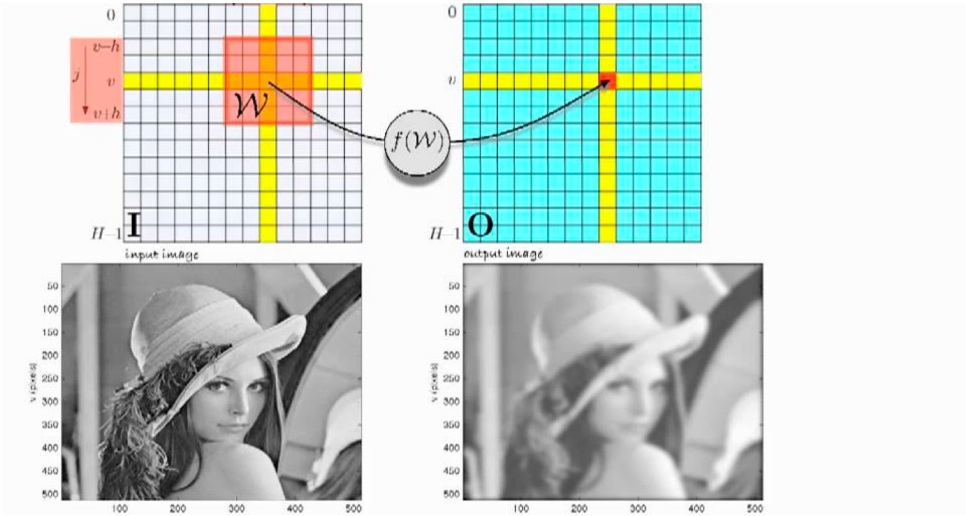
- Difficult problem can be handled effectively
  - Representation
    - 3D points, lines, planes in the 3D space
      - Distance?

What is linear algebra?

- Difficult problem can be handled effectively
  - Representation
  - Coordinates will be used
    - Perform geometrical transformations/computation
      - The rotation of a 3D object
      - The intersection of 3 non-parallel planes
    - Associate 3D with 2D points
      - The projection of a 3D object onto a plane

What is linear algebra?

- Difficult problem can be handled effectively
  - Representation
  - Coordinates will be used
  - Images are matrices of numbers
    - Find properties of these number



What is linear algebra?

- Difficult problem can be handled effectively
  - Representation
  - Coordinates will be used
  - Images are matrices of numbers
- Linear algebra
  - Organize information in cases where certain mathematical structures are present
  - The study of those structures
    - Vectors and linear functions

Scope of the lectures

Part 1: Vectors, matrices, matrix/vector arithmetic, geometric transformations

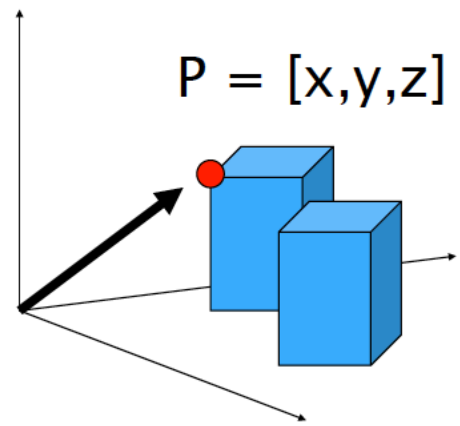
Part 2: Linear systems, linear-least squares

Part 3: Eigen values/vectors, singular value decomposition

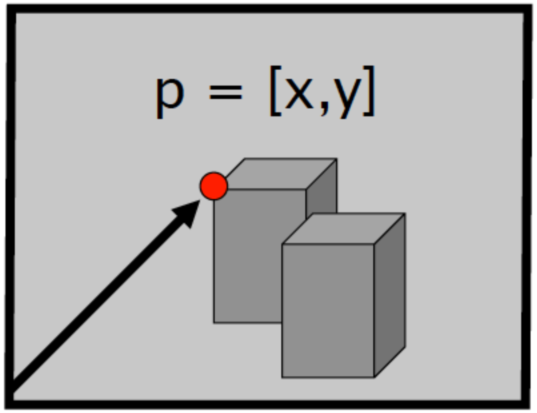
Linear Algebra - Part 1
Vectors, matrices, matrix/vector arithmetic, geometric transformations

Liangliang Nan

Vectors (i.e., 2D and 3D vectors)



3D world



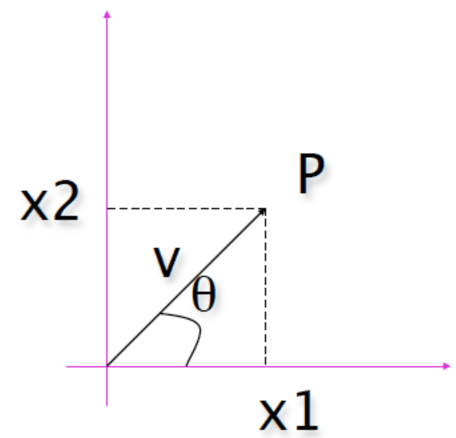
Image

Vectors (i.e., 2D vectors)

$$\mathbf{v} = (x_1, x_2)$$

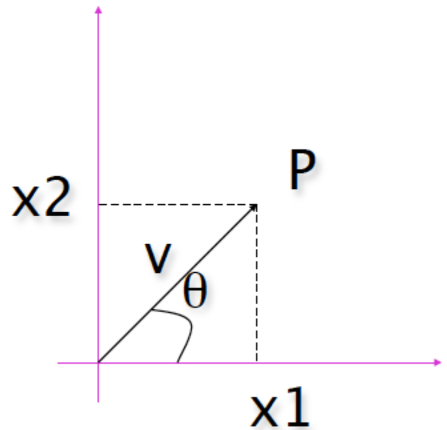
Magnitude: $\|\mathbf{v}\| = \sqrt{x_1^{\,2} + x_2^{\,2}}$

If $\|\mathbf{v}\| = 1$, $\mathbf{v}$ Is a UNIT vector



Vectors (i.e., 2D vectors)

$$\mathbf{v} = (x_1, x_2)$$



Magnitude: $\|\mathbf{v}\| = \sqrt{{x_1}^2 + {x_2}^2}$

If $\|\mathbf{v}\| = 1$, $\mathbf{v}$ Is a UNIT vector

What is the unit vector that has the same direction as **v?**

Vectors (i.e., 2D vectors)

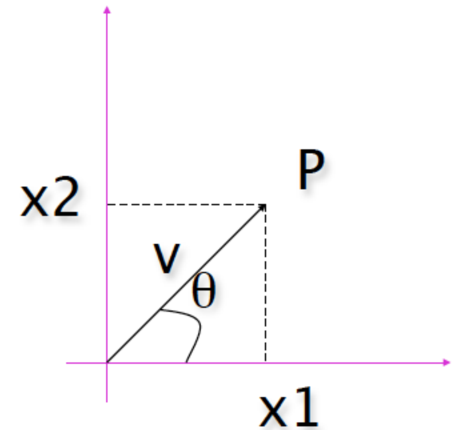
$$\mathbf{v} = (x_1, x_2)$$

Magnitude: $\|\mathbf{v}\| = \sqrt{x_1^{\,2} + x_2^{\,2}}$

If $\|\mathbf{v}\| = 1$, $\mathbf{v}$ Is a UNIT vector

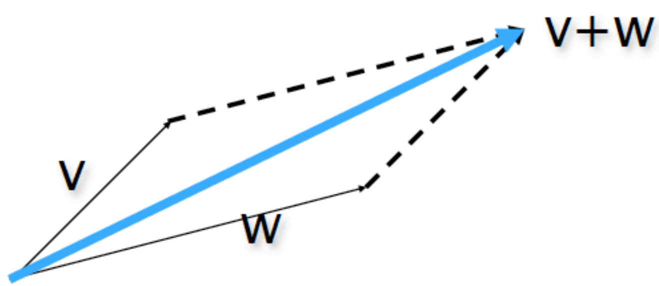
$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\frac{x_1}{\|\mathbf{v}\|}, \frac{x_2}{\|\mathbf{v}\|}\right)$$ Is a unit vector

Orientation: $\theta = \tan^{-1}\left(\frac{x_2}{x_1}\right)$



Vector Addition

$$\mathbf{v} + \mathbf{w} = (x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$$



A parallelogram

Compute

(0, 1) + (1, 0) = ?

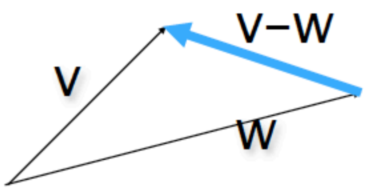
(1, 1) + (1, 1) = ?

(1, 0) + (-1, 0) = ?

Vector Subtraction

- Inverse operation of addition

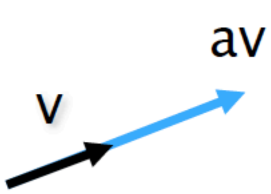
$$\mathbf{v} - \mathbf{w} = (x_1, x_2) - (y_1, y_2) = (x_1 - y_1, x_2 - y_2)$$



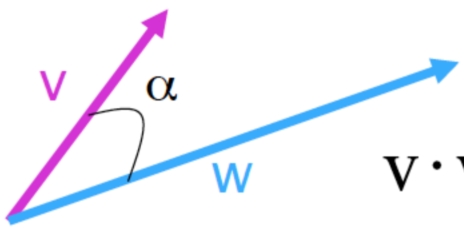
Scalar Product

- Length is changed
- Direction is preserved

$$a\mathbf{v} = a(x_1, x_2) = (ax_1, ax_2)$$



Inner (dot) Product

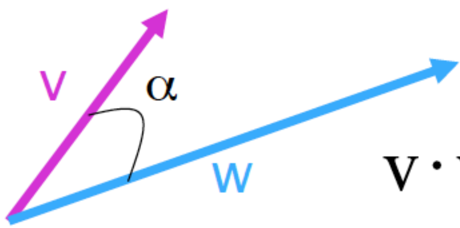


$$v \cdot w = (x_1, x_2) \cdot (y_1, y_2) = x_1 y_1 + x_2 y_2$$

The inner product is a SCALAR!

$$v \cdot w = (x_1, x_2) \cdot (y_1, y_2) = \| v \| \cdot \| w \| \cos \alpha$$

Inner (dot) Product



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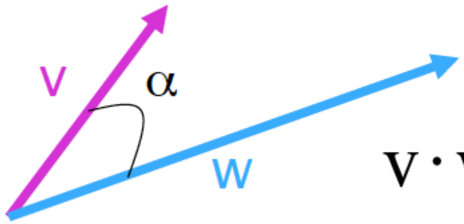
$$\text{if} \quad v \perp w, \quad v \cdot w =$$

Inner (dot) Product



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The inner product is a SCALAR!

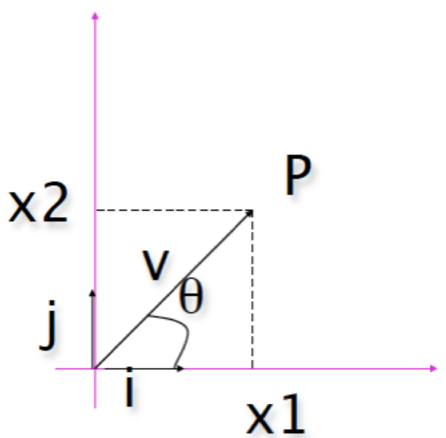
$$v \cdot w = (x_1, x_2) \cdot (y_1, y_2) = \| v \| \cdot \| w \| \cos \alpha$$

Compute the inner product of

(0, 1) · (1, 0) = ?
(1, 1) · (1, 1) = ?
(1, 0) · (-1, 0) = ?

Orthonormal Basis

- Unit vectors; orthogonal to each other
- Any vector -> linear combination of the basis



$$\mathbf{i} = (1,0) \qquad \|\mathbf{i}\| = 1$$

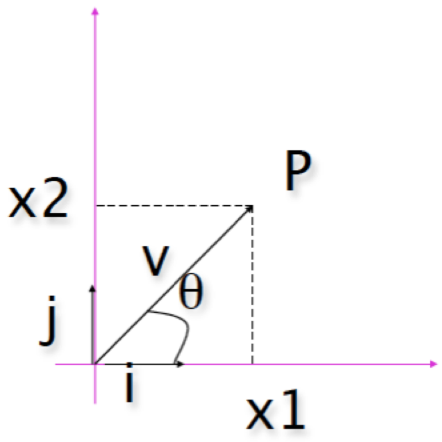
$$\mathbf{j} = (0,1) \qquad \|\mathbf{j}\| = 1$$

$$\mathbf{i} \cdot \mathbf{j} = 0$$

$$\mathbf{v} = (x_1, x_2) \qquad \mathbf{v} = \mathrm{x}_1\mathbf{i} + \mathrm{x}_2\mathbf{j}$$

Orthonormal Basis

- Unit vectors; orthogonal to each other
- Any vector -> linear combination of the basis



$$\mathbf{i} = (1,0) \qquad \|\mathbf{i}\| = 1 \qquad \mathbf{i} \cdot \mathbf{j} = 0$$

$$\mathbf{j} = (0,1) \qquad \|\mathbf{j}\| = 1$$

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$$\mathbf{v} \cdot \mathbf{i} =$$

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Orthonormal Basis

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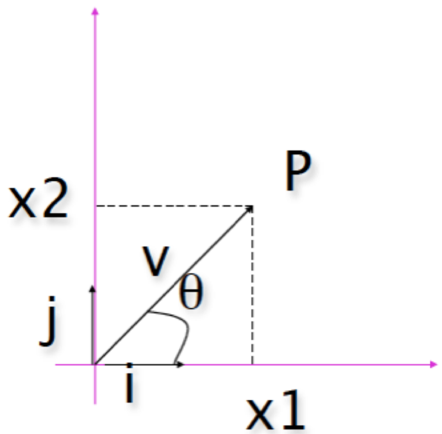

$$\mathbf{i} = (1,0) \qquad \|\mathbf{i}\| = 1$$

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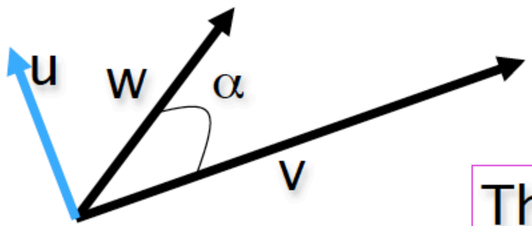
$$\mathbf{i} \cdot \mathbf{j} = 0$$

$$\mathbf{v} = (x_1, x_2) \qquad \mathbf{v} = \mathrm{x}_1\mathbf{i} + \mathrm{x}_2\mathbf{j}$$

$$\mathbf{v} \cdot \mathbf{i} = ? \;\; = (\mathrm{x}_1\mathbf{i} + \mathrm{x}_2\mathbf{j}) \cdot \mathbf{i} = \mathrm{x}_1 1 + \mathrm{x}_2 0 = \mathrm{x}_1$$

$$\mathbf{v} \cdot \mathbf{j} = (\mathrm{x}_1\mathbf{i} + \mathrm{x}_2\mathbf{j}) \cdot \mathbf{j} = \mathrm{x}_1.0 + \mathrm{x}_2.1 = \mathrm{x}_2$$

Vector (cross) Product

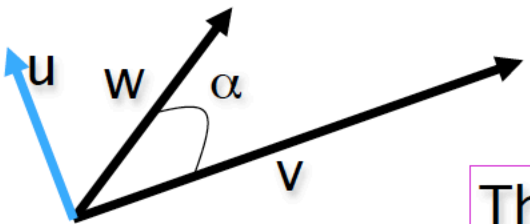


$$u = v \times w$$

The cross product is a VECTOR!

$$\text{Magnitude:} \|u\| = \|v \times w\| = \|v\| \|w\| \sin \alpha$$

Vector (cross) Product



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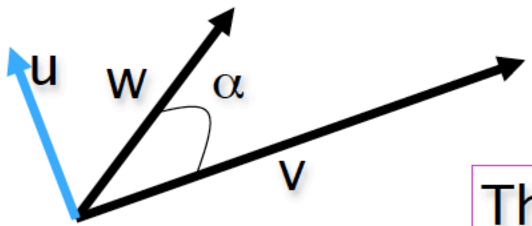
Orientation:

$$u \perp v \Rightarrow u \cdot v =$$

$$u \perp w \Rightarrow u \cdot w =$$

Vector (cross) Product



$$u = v \times w$$

The cross product is a VECTOR!

$$\text{Magnitude:} \|u\| = \|v \times w\| = \|v\| \|w\| \sin \alpha$$

Orientation:

$$u \perp v \Rightarrow u \cdot v = (v \times w) \cdot v = 0$$

$$u \perp w \Rightarrow u \cdot w = (v \times w) \cdot w = 0$$

if $v \,//\, w$? $\rightarrow u =$

Vector Product Computation

- Orthonormal basis

$$\mathbf{i} = (1,0,0) \qquad \|\mathbf{i}\| = 1 \qquad \mathbf{i} = \mathbf{j} \times \mathbf{k}$$

$$\mathbf{j} = (0,1,0) \qquad \|\mathbf{j}\| = 1 \qquad \mathbf{j} = \mathbf{k} \times \mathbf{i}$$

$$\mathbf{k} = (0,0,1) \qquad \|\mathbf{k}\| = 1 \qquad \mathbf{k} = \mathbf{i} \times \mathbf{j}$$

$$\mathbf{u} = \mathbf{v} \times \mathbf{w} = (x_1, x_2, x_3) \times (y_1, y_2, y_3)$$

Vector Product Computation

- Orthonormal basis

$$\mathbf{i} = (1,0,0) \qquad \|\mathbf{i}\| = 1 \qquad \mathbf{i} = \mathbf{j} \times \mathbf{k}$$

$$\mathbf{j} = (0,1,0) \qquad \|\mathbf{j}\| = 1 \qquad \mathbf{j} = \mathbf{k} \times \mathbf{i}$$

$$\mathbf{k} = (0,0,1) \qquad \|\mathbf{k}\| = 1 \qquad \mathbf{k} = \mathbf{i} \times \mathbf{j}$$

$$\mathbf{u} = \mathbf{v} \times \mathbf{w} = (x_1, x_2, x_3) \times (y_1, y_2, y_3)$$

$$= (x_2 y_3 - x_3 y_2)\mathbf{i} + (x_3 y_1 - x_1 y_3)\mathbf{j} + (x_1 y_2 - x_2 y_1)\mathbf{k}$$

Matrices

$$A_{n\times m} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix} \longleftrightarrow$$

Pixel's intensity value

Matrix addition

$$A_{n\times m} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix} \longleftrightarrow$$

Pixel's intensity value

Sum: $C_{n\times m} = A_{n\times m} + B_{n\times m} \qquad c_{ij} = a_{ij} + b_{ij}$

A and B must have the same dimensions!

Example: $\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 4 & 6 \end{bmatrix}$

Matrix product

$$A_{n\times m} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix} \quad \mathbf{a}_i$$

$$B_{m\times p} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1p} \\ b_{21} & b_{22} & \dots & b_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mp} \end{bmatrix}$$

$$\mathbf{b}_j$$

Product:

$$C_{n\times p} = A_{n\times m} B_{m\times p}$$

$$c_{ij} = \mathbf{a}_i \cdot \mathbf{b}_j = \sum_{k=1}^{m} a_{ik} b_{kj}$$

A and B must have compatible dimensions!

$$A_{n\times n} B_{n\times n} \overset{?}{=} B_{n\times n} A_{n\times n}$$

Matrix transpose

Transpose:

$$C_{m\times n} = A^T{}_{n\times m} \qquad (A+B)^T = A^T + B^T$$

$$c_{ij} = a_{ji} \qquad (AB)^T = B^T A^T$$

If $A^T = A$ A is symmetric

Examples:

$$\begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 2 & 5 \end{bmatrix} \qquad \begin{bmatrix} 6 & 2 \\ 1 & 5 \\ 3 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 & 3 \\ 2 & 5 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 2 & 7 \end{bmatrix}$$

Matrix determinant

$$\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

$$\det\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(alternate signs)

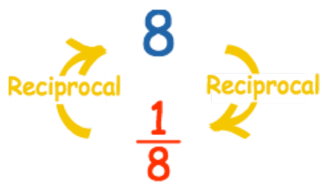
A must be square

Example: $\det\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} = ?$

$$\begin{vmatrix} 1 & 2 & 1 \\ 3 & 0 & 4 \\ 8 & 4 & 10 \end{vmatrix} = ?$$

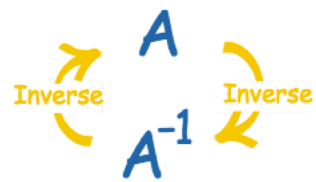
Matrix inverse

- Reciprocal



$$8 \times (^1/_8) = \mathbf{1}$$

- Inverse of a matrix



$$A \times A^{-1} = \mathbf{I}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity Matrix

When the inverse comes first:

$$(^1/_8) \times 8 = \mathbf{1}$$

$$A^{-1} \times A = \mathbf{I}$$

Matrix inverse: compute

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\underbrace{ad-bc}_{\text{determinant}}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

1. **Swap** the positions of a and d
2. Put **negatives** in front of b and c
3. **Divide** everything by the [determinant]

Matrix inverse: always exist?

- A must be square

$$\begin{bmatrix} 6 & 1 & 3 \\ 2 & 5 & 8 \end{bmatrix}$$

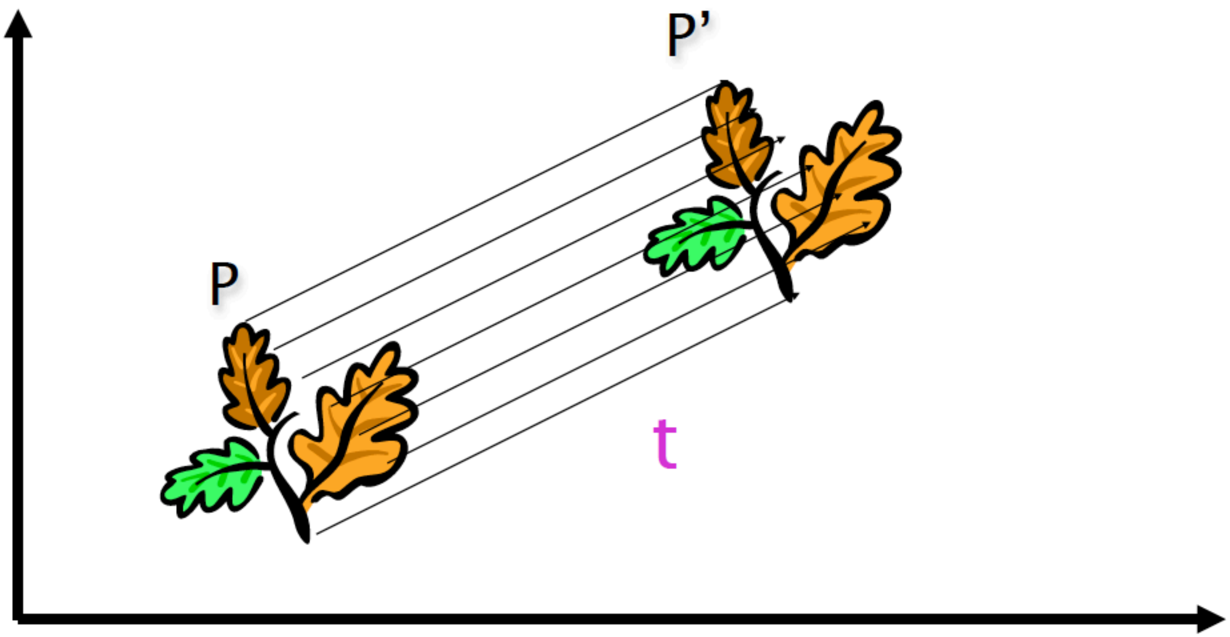
Matrix inverse: always exist?

- A must be square
- A must be non-singular

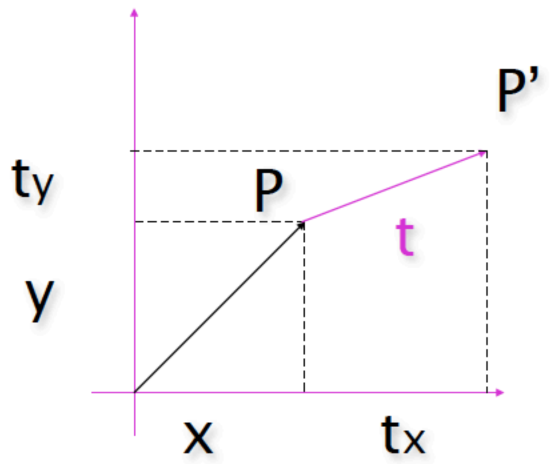
$$\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}^{-1}$$

2D Geometric Transformations

2D Translation



2D Translation Equation

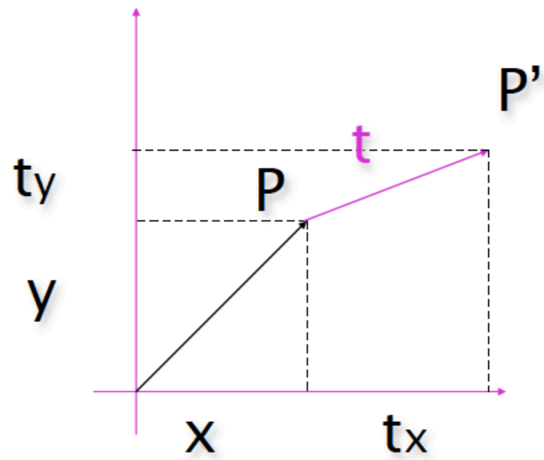


$$\mathbf{P} = (x, y)$$

$$\mathbf{t} = (t_x, t_y)$$

$$\mathbf{P}' = \mathbf{P} + \mathbf{t} = (x + t_x, y + t_y)$$

2D Translation using Matrices



$$\mathbf{P} = (x, y)$$

$$\mathbf{t} = (t_x, t_y)$$

$$\mathbf{P'} \rightarrow \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Not square

What is the inverse transformation?

2D Translation using Matrices

- Homogeneous coordinates
  - Multiply the coordinates by a non-zero scalar and add an extra coordinate equal to that scalar

$$(x, y) \rightarrow (x \cdot z, y \cdot z, z) \quad z \neq 0$$

$$(x, y, z) \rightarrow (x \cdot w, y \cdot w, z \cdot w, w) \quad w \neq 0$$

2D Translation using Matrices

- Homogeneous coordinates
  - Multiply the coordinates by a non-zero scalar and add an extra coordinate equal to that scalar

$$(x, y) \rightarrow (x \cdot z, y \cdot z, z) \quad z \neq 0$$

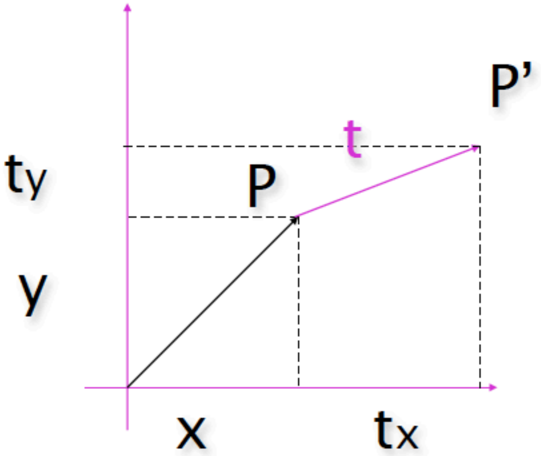
$$(x, y, z) \rightarrow (x \cdot w, y \cdot w, z \cdot w, w) \quad w \neq 0$$

- Back to Cartesian coordinates
  - Divide by the last coordinate and eliminate it

$$(x, y, z) \quad z \neq 0 \rightarrow (x / z, y / z)$$

$$(x, y, z, w) \quad w \neq 0 \rightarrow (x / w, y / w, z / w)$$

2D Translation using Homogeneous Coordinates



$$\mathbf{P} = (x, y) \rightarrow (x, y, 1)$$

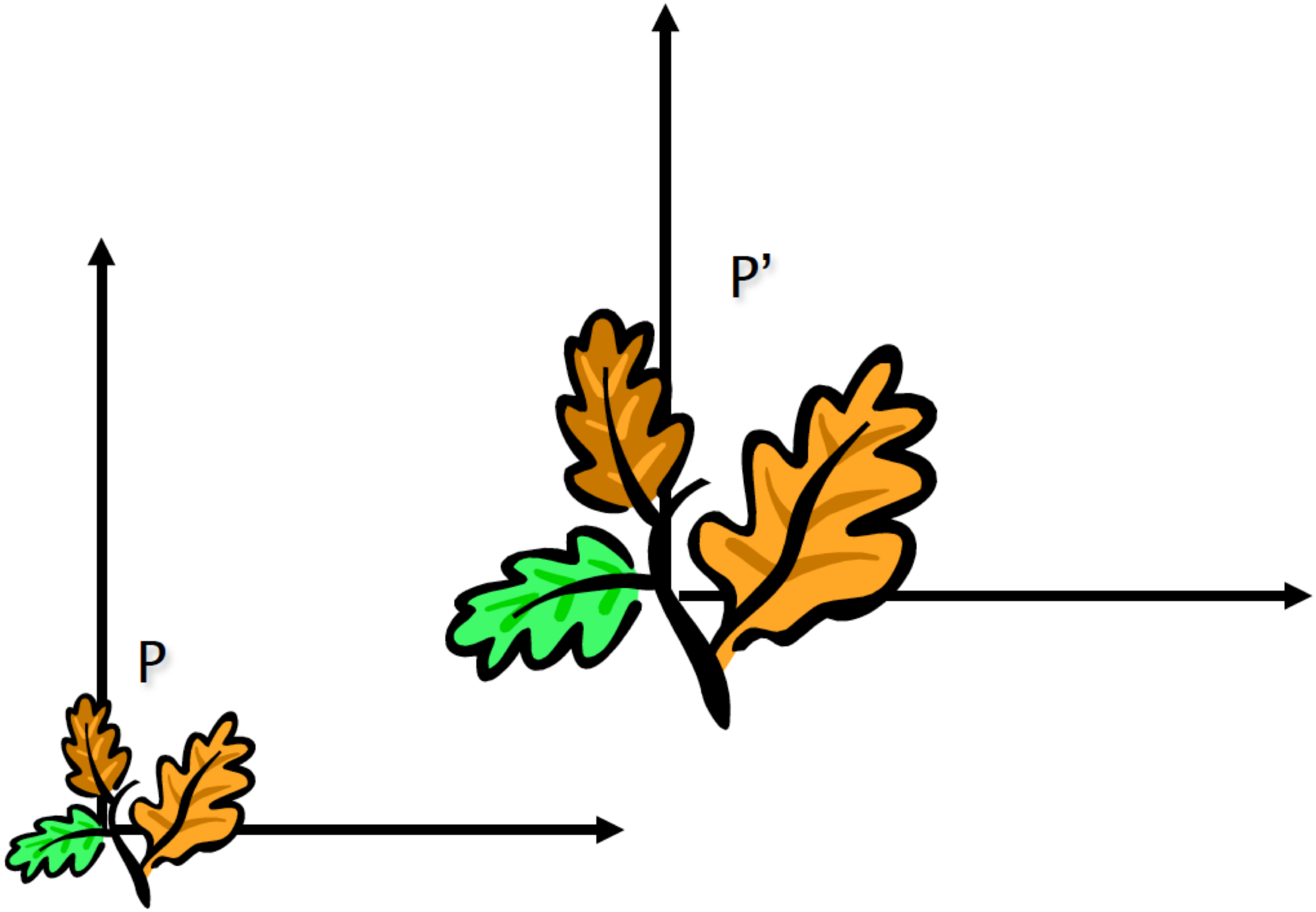
$$\mathbf{t} = (t_x, t_y) \rightarrow (t_x, t_y, 1)$$

t P

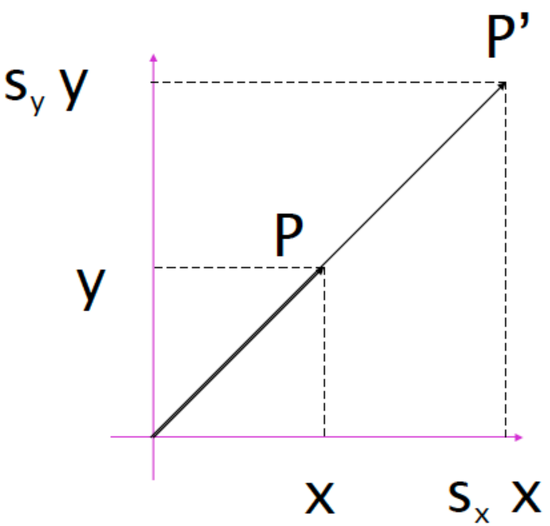
$$\mathbf{P'} \rightarrow \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{I} & \mathbf{t} \\ 0 & 1 \end{bmatrix} \cdot \mathbf{P} = \mathbf{T} \cdot \mathbf{P}$$

Scaling



Scaling Equation



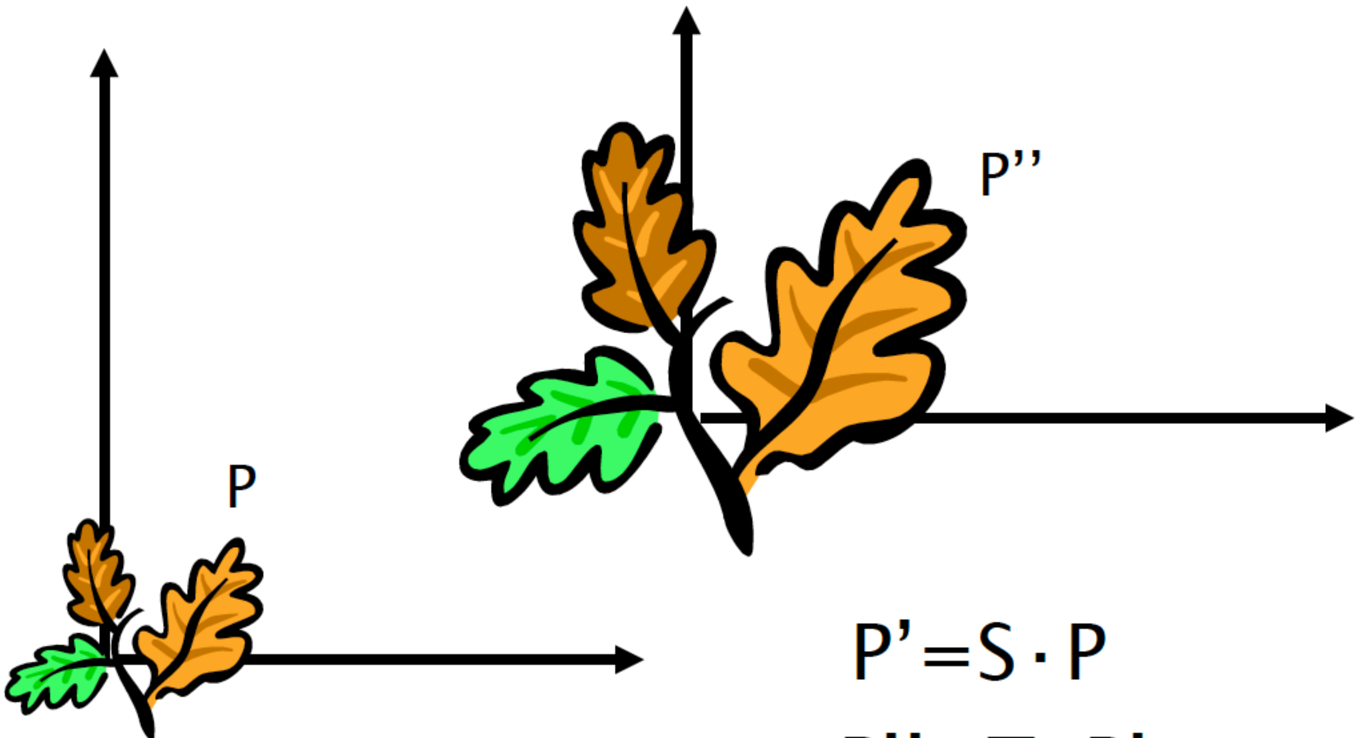
$$\mathbf{P} = (x, y) \rightarrow \mathbf{P}' = (s_x x, s_y y)$$

$$\mathbf{P} = (x, y) \rightarrow (x, y, 1)$$

$$\mathbf{P}' = (s_x x, s_y y) \rightarrow (s_x x, s_y y, 1)$$

$$\mathbf{P}' \rightarrow \begin{bmatrix} s_x x \\ s_y y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{S}} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{S}' & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \cdot \mathbf{P} = \mathbf{S} \cdot \mathbf{P}$$

Scaling & Translation



$$P' = S \cdot P$$

$$P'' = T \cdot P'$$

$$P'' = T \cdot P' = T \cdot (S \cdot P) = (T \cdot S) \cdot P = A \cdot P$$

Scaling & Translation

$$\mathbf{P''} = \mathbf{T} \cdot \mathbf{S} \cdot \mathbf{P} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} =$$

$$= \underbrace{\begin{bmatrix} \quad ? \quad \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scaling & Translation

$$\mathbf{P''} = \mathbf{T} \cdot \mathbf{S} \cdot \mathbf{P} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} =$$

$$= \underbrace{\begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

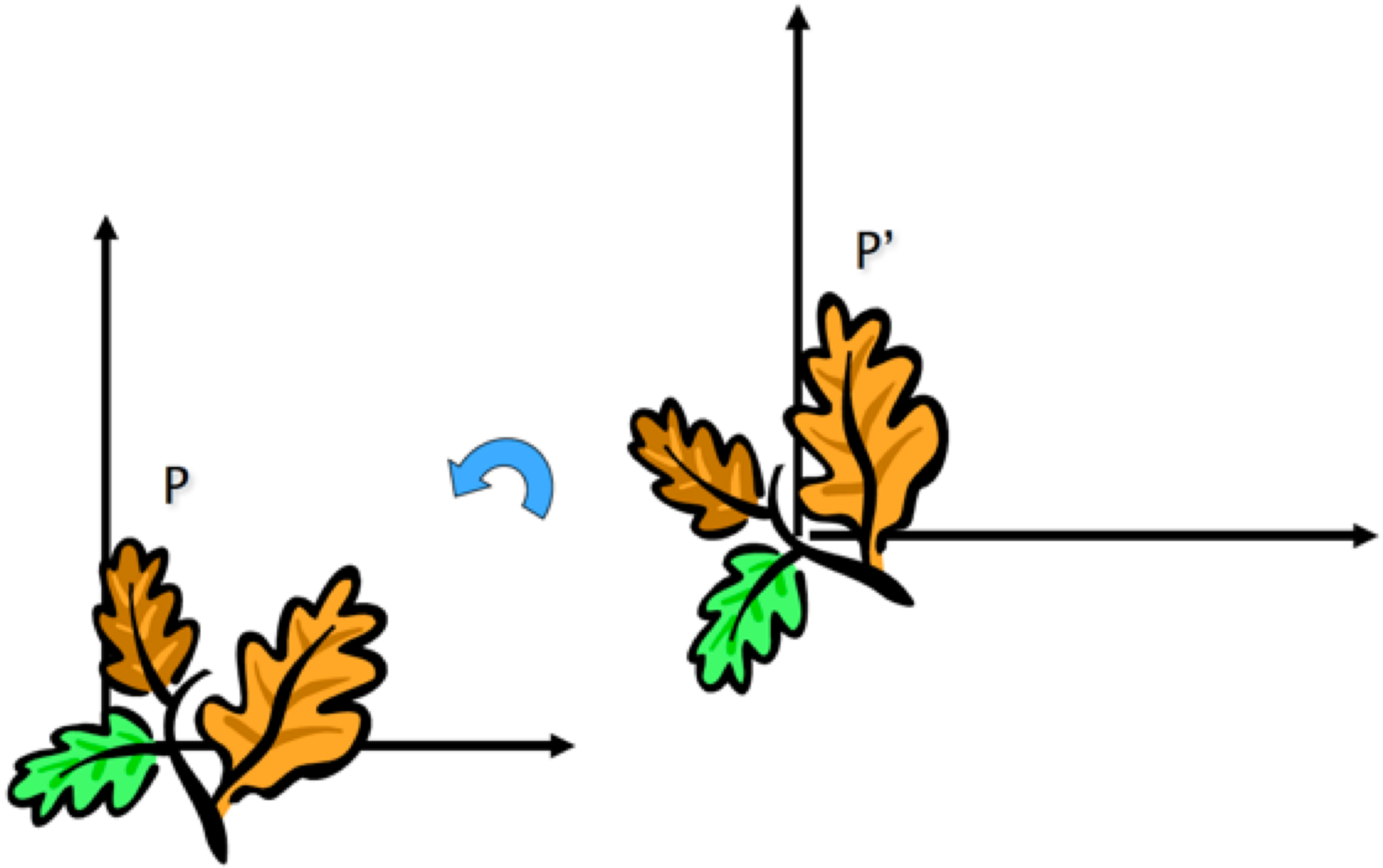
Translation & Scaling = Scaling & Translation ?

$$\mathbf{P'''} = \mathbf{T} \cdot \mathbf{S} \cdot \mathbf{P} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x x + t_x \\ s_y y + t_y \\ 1 \end{bmatrix}$$

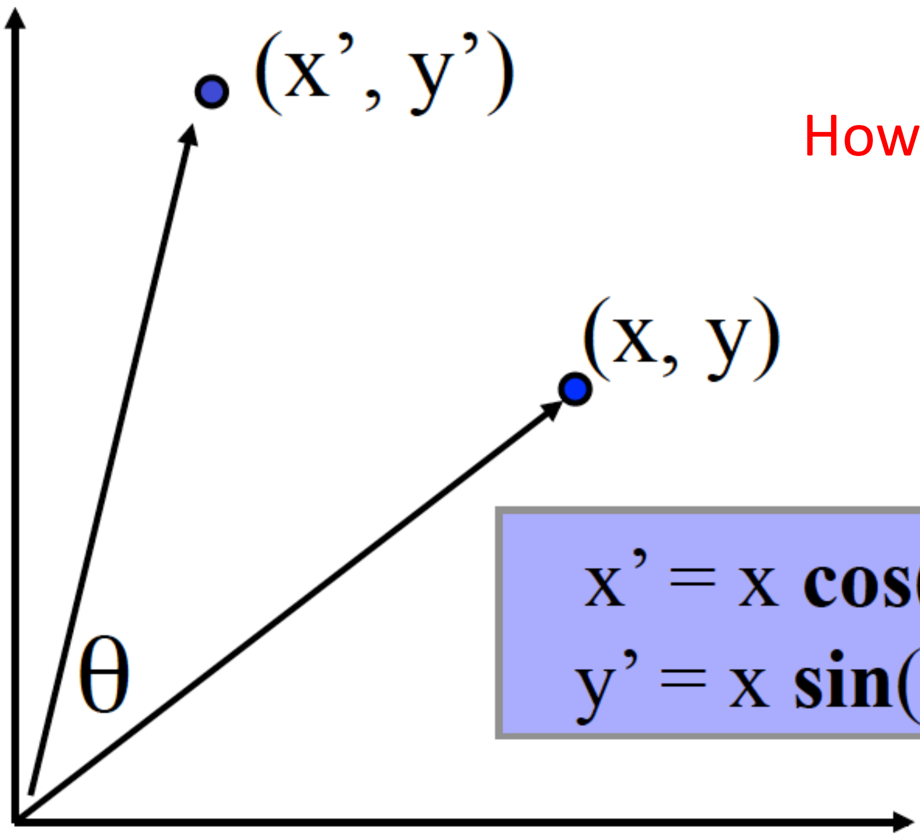
$$\mathbf{P'''} = \mathbf{S} \cdot \mathbf{T} \cdot \mathbf{P} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} s_x & 0 & s_x t_x \\ 0 & s_y & s_y t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x x + s_x t_x \\ s_y y + s_y t_y \\ 1 \end{bmatrix}$$

Rotation



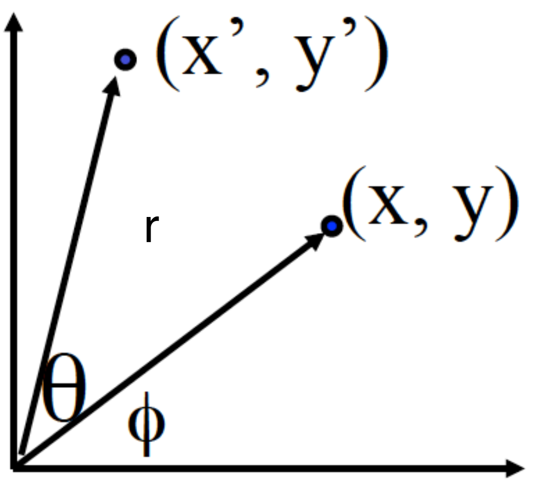
Rotation Equations



How to derive this equation?

$$x' = x\,\mathbf{cos}(\theta) - y\,\mathbf{sin}(\theta)$$
$$y' = x\,\mathbf{sin}(\theta) + y\,\mathbf{cos}(\theta)$$

Rotation Equations



$$x = r \cos(\phi)$$
$$y = r \sin(\phi)$$
$$x' = r \cos(\phi + \theta)$$
$$y' = r \sin(\phi + \theta)$$

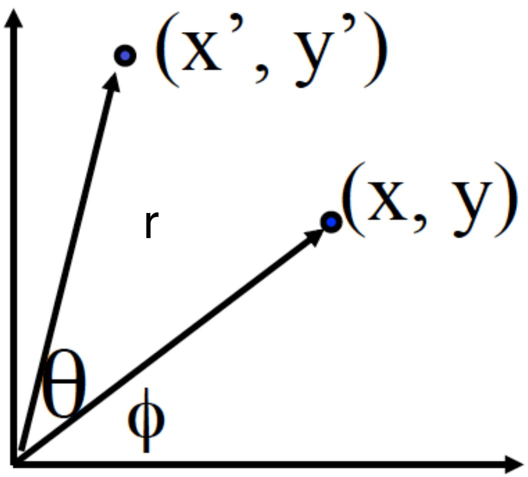
Expand

$$x' = r \cos(\phi)\cos(\theta) - r \sin(\phi)\sin(\theta)$$
$$y' = r \sin(\phi)\cos(\theta) + r \cos(\phi)\sin(\theta)$$

Substitute

$$x' = x \cos(\theta) - y \sin(\theta)$$
$$y' = x \sin(\theta) + y \cos(\theta)$$

Rotation Equations



This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

What is the inverse transformation ?

- Rotation by –θ

R has many interesting properties:

$$\mathbf{R}^{-1} = \mathbf{R}^{T} \qquad \mathbf{R} \cdot \mathbf{R}^{\mathrm{T}} = \mathbf{R}^{\mathrm{T}} \cdot \mathbf{R} = \mathbf{I} \qquad \det(\mathbf{R}) = 1$$

Translation + Rotation + Scaling

$$\mathbf{P}' = \mathbf{T}\cdot \mathrm{R}\cdot \mathbf{S}\cdot \mathbf{P} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} R' & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} R'S & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

If $s_x = s_y$, this is a similarity transformation

Assignment 1

Matrix/vector arithmetic

Next Lecture

Part 2: Linear systems, linear least squares