

# Ready to follow GEO5017?

## Self-assessment of math fundamentals

### Calculus

1. What is the derivative of function  $f(x) = x^3 + 3 \sin(x)$ ?
2. What is the derivative of function  $\sigma(x) = \frac{1}{1+e^{-x}}$ ?
3. What are the partial derivatives of  $f(x, y) = 2x^2 \cos(y^3)$ ?
4. Let  $z = f(x, y)$ , where  $x = g(r, s)$  and  $y = h(r, s)$ , so that  $z$  is a function of  $r$  and  $s$ . Then what are the partial derivatives of  $z$  with respect to  $r$  and  $s$ ?
5. Is the function  $f(x) = |x|$  continue at every point? Is it differentiable at every point?
6. Is the function  $f(x) = x^2 - 3x + 2$  continue at every point? Is it differentiable at every point?
7. What is the gradient of function  $f(x, y) = 2x^3 + 3xy^2$ ?
8. Is the function  $y = (x - 4)(x + 1)(x - 8)$  monotonic increasing? Strictly increasing?
9. Given a function  $f(x)$  differential every point, what are the sufficient conditions for  $f(x)$  to have a global minimum value?

### Linear Algebra

1. Solve the following system of linear equations for  $x$ ,  $y$ , and  $z$ :

$$\begin{aligned}2x + 3y - z &= 7 \\4x - 2y + 3z &= 12 \\-x + 5y + 2z &= -5\end{aligned}$$

2. Find an upper triangular matrix  $A$  such that  $A^3 = \begin{bmatrix} 8 & -57 \\ 0 & 27 \end{bmatrix}$
3. Prove that the following matrices are orthogonal:  $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ ,  $\begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix}$
4. Are the vectors  $\mathbf{x} = (1, 2)$  and  $\mathbf{y} = (2, 3)$  linearly independent? What does it mean (geometrically) that two vectors are linearly dependent?

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5. Express  $v = (1, -2, 5)$  in  $R^3$  as a linear combination of following vectors:

$$u_1 = (1, -3, 2), u_2 = (2, -4, 1), u_3 = (1, -5, 7)$$

6. What does it mean that a matrix is invertible?

7. Suppose a square matrix has a null (0) determinant. What can you conclude?

8. Let  $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ . Find  $A^{-1} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$  and  $\det(A)$

9. What are the eigenvalues and eigenvectors? Calculate eigenvalues and eigenvectors

of a 3 by 3 matrix  $\begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$

### Statistics

1. You roll two dice. What is the probability that the sum is 4?

2. A coin is weighted in a way such that the head ( $H$ ) is twice as likely to appear as the tail ( $T$ ). Find  $P(T)$  and  $P(H)$ .

3. A die is weighted so that the outcomes produce the following probability distribution:

Outcome	1	2	3	4	5	6
Probability	0.1	0.3	0.2	0.1	0.1	0.2

Consider events  $A = \{ \text{even number} \}$ ,  $B = \{2, 3, 4, 5\}$ ,  $C = \{x : x < 3\}$ , and  $D = \{x : x > 7\}$ , find the following probabilities:  $P(A)$ ,  $P(B)$ ,  $P(C)$ , and  $P(D)$ .

4. What does “two random events A and B are independent” mean?

5. What are mutually exclusive (disjoint) events?

6. What does  $P(A|B) = 0.6$  mean?

7. A bag contains 6 black marbles and 4 blue marbles. Another bag contains 3 black marbles and 7 blue marbles. Suppose you randomly pick one of the bags with equal probability; from that bag, you randomly select a marble. What is the probability that you choose a blue marble?

8. In a certain city, 40 percent of the people consider themselves Conservatives ( $C$ ), 35 percent consider themselves to be Liberals ( $L$ ), and 25 percent consider themselves to be Independents ( $I$ ). During a particular election, 45 percent of the Conservatives voted, 40 percent of the Liberals voted, and 60 percent of the Independents voted. If a randomly selected person voted, find the probability that the voter is (a) Conservative, (b) Liberal, (c) Independent.

9. A fair coin is tossed 6 times. This is a binomial experiment with  $n = 6$  and  $p = q = \frac{1}{2}$ . Calculate the probability that the head occurs exactly twice (i.e.,  $k = 2$ ).