TUDelft
3Dgeoinfo

Lecture
Surface Reconstruction
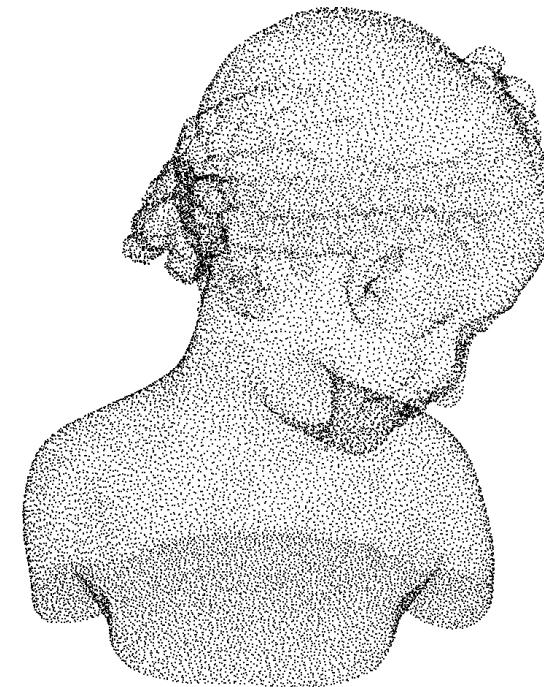
Liangliang Nan

Today's Agenda

- Introduction

- Smooth object reconstruction
  - The pioneer work of Hoppe *et al.* (1992)
  - Poisson reconstruction [Kazhdan *et al.* 2006]
  - Piecewise smooth reconstruction

- Piecewise planar object reconstruction [Nan and Wonka. 2017]

Introduction

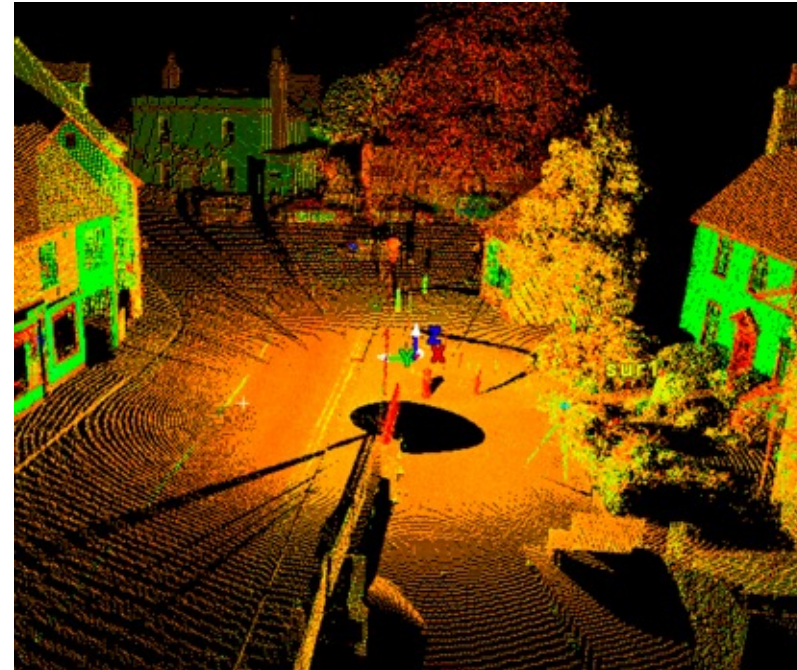- Data sources
  - Laser scanning with a turntable

Introduction

- Data sources
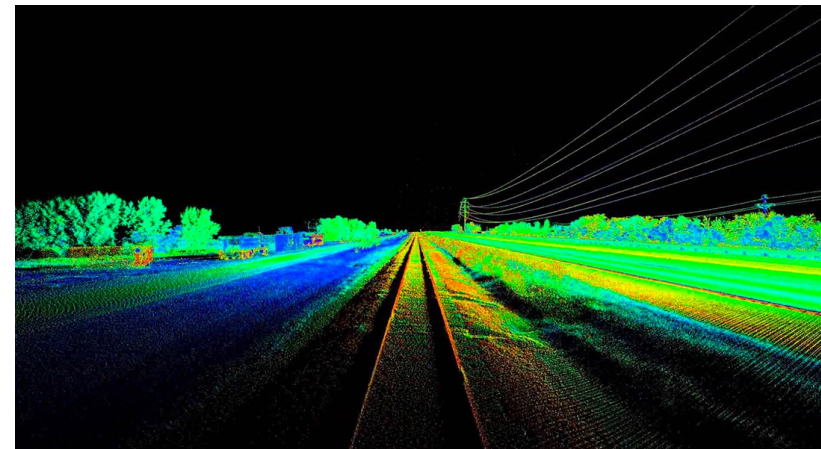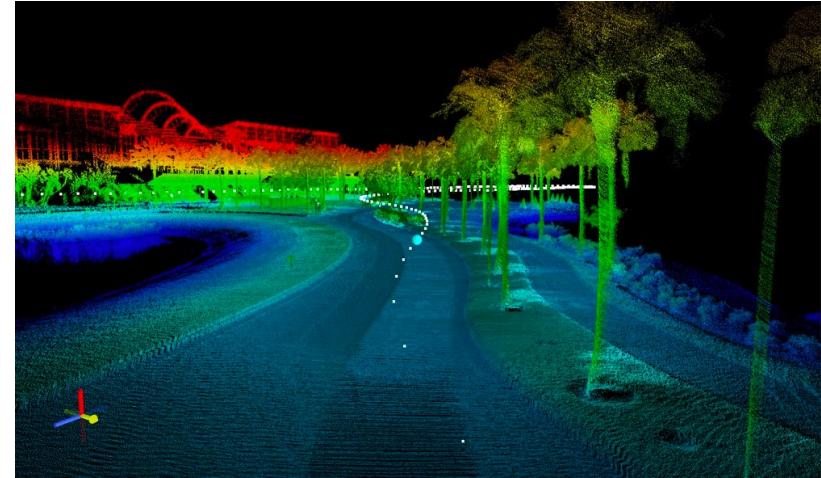 - Laser scanning with a hand-held scanner

Introduction

- Data sources
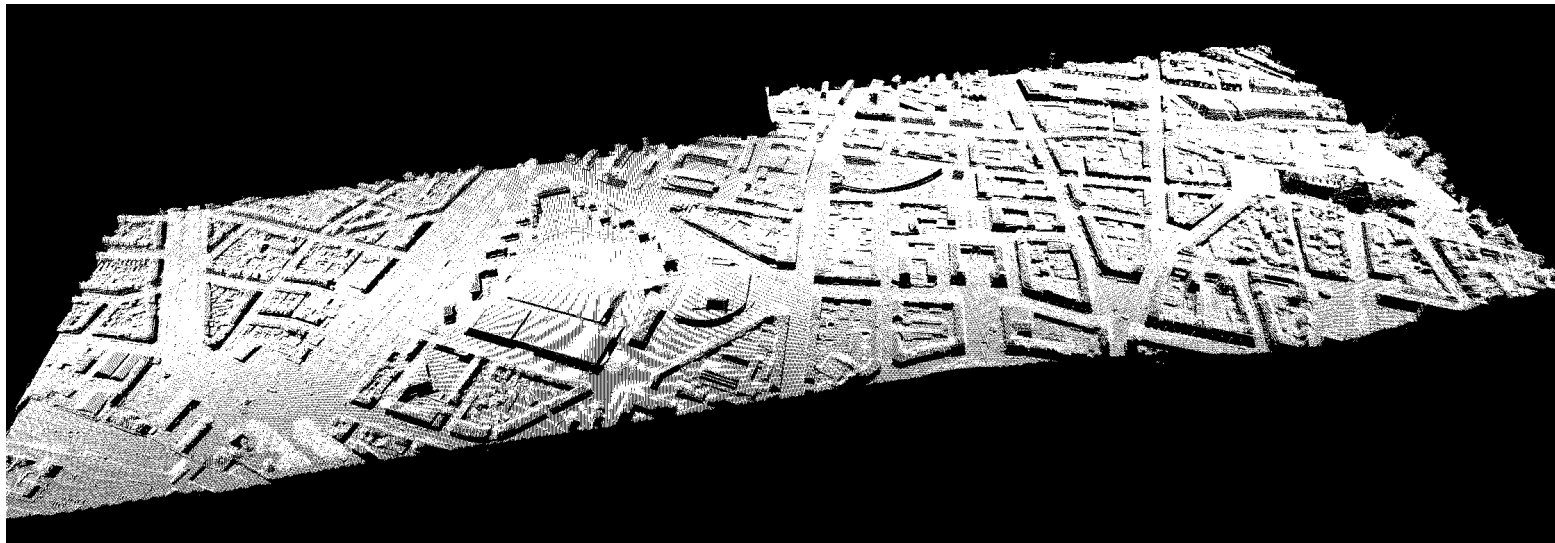 - Laser scanning with static laser scanner (range of 100, 200… meters)

Introduction

- Data sources
  - Laser scanning – mobile scanners

Introduction

- ## Data sources
 - Laser scanning – airborne LiDAR

Introduction

- Data sources
  - Laser scanning
 - Structure from Motion (SfM) and Multi-view stereo (MVS)

Introduction

- ## Surface reconstruction
  - – Input: point set *P* sampled over a surface *S*
    - Non-uniform sampling
    - With holes
    - With uncertainty (noise)
  - – Output: surface approximating *S* in terms of topology and geometry
    - Desired
      - – Watertight
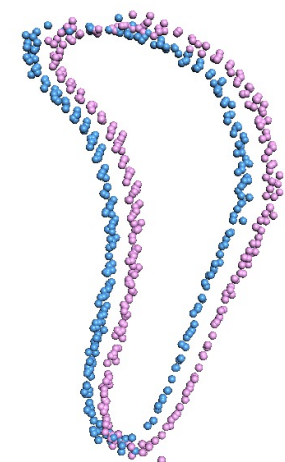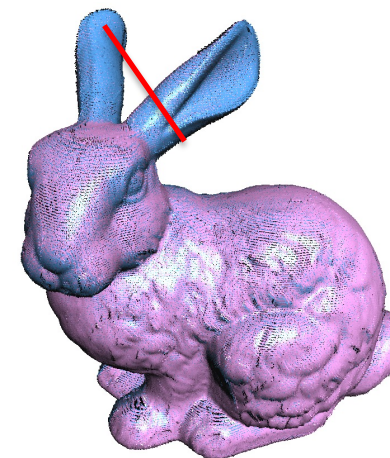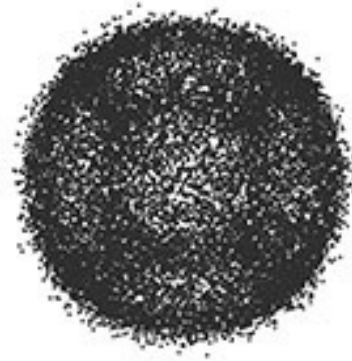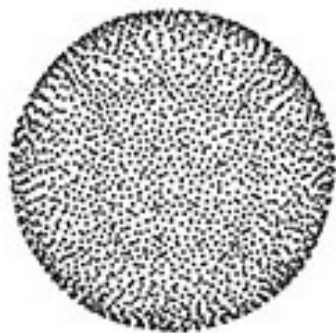   - – Intersection free
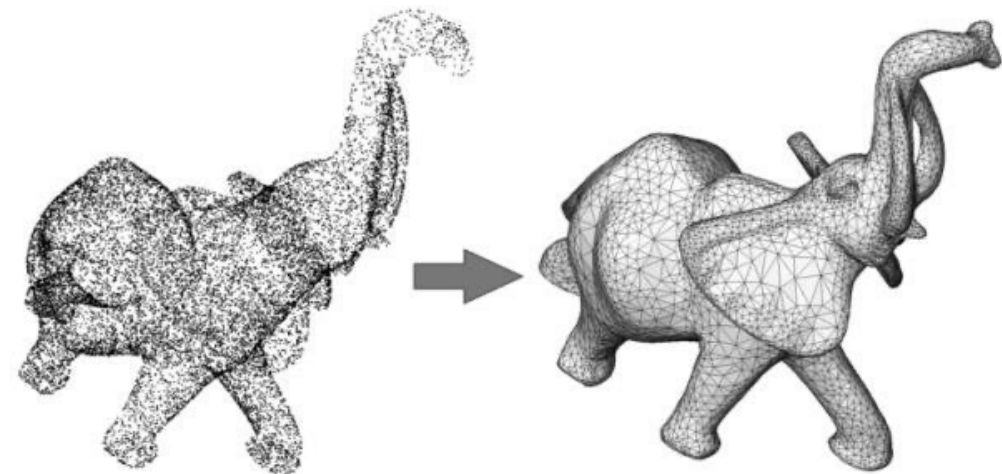
Introduction

- Challenges
  - The point samples may not be uniformly distributed
    - Oblique scanning angles
    - Laser energy attenuation
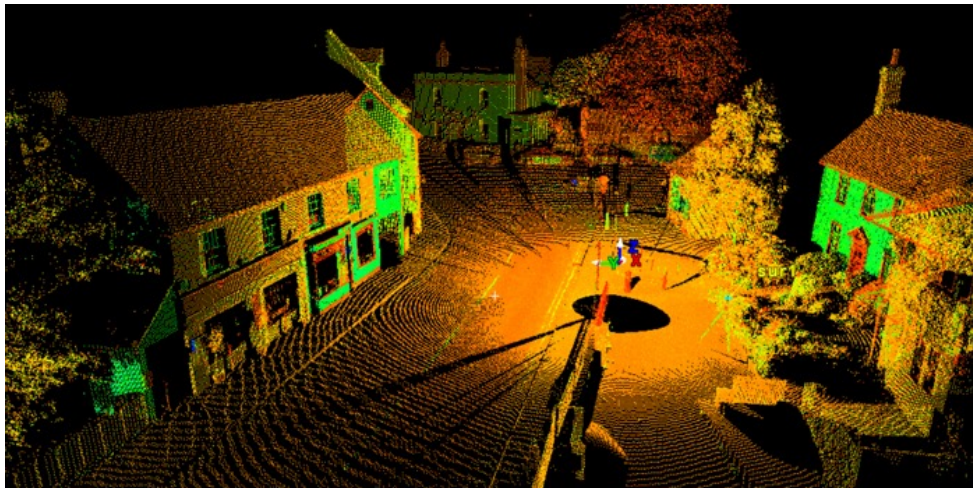
Introduction

- Challenges
  - The point samples may not be uniformly distributed
  - The positions and normals are generally noisy
    - Sampling inaccuracy
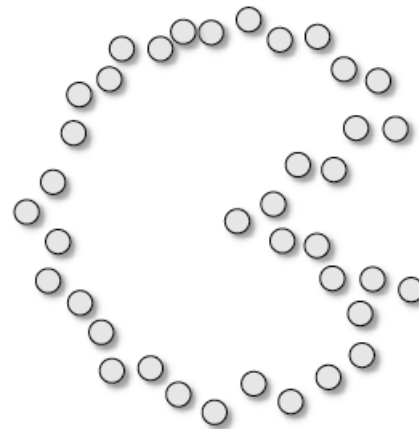   - Scan misregistration

Introduction

- Challenges
  - The point samples may not be uniformly distributed
  - The positions and normals are generally noisy
  - Missing data
    - Material properties, inaccessibility, occlusion, etc.

Introduction

- Challenges
  - The point samples may not be uniformly distributed
  - The positions and normals are generally noisy
  - Missing data
  - Ill-posed problem

Many candidate surfaces for the reconstruction problem!

Introduction

- Challenges
  - The point samples may not be uniformly distributed
  - The positions and normals are generally noisy
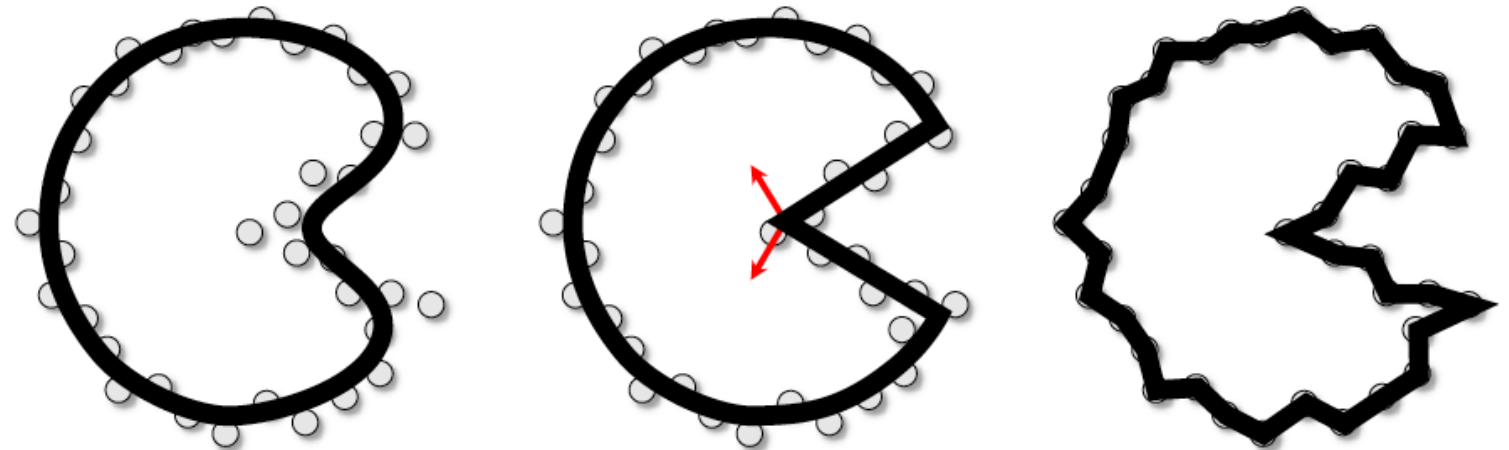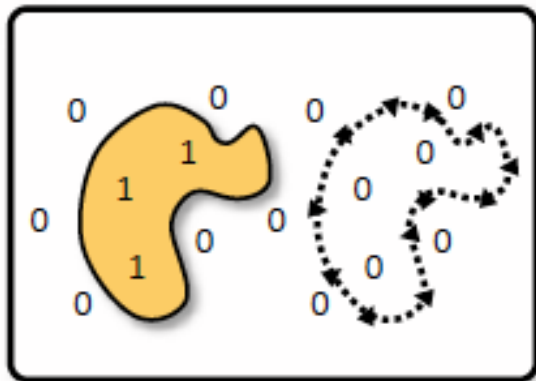 - Missing data
  - Ill-posed problem

How to pick?

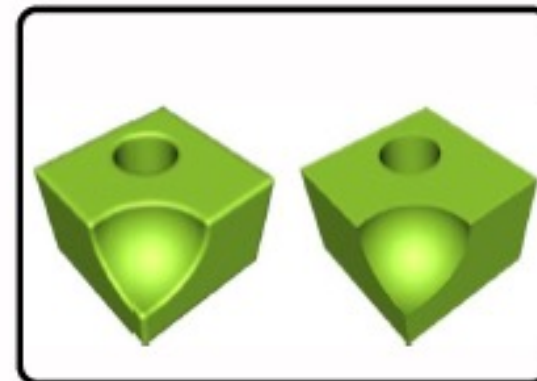Many candidate surfaces for the reconstruction problem!

General Ideas

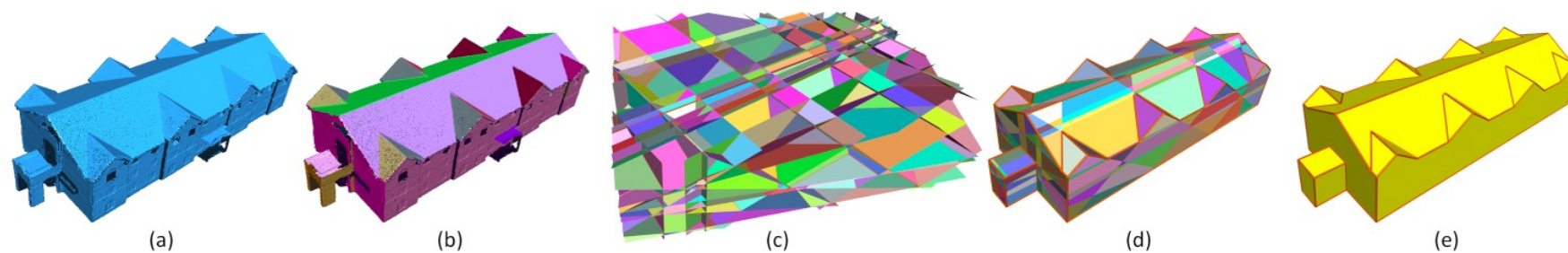- Surface Smoothness Priors

Global Smoothness

Piecewise Smoothness

General Ideas

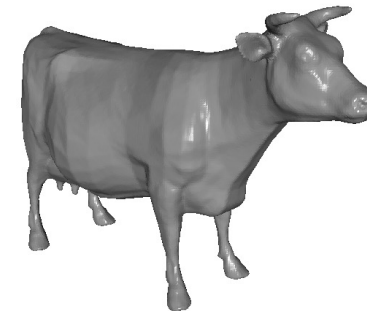- ## Domain-Specific Priors

[Verdie et al, 2015]

(a) (b) (c) (d) (e)
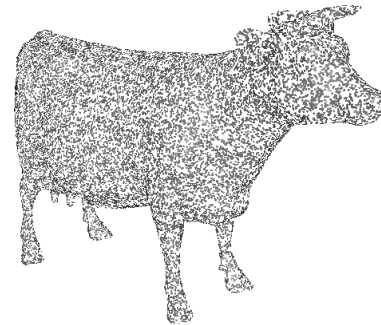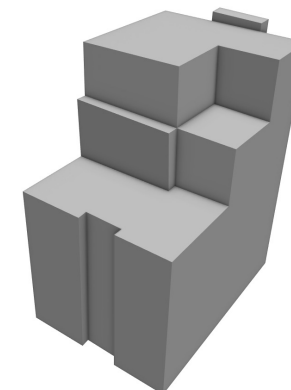
[Nan and Wonka 2017]

Introduction

- Smooth surface reconstruction


- Piecewise-planar object reconstruction

Today's Agenda

- Introduction
- Smooth object reconstruction
  - The pioneer work of Hoppe *et al.* (1992)
  - Poisson reconstruction [Kazhdan *et al.* 06]
  - Piecewise smooth reconstruction
- Piecewise planar object reconstruction [Nan and Wonka. 2017]

The pioneer work of Hoppe *et al.* (1992)

- Two main steps
  - Estimate signed geometric distance to the unknown surface
 - Extract the zero-set of the distance field using a contouring algorithm

Sample points

$F(q)$

< 0   0   > 0

contouring

< 0   0   > 0
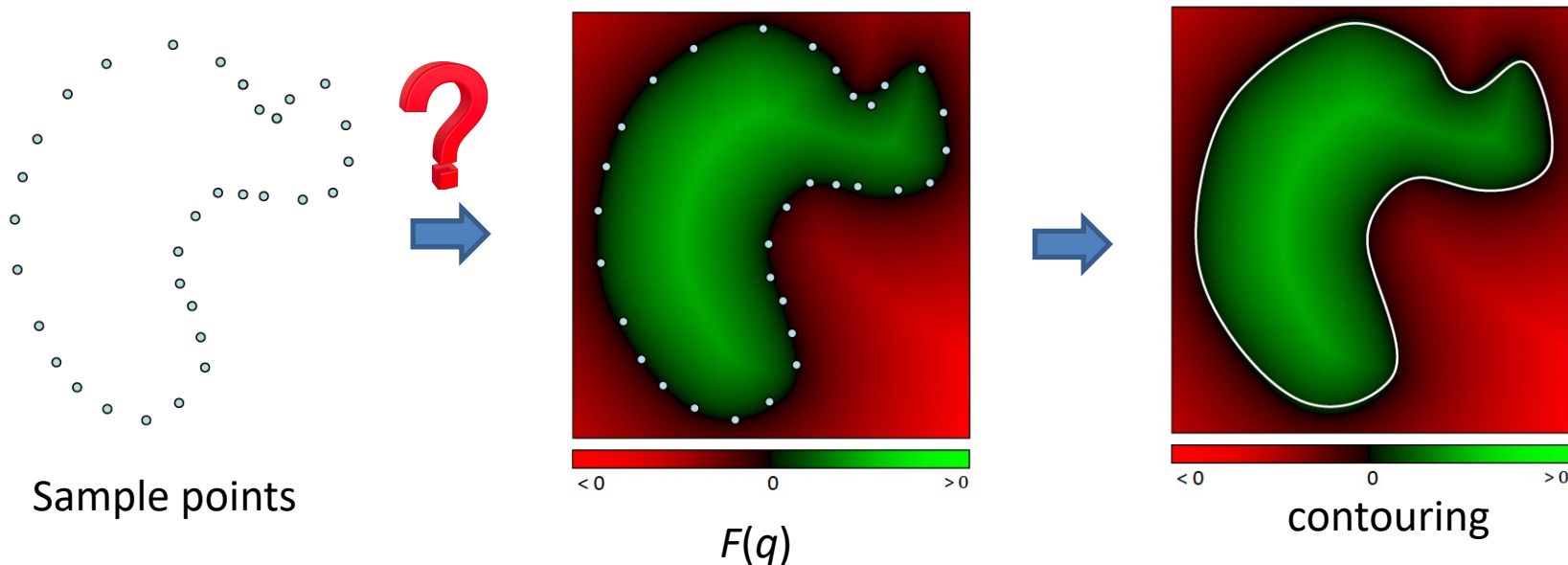
The pioneer work of Hoppe *et al.* (1992)

- ## Define a signed distance function (SDF)
  - Associate an oriented plane (tangent plane) with each of the data points
    - Tangent plane is a local linear approximation to the surface.
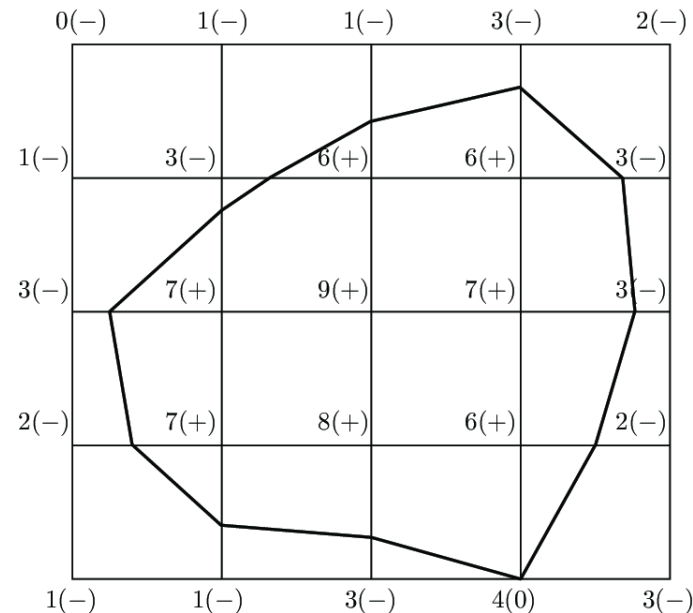   - Used to define signed distance function to surface.
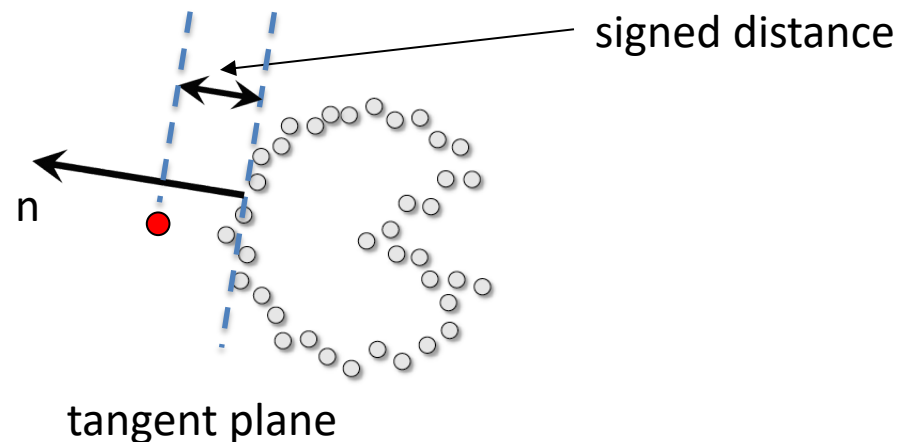
signed distance

n

tangent plane

The pioneer work of Hoppe *et al.* (1992)

- Contour tracing
  - Extract 0-set iso-surface from the scalar field
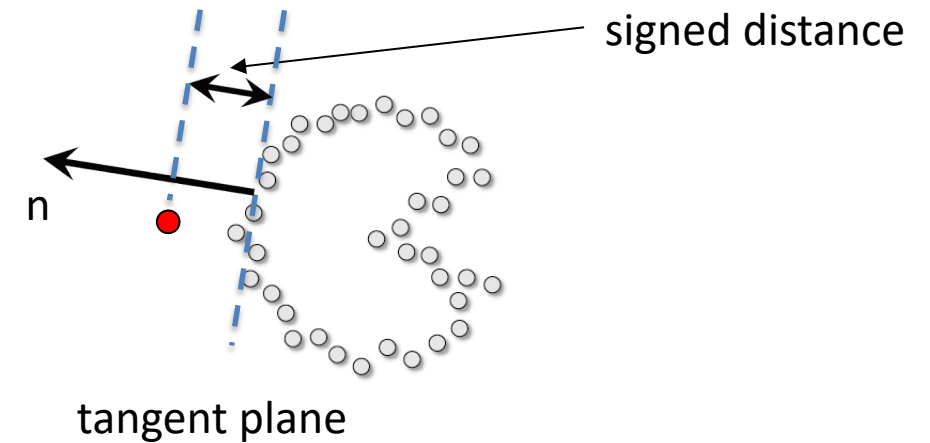   - Marching cubes
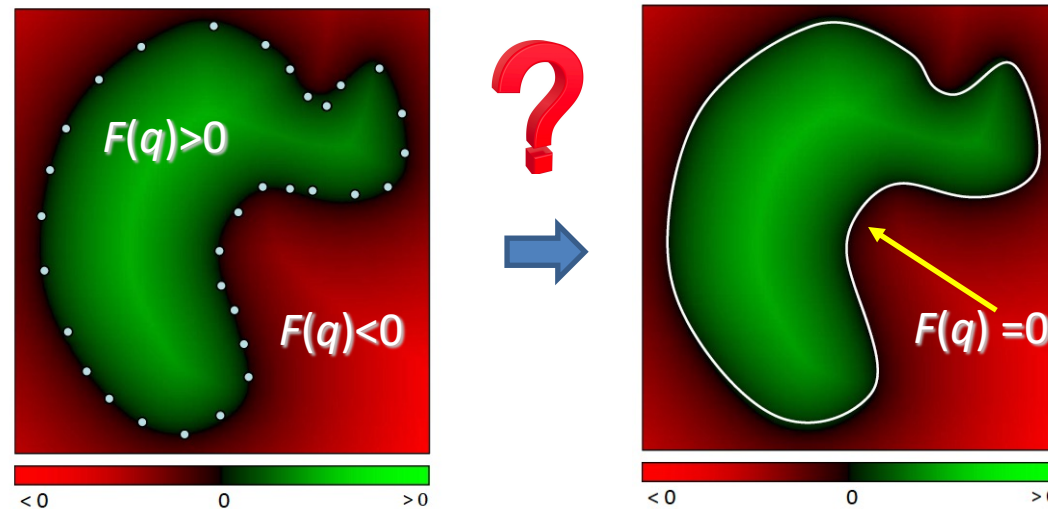
The pioneer work of Hoppe *et al.* (1992)

- ## Contour tracing
  - Extract 0-set iso-surface from the scalar field
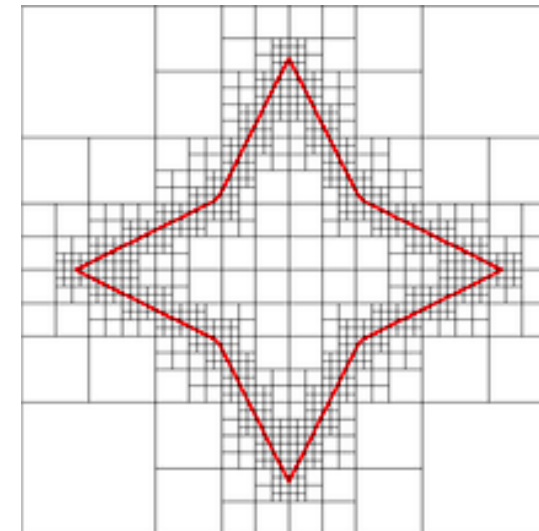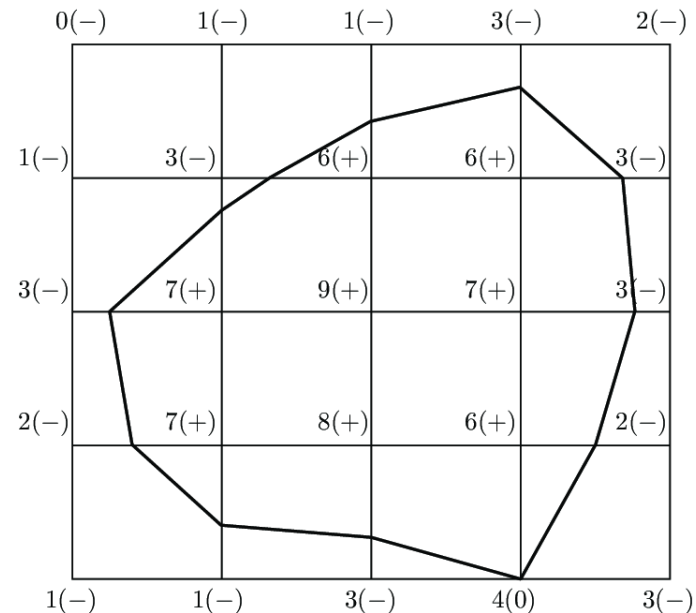   - Marching cubes

Marching squares (2D version of marching cubes)

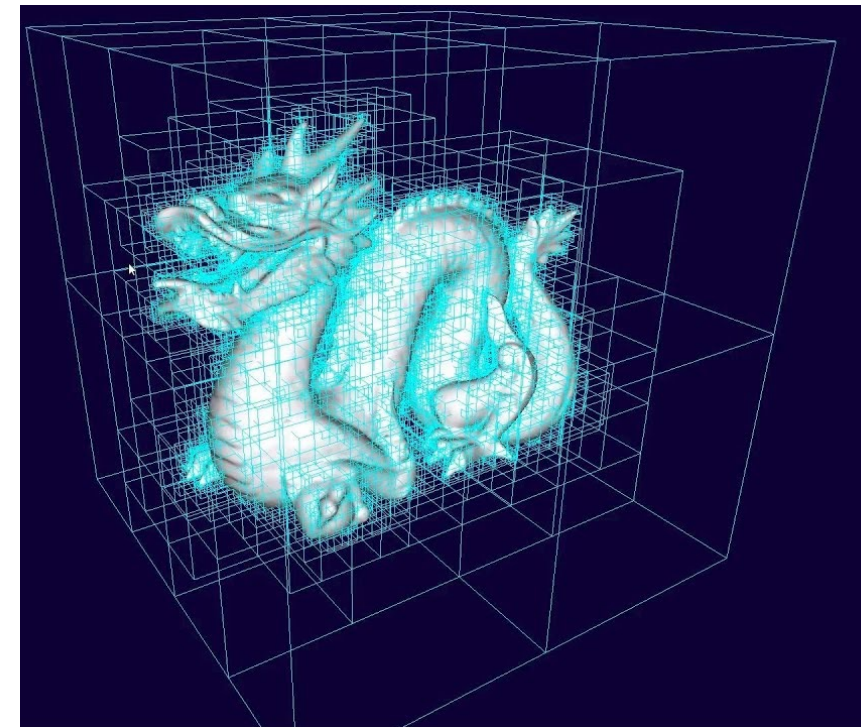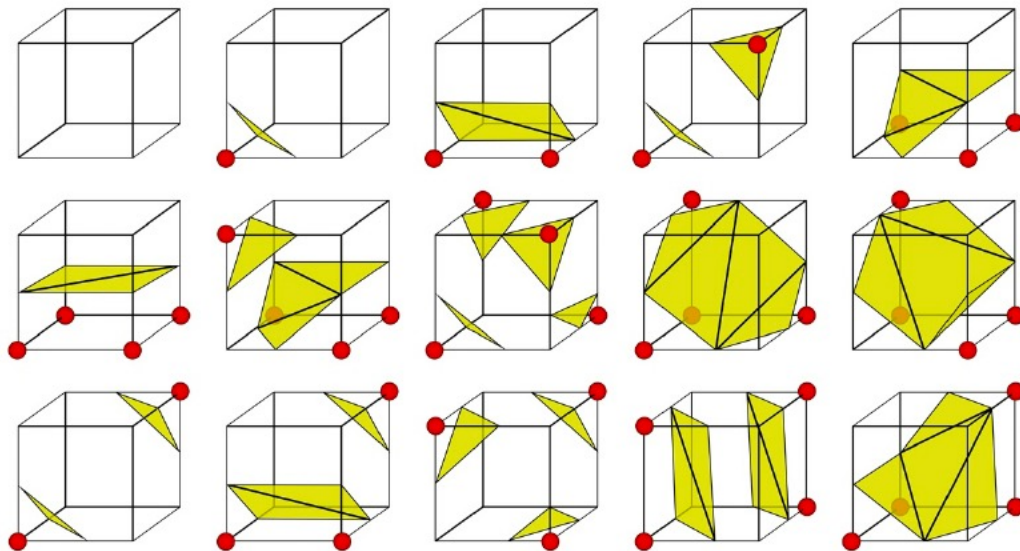# The pioneer work of Hoppe *et al.* (1992)

- Contour tracing
  - Extract 0-set iso-surface from the scalar field
    - Marching cubes
    - Irregular grid

Marching squares (2D version of marching cubes)
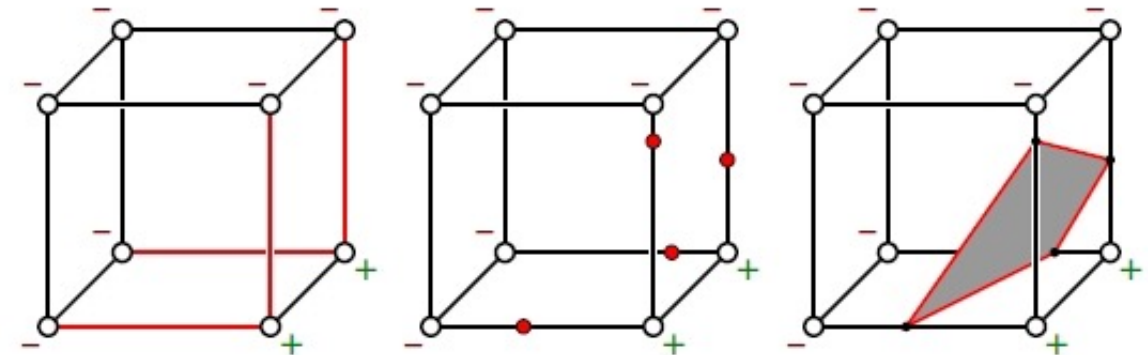
The pioneer work of Hoppe *et al.* (1992)
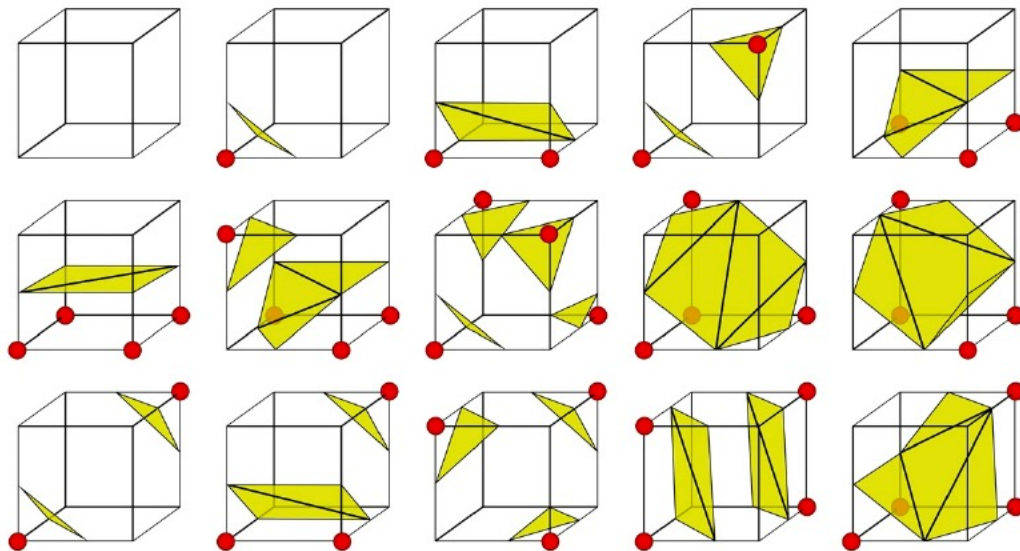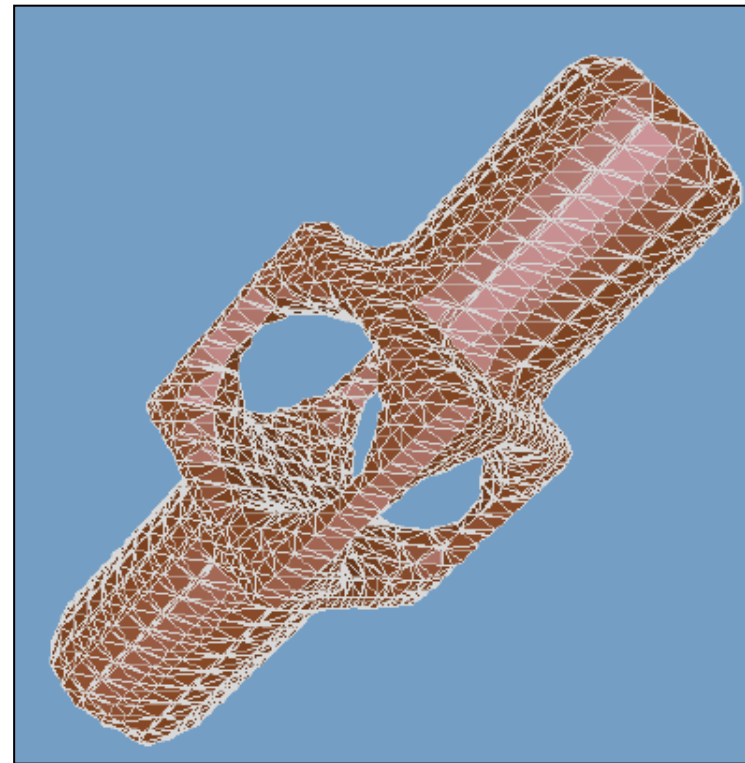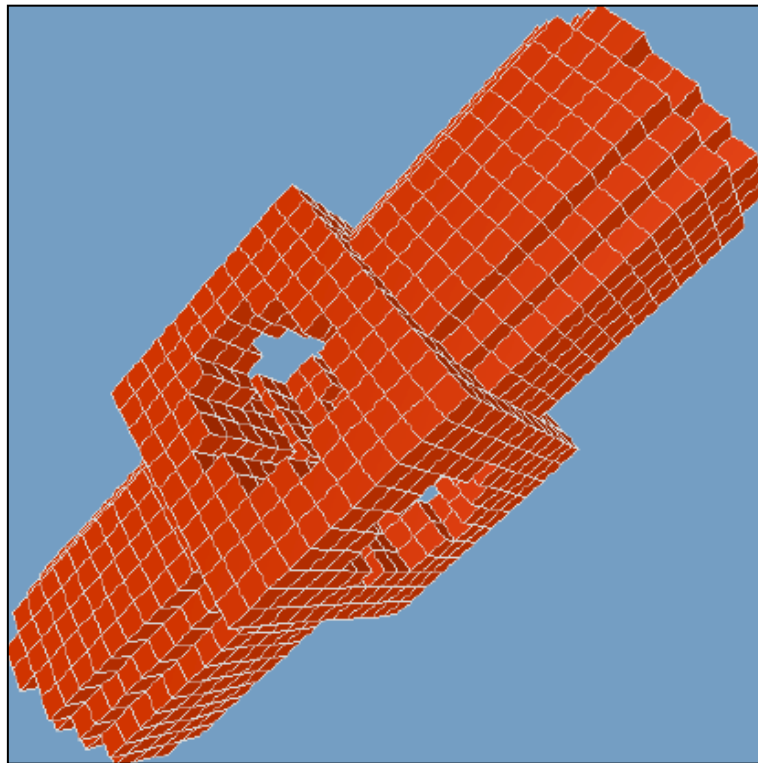
- Contour tracing
  - Extract 0-set iso-surface from the scalar field
  - Marching cubes

Marching cubes

The pioneer work of Hoppe *et al.* (1992)

- Contour tracing
  - Extract 0-set iso-surface from the scalar field
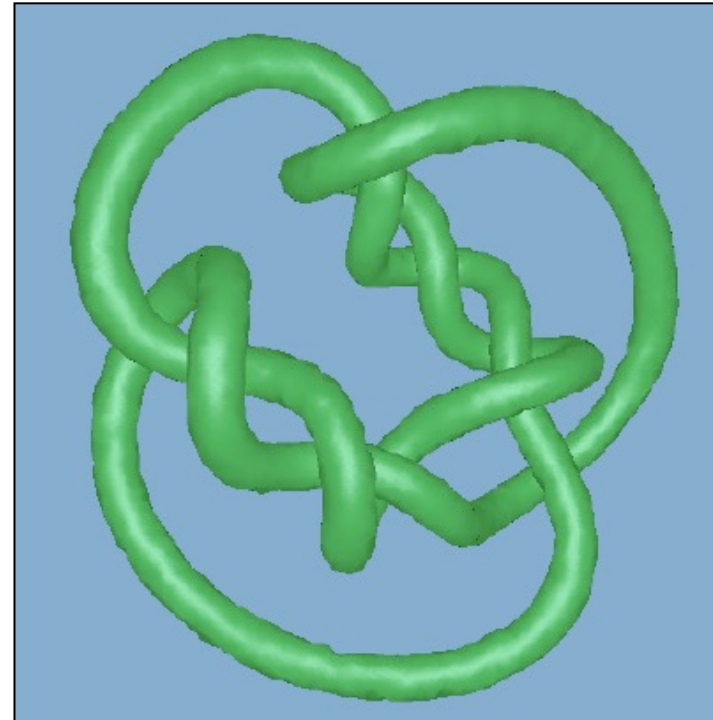 - Marching cubes
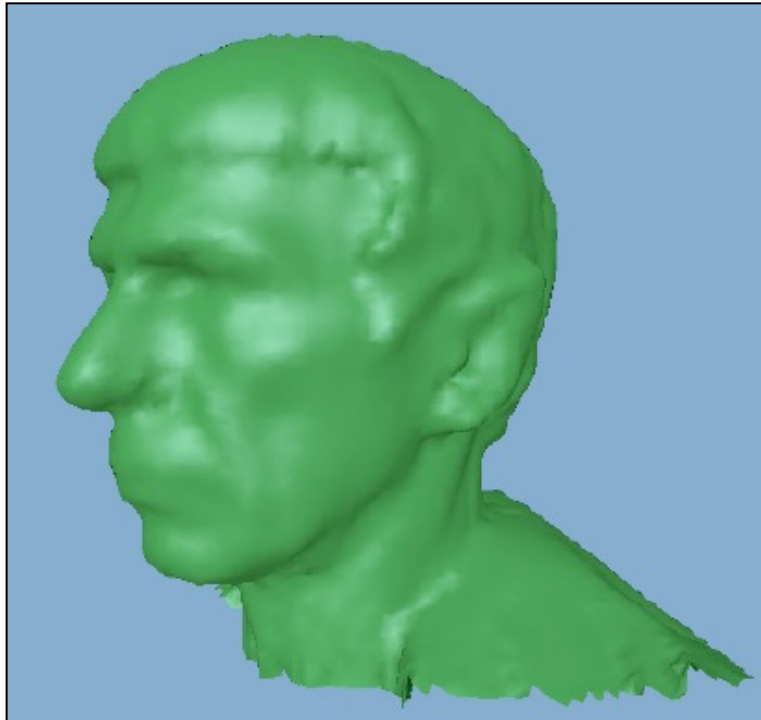

Marching cubes

The pioneer work of Hoppe *et al.* (1992)

- Contour tracing

# The pioneer work of Hoppe *et al.* (1992)

- Reconstruction results

Today's Agenda

- Introduction

- Smooth object reconstruction

  – The pioneer work of Hoppe *et al.* (1992)

  – Poisson reconstruction [Kazhdan *et al.* 06]

  – Piecewise smooth reconstruction

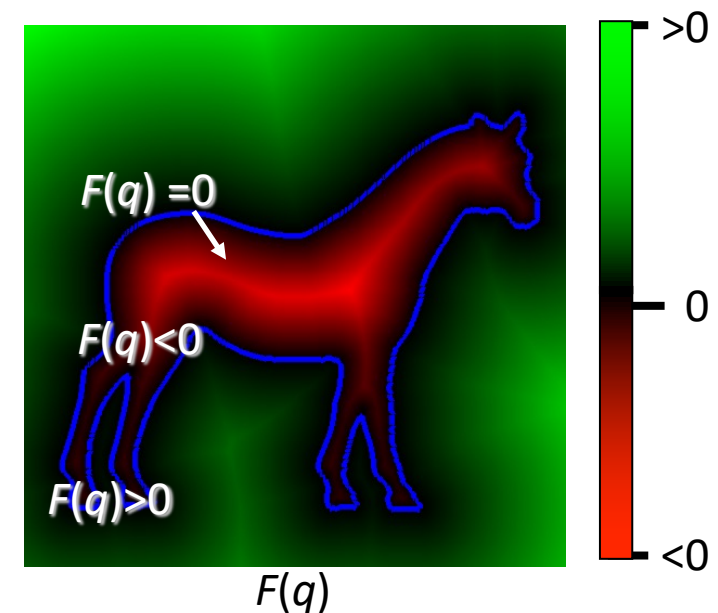- Piecewise planar object reconstruction [Nan and Wonka. 2017]

Poisson Reconstruction

- Inherited idea of from Hoppe 1992

- Discrete SDF -> Implicit function fitting
  - Define a 3D scalar function
    - Zero values at the points
    - Positive values outside
    - Negative values inside
  - Extract the zero isosurface

Sample points

$F(q) = 0$

$F(q) < 0$

$F(q) > 0$

>0

0

<0

$F(q)$
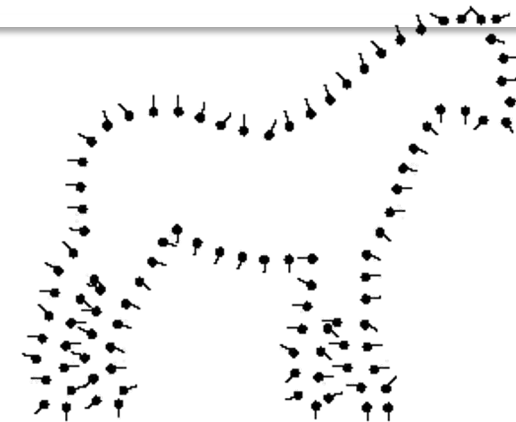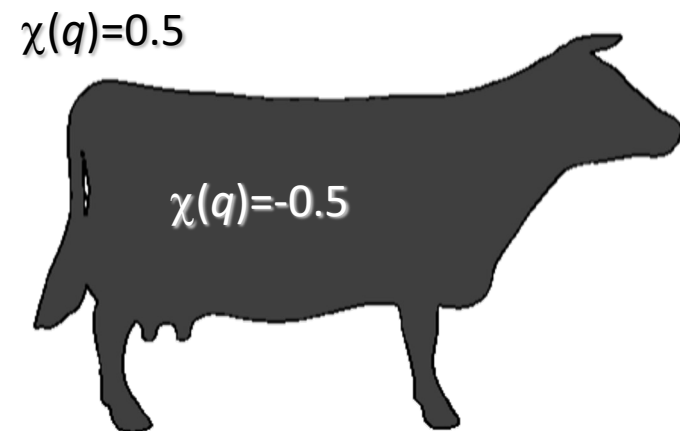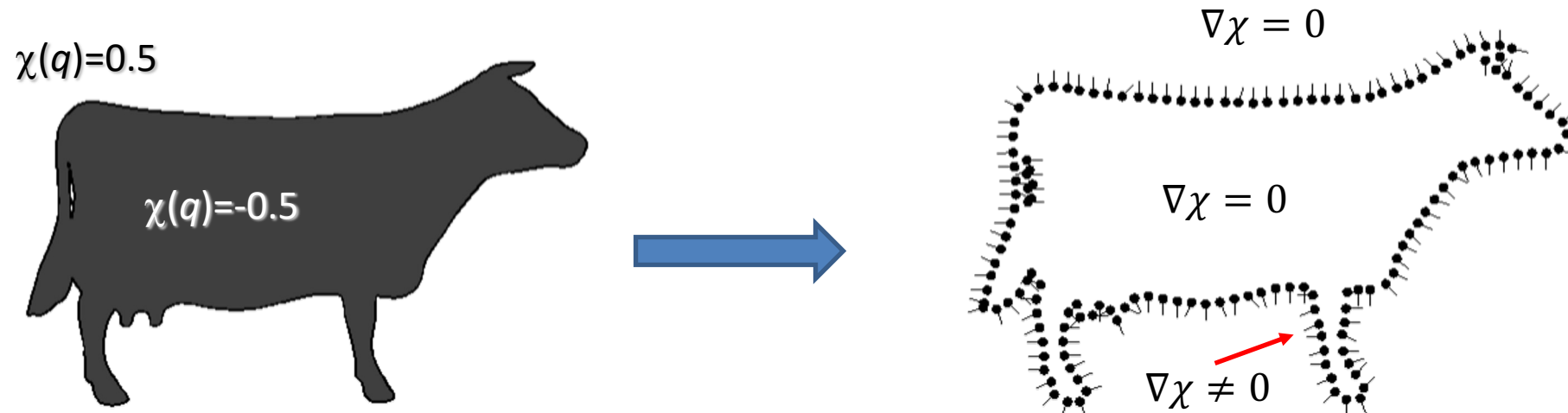
Poisson Reconstruction

- The idea
  - The indicator function (χ)
    - Interior: constant negative value
    - Exterior: constant positive value
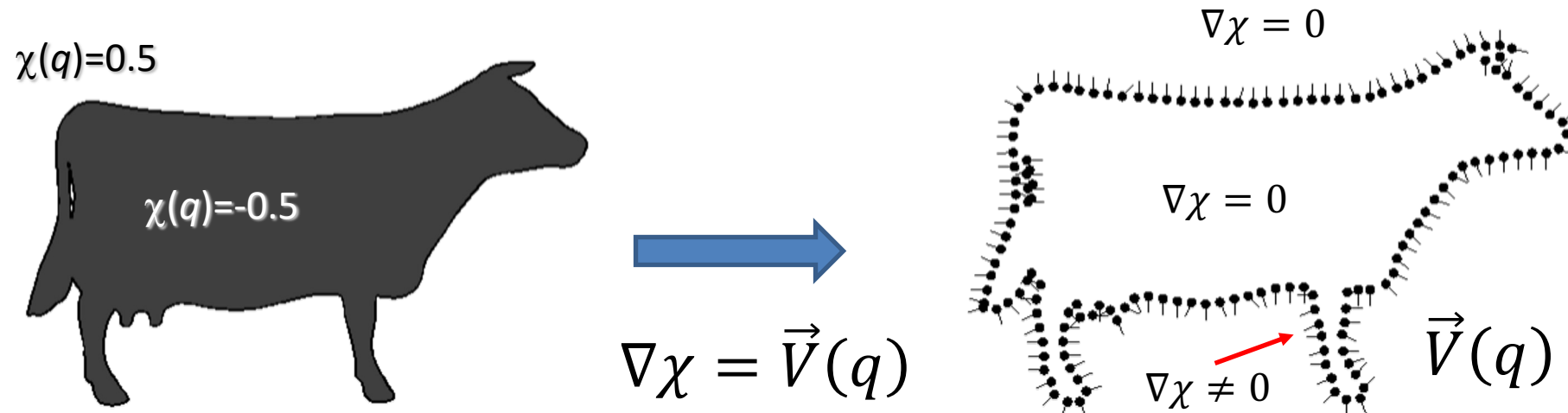
$\chi(q)=0.5$

$\chi(q)=-0.5$

Poisson Reconstruction

- ## The idea
  - The indicator function (χ)
  - The gradient of the indicator function ($\nabla\chi$)
    - Zero everywhere except close to the boundary

$\chi(q)$=0.5

$\chi(q)$=-0.5

$\nabla\chi = 0$

$\nabla\chi = 0$

$\nabla\chi \neq 0$

Poisson Reconstruction

- The idea
  - The indicator function (χ)
  - The gradient of the indicator function ($\nabla\chi$)
  - Oriented points $\vec{V}(q)\ \approx$ discretization of gradient of indicator function

$\chi(q)=0.5$

$\chi(q)=-0.5$

$\nabla\chi = \vec{V}(q)$

$\nabla\chi = 0$

$\nabla\chi = 0$

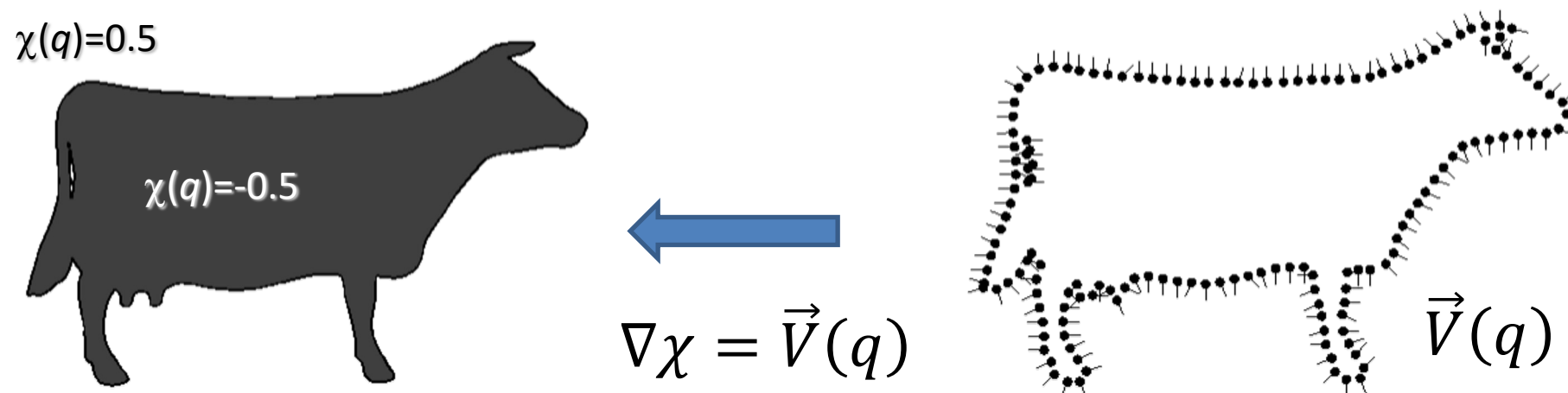$\nabla\chi \neq 0$

$\vec{V}(q)$

Poisson Reconstruction

- The idea
  - The indicator function (χ)
  - The gradient of the indicator function ($\nabla \chi$)
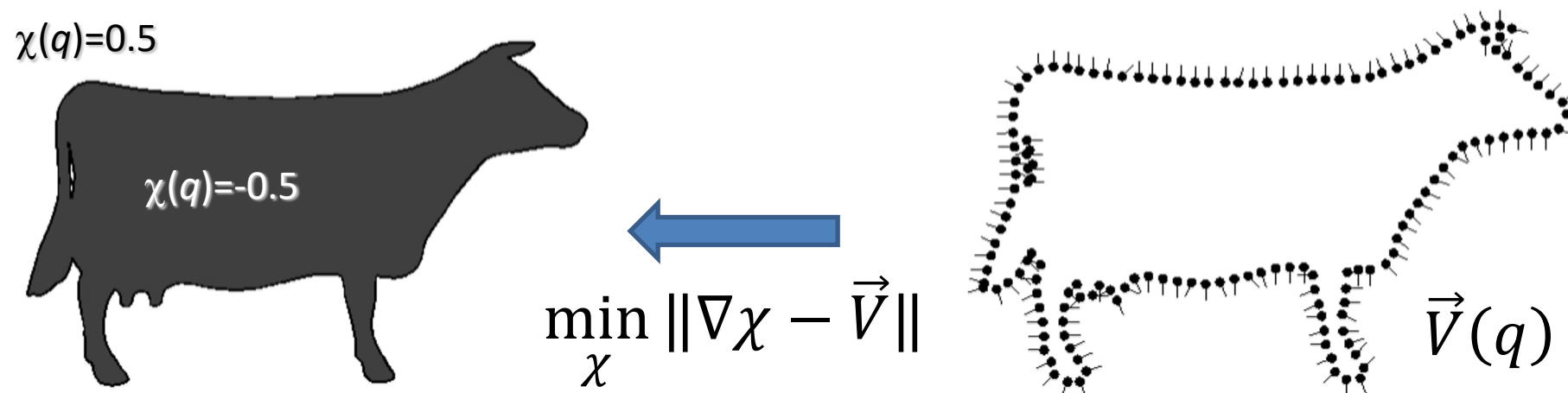 - Oriented points $\vec{V}(q)$ $\approx$ discretization of gradient of indicator function
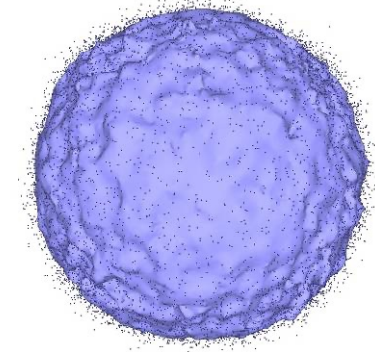 - Reconstruction

$\chi(q)$=0.5

$\chi(q)$=-0.5

$$\nabla \chi = \vec{V}(q)$$

$$\vec{V}(q)$$

Poisson Reconstruction

- ## The idea
  - The indicator function (χ)
  - The gradient of the indicator function ($\nabla\chi$)
  - Oriented points $\vec{V}(q)$ $\approx$ discretization of gradient of indicator function
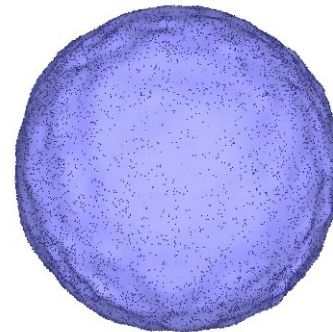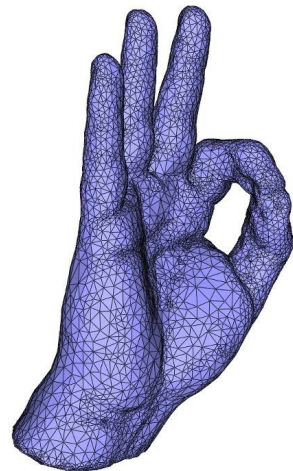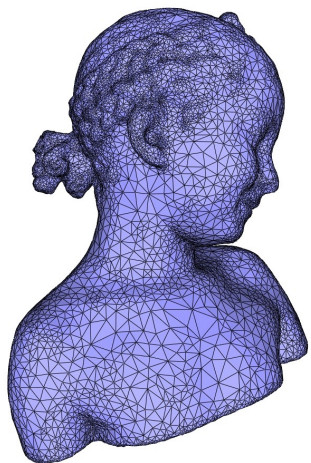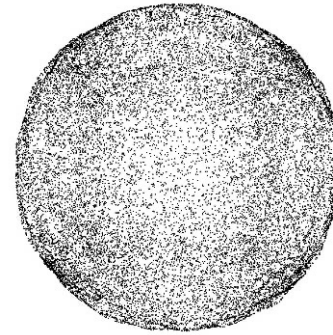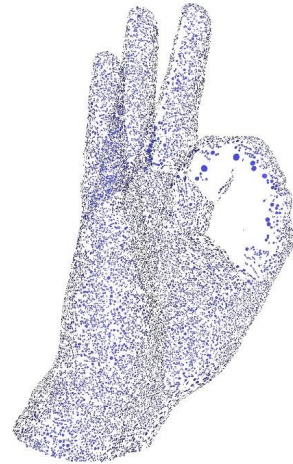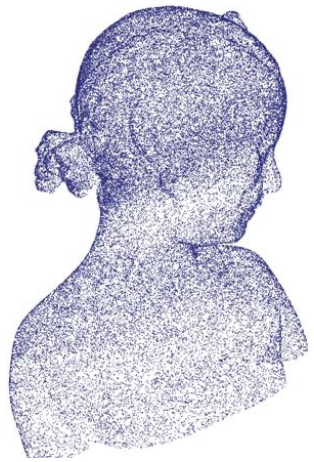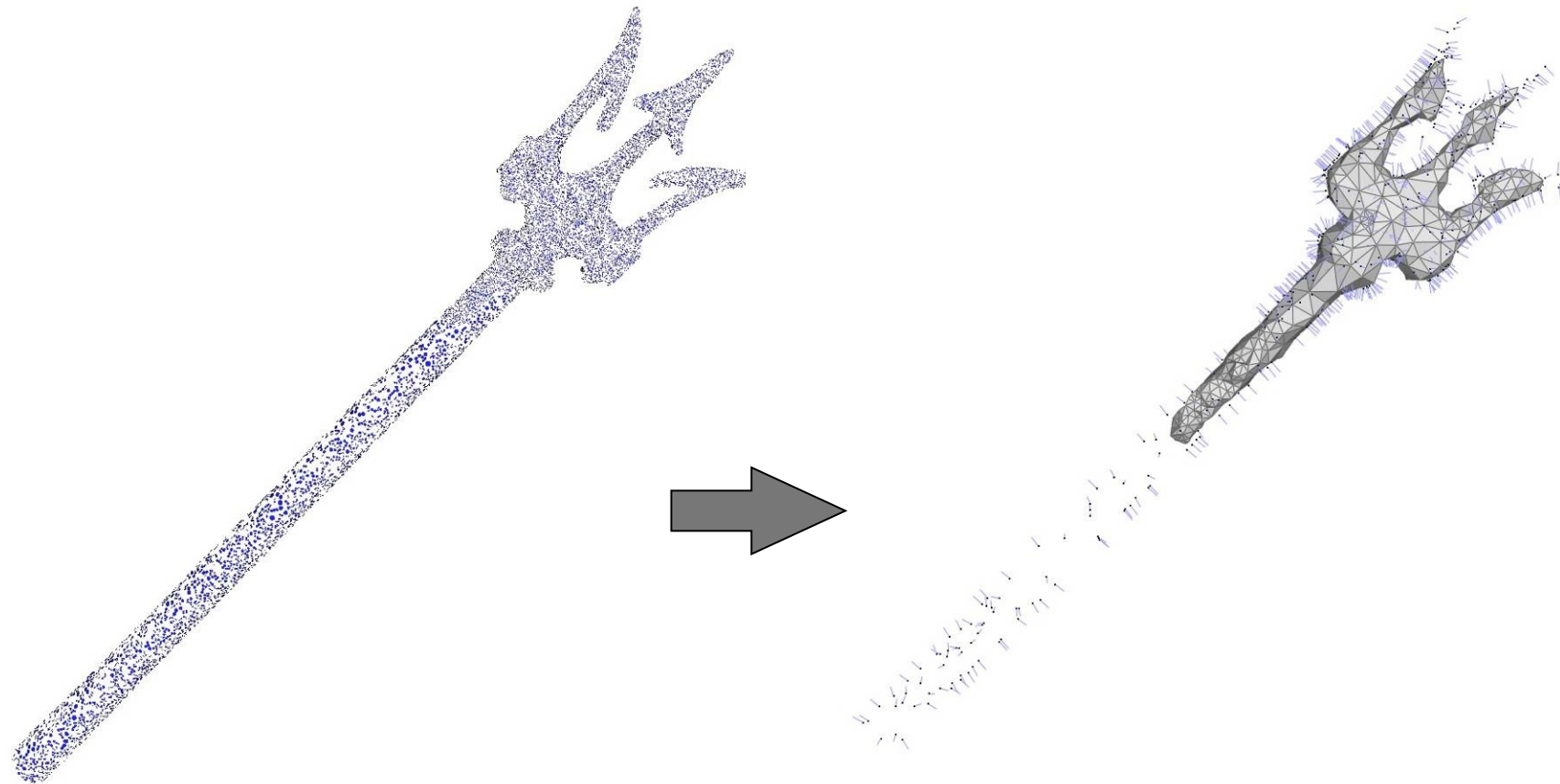 - Reconstruction: finding the indicator function + iso-surface extraction

$\chi(q)=0.5$

$\chi(q)=-0.5$

$$\min_{\chi} \|\nabla\chi - \vec{V}\|$$

$\vec{V}(q)$
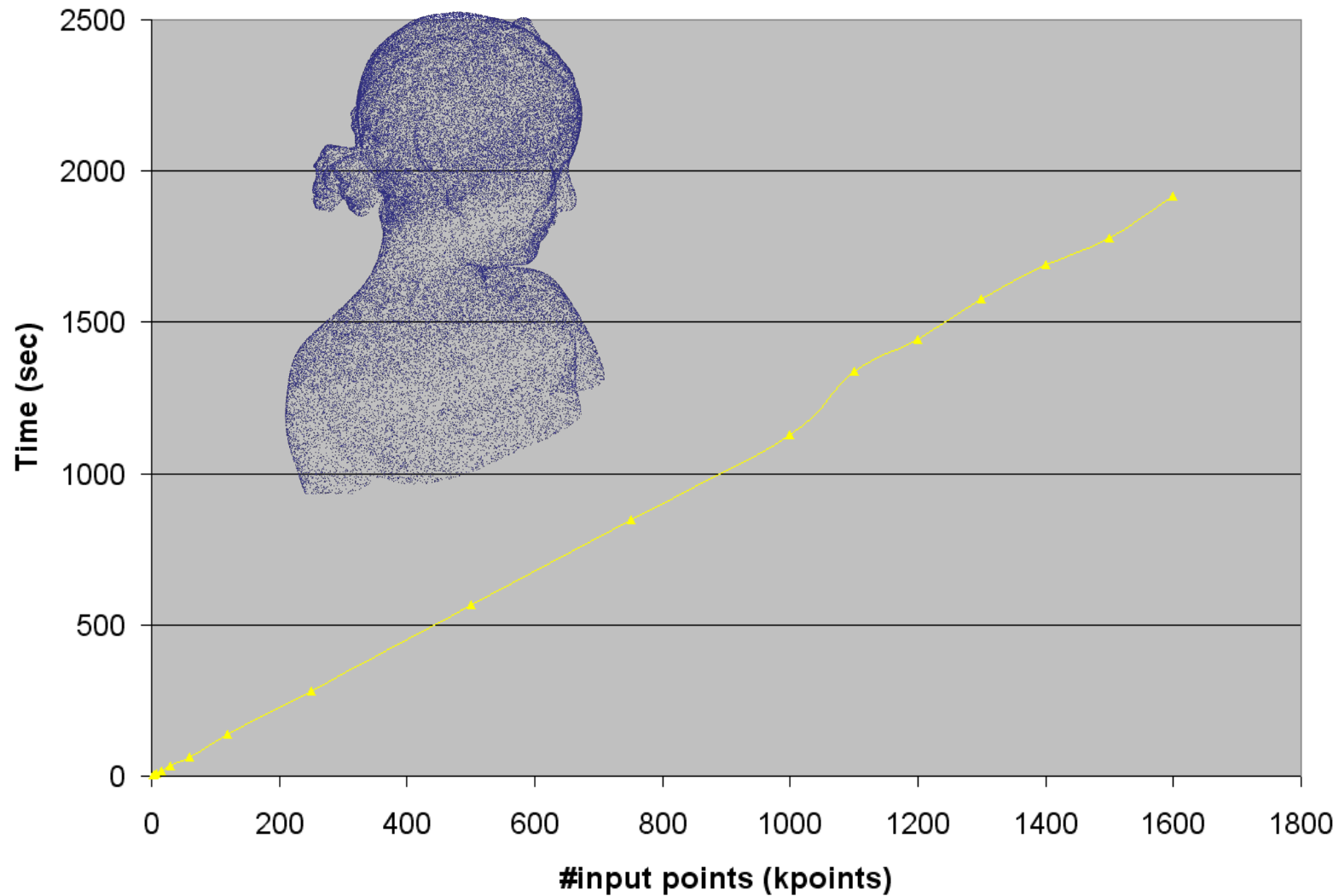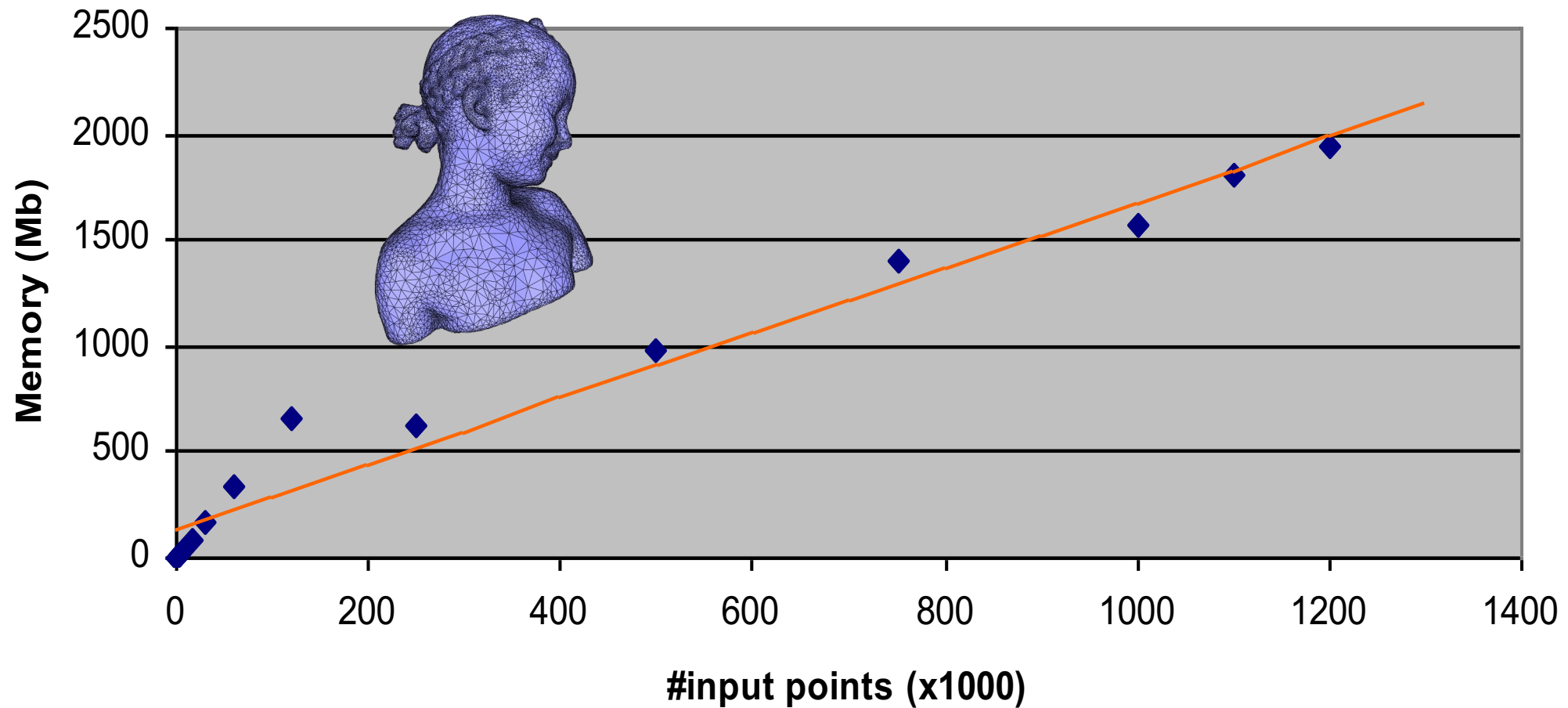
Poisson Reconstruction
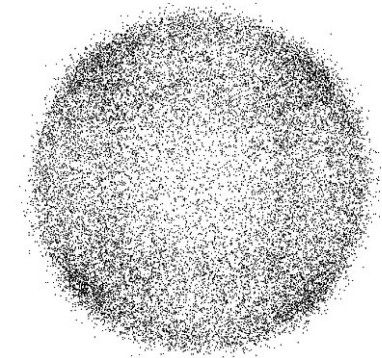
Poisson Reconstruction

Left: 50K points sampled on
Neptune trident

Right: point set simplified to 1K
then reconstructed

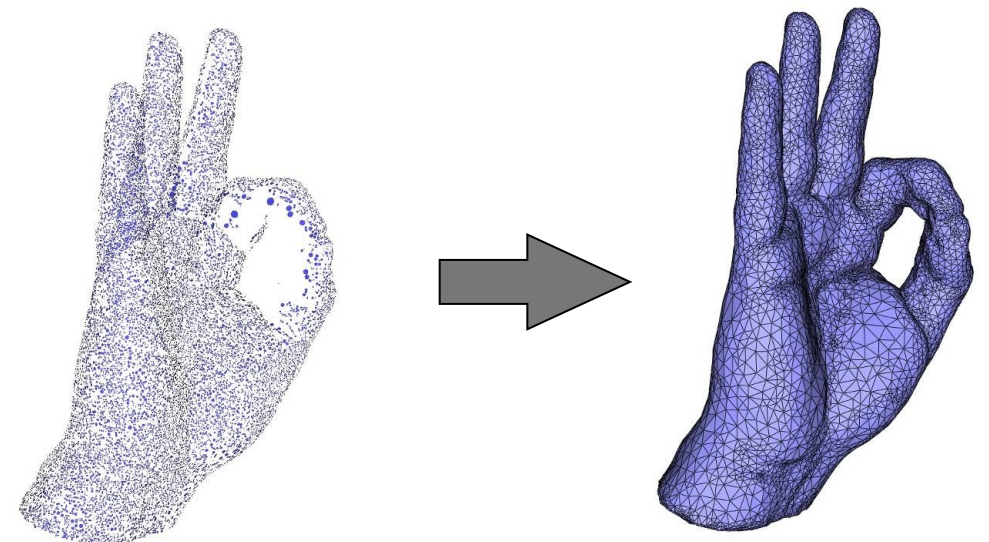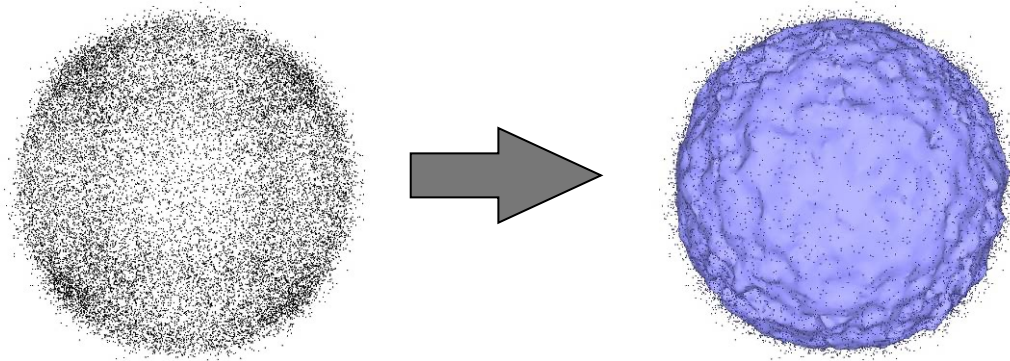# Poisson duration wrt #input points

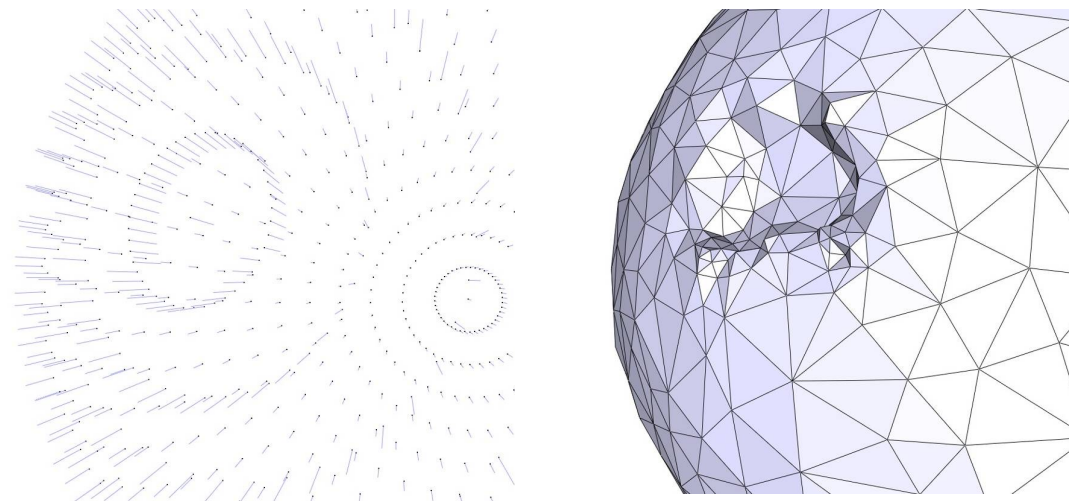# Memory wrt #input points

Poisson Reconstruction

- Properties
  - ✓ Supports noisy, non-uniform data
  - ✓ Can fill reasonably large holes

Poisson Reconstruction

- Properties
  - ✓ Supports noisy, non-uniform data
  - ✓ Can fill reasonably large holes

- Limitations
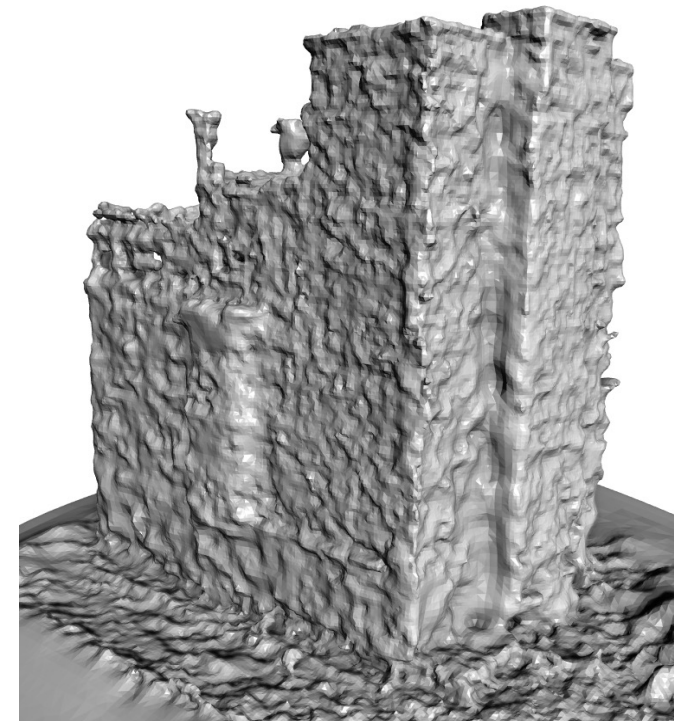 - It requires good normal information

Poisson Reconstruction

- ## Properties

  - ✓ Supports noisy, non-uniform data

  - ✓ Can fill reasonably large holes

- ## Limitations

  - It requires good normal information

  - Sharp features are oversmoothed

    - Not good for piecewise planar objects

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- Introduction
- Smooth object reconstruction
  - The pioneer work of Hoppe *et al.* (1992)
  - Poisson reconstruction [Kazhdan *et al.* 06]
  - Piecewise smooth reconstruction
- Piecewise planar object reconstruction [Nan and Wonka. 2017]

Piecewise Smooth Reconstruction

- Piecewise-smooth

smooth

piecewise-smooth

Piecewise Smooth Reconstruction

- Feature detection
  - Extract a set of sharp features
  - Decompose the point cloud into smooth patches
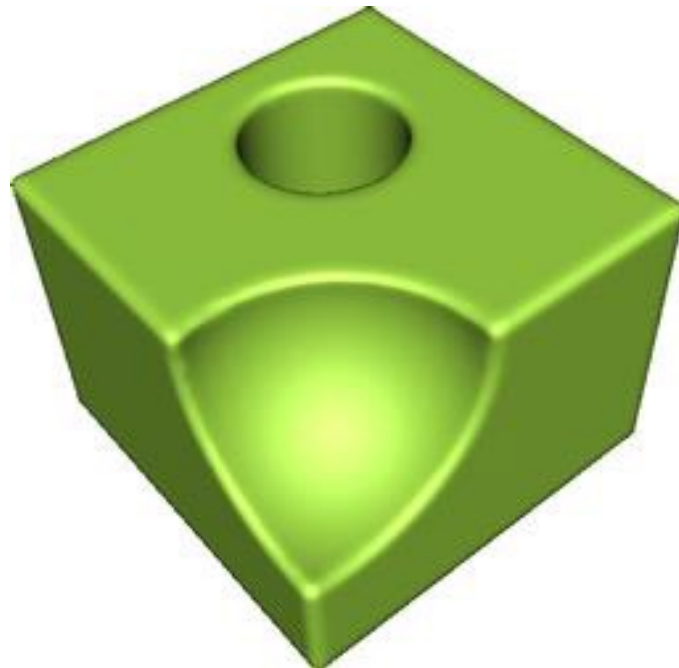- Smooth reconstruction patch by patch
- Stitch the patches
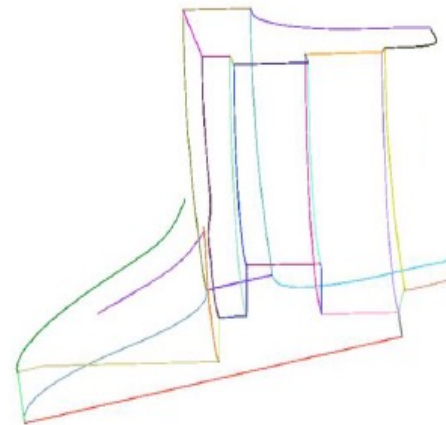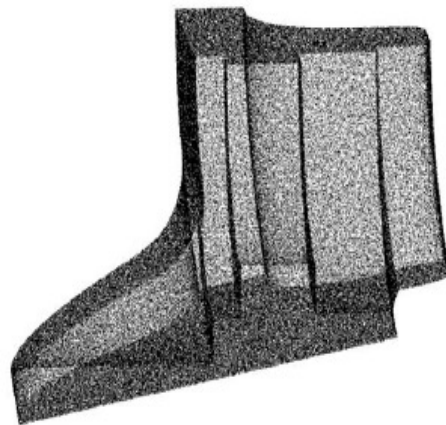
Today's Agenda
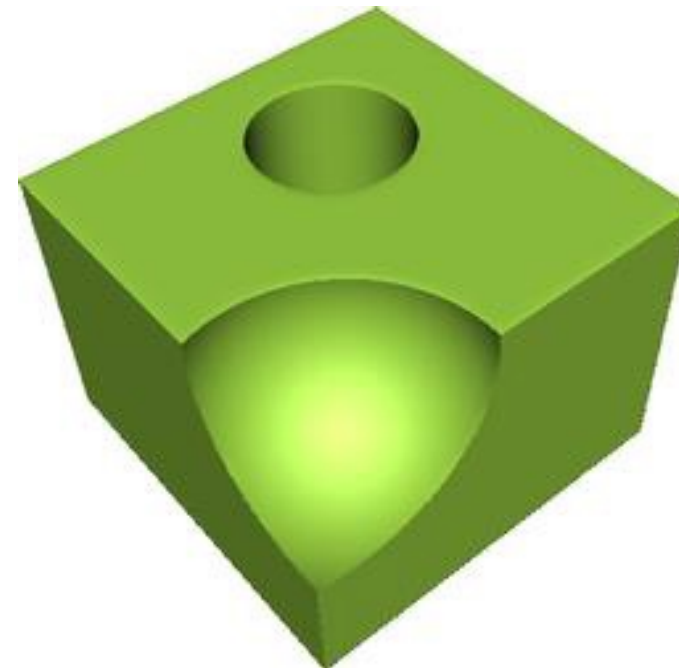
- Introduction
- Smooth object reconstruction
  - The pioneer work of Hoppe *et al.* (1992)
  - Poisson reconstruction [Kazhdan *et al.* 06]
  - Piecewise smooth reconstruction
- Piecewise planar object reconstruction

Polygonal Surface Reconstruction

- Surface Reconstruction Methods
  - Smooth surfaces
    - Fit noisy data; robust to non-uniform distribution; fill (small) holes
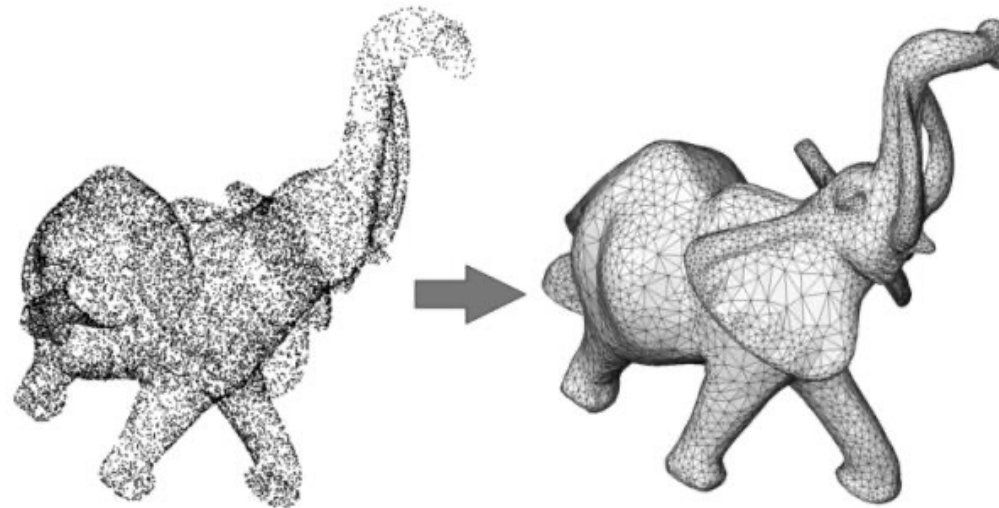
Poisson Surface Reconstruction [Kazhdan *et al.* 06]

Polygonal Surface Reconstruction

- Surface Reconstruction Methods
  – Smooth surfaces
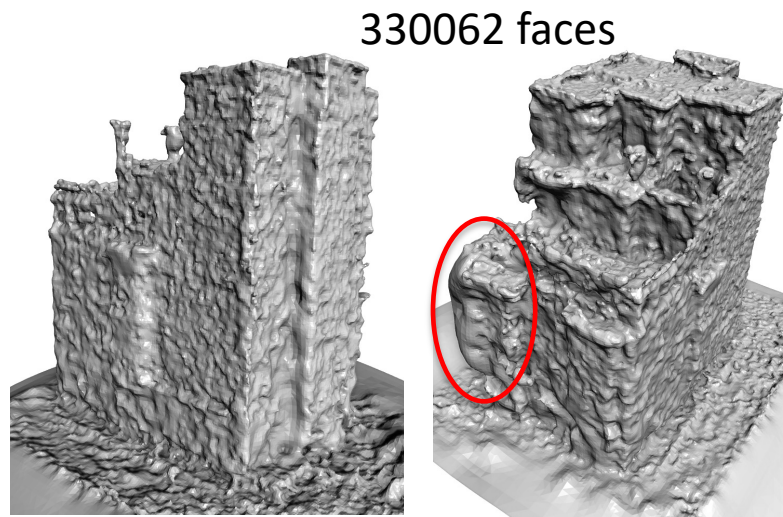 – Piecewise planar objects

Polygonal Surface Reconstruction

- Surface Reconstruction Methods
  - Smooth surfaces
  - Piecewise planar objects

Polygonal Surface Reconstruction

- Surface Reconstruction Methods
  - Smooth surfaces
  - Piecewise planar objects
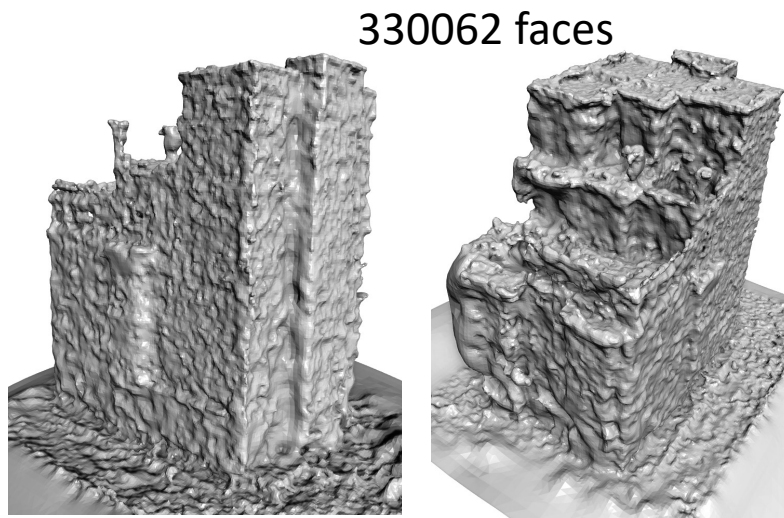
330062 faces

Result of [Kazhdan *et al.* 06]

- Unsatisfied results
  - Bumpy
  - Large number of faces
  - Unacceptable hole filling
- Rare direct applications
  - Post-processing required
    - Topologically correct
    - Simplified

Polygonal Surface Reconstruction

- Surface Reconstruction Methods
  - Smooth surfaces
  - Piecewise planar objects
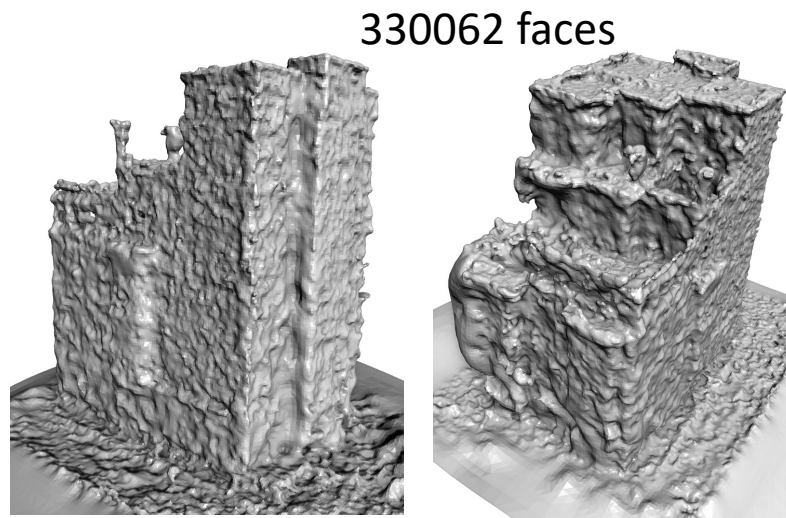
330062 faces

Result of [Kazhdan *et al.* 06]
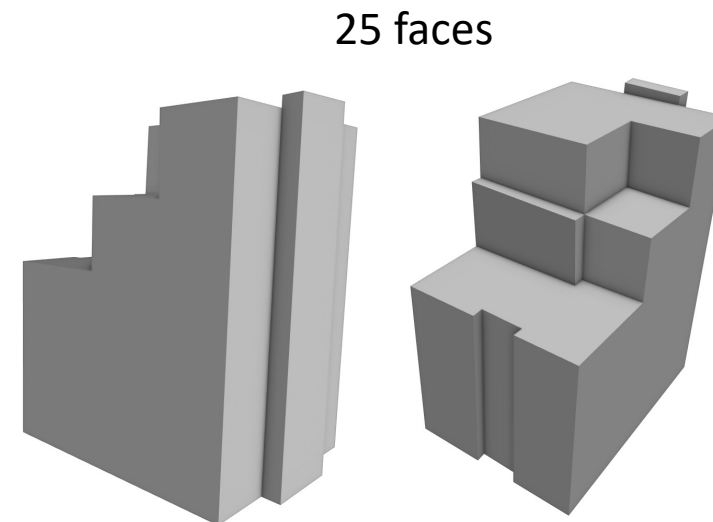

How to recover the sharp features?

Polygonal Surface Reconstruction

- Surface Reconstruction Methods
  - Smooth surfaces
  - Piecewise planar objects

330062 faces

25 faces

Result of [Kazhdan *et al.* 06]

[Nan and Wonka 17]

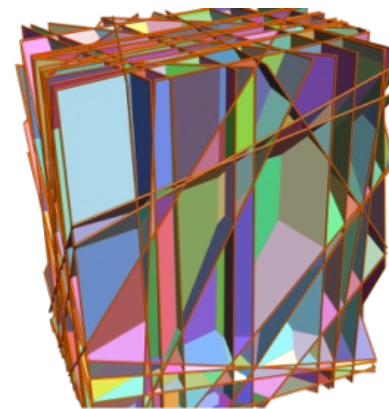# Polygonal Surface Reconstruction

- Overview
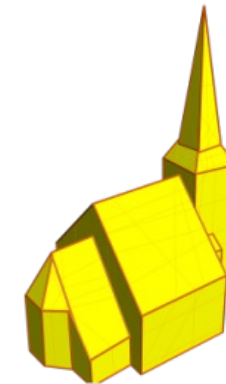

Input   Planar segments   Candidate faces   Result

Polygonal Surface Reconstruction

- Overview

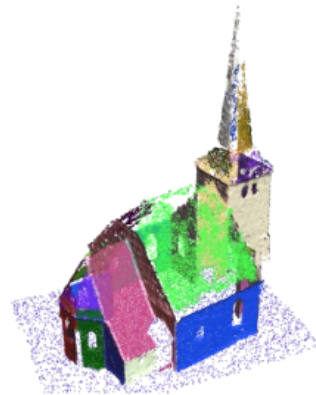
Input   Planar segments

RANSAC algorithm
- Random 3 points -> plane
- Scoring, accept or reject
- Repeat
  - Plane from the remaining points
  - Stop if no plane can be extracted

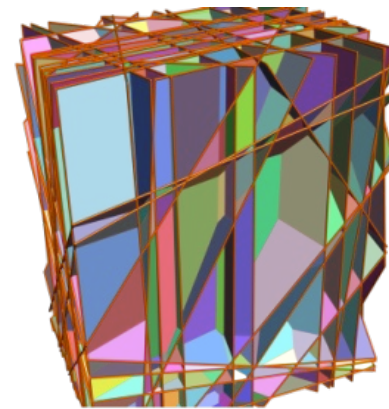# Polygonal Surface Reconstruction

- ## Candidate Generation
  - Supporting plane clipping
  - Pairwise intersection

Input　　　　Planar segments　　　　Candidate faces

Polygonal Surface Reconstruction
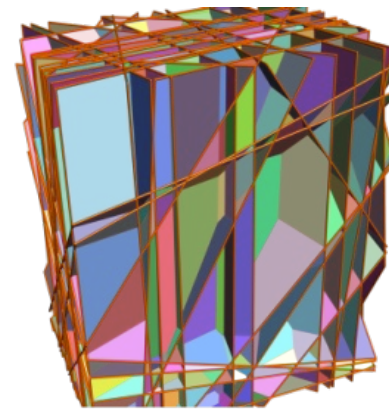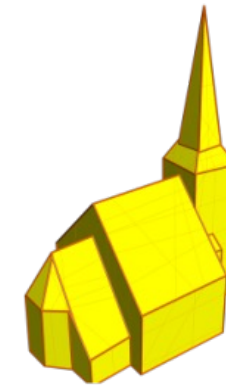
- Face Selection
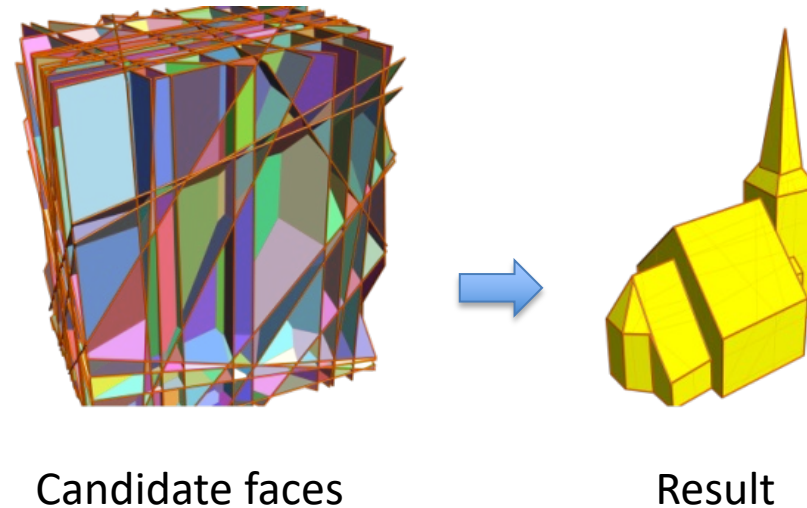

Input   Planar segments   Candidate faces   Result

Polygonal Surface Reconstruction

- Face Selection
  – Labeling problem
  – Linear integer program

Candidate faces   Result

N candidate faces $F = \{f_i | 1 \leq i \leq N\}$

Variables: $x_i = \begin{cases} 1, & \text{face } f_i \text{ will be chosen} \\ 0, & \text{face } f_i \text{ will } \textbf{not} \text{ be chosen} \end{cases}$

Polygonal Surface Reconstruction

- ## Objective Function
  - ### Data fitting
    - Favors selecting faces with more support
    - Percentage of unused points

$$E_f = 1 - \frac{1}{|P|} \sum_{i=1}^{N} x_i \cdot support(f_i)$$

Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
    - Favors selecting faces with more support
    - Percentage of unused points

$$E_f = 1 - \frac{1}{|P|} \sum_{i=1}^{N} x_i \cdot support(f_i)$$

Confidence weighted number of supporting point

$$support(f) = \sum_{p,f|dist(p,f)<\varepsilon} (1 - \frac{dist(p,f)}{\varepsilon}) \cdot conf(p)$$

Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
    - Favors selecting faces with more support
    - Percentage of unused points

$$E_f = 1 - \frac{1}{|P|} \sum_{i=1}^{N} x_i \cdot support(f_i)$$
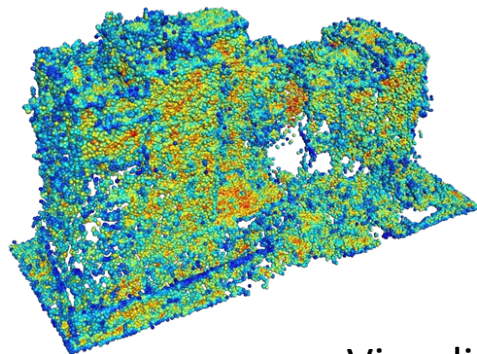
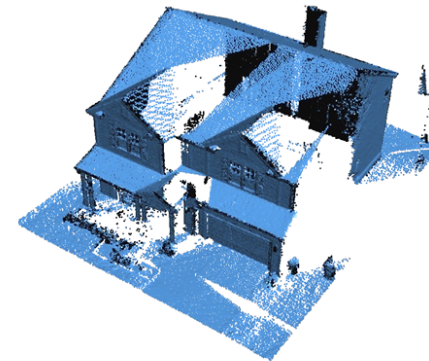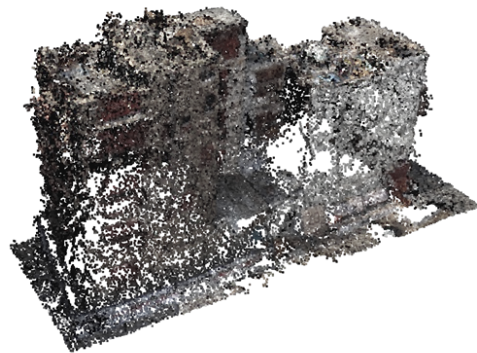Confidence weighted number of supporting point
$$support(f) = \sum_{p,f|dist(p,f)<\varepsilon} (1 - \frac{dist(p,f)}{\varepsilon}) \cdot conf(p)$$

Point confidence
$$conf(p) = \frac{1}{3} \sum_{i=1}^{3} (1 - \frac{3\lambda_i^1}{\lambda_i^1 + \lambda_i^2 + \lambda_i^3}) \cdot \frac{\lambda_i^2}{\lambda_i^3} \qquad \lambda_i^1 \leq \lambda_i^2 \leq \lambda_i^3$$

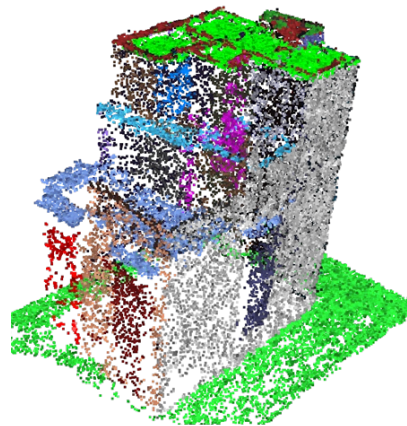# Polygonal Surface Reconstruction
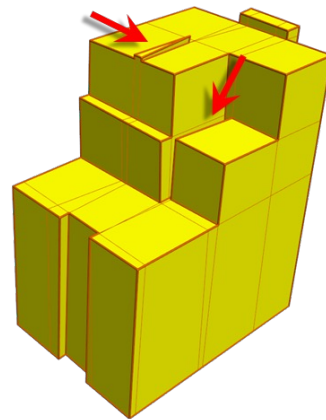
- Objective Function
  - Data fitting
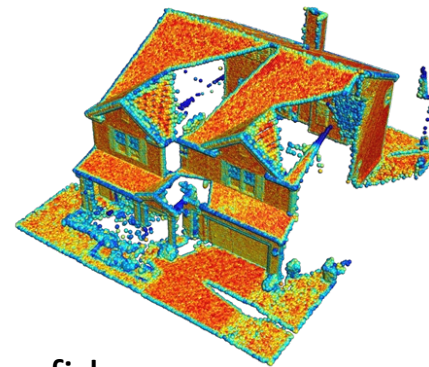
Visualization of point confidences

Polygonal Surface Reconstruction
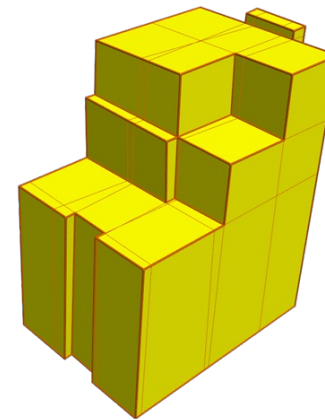
- Objective Function
  - Data fitting
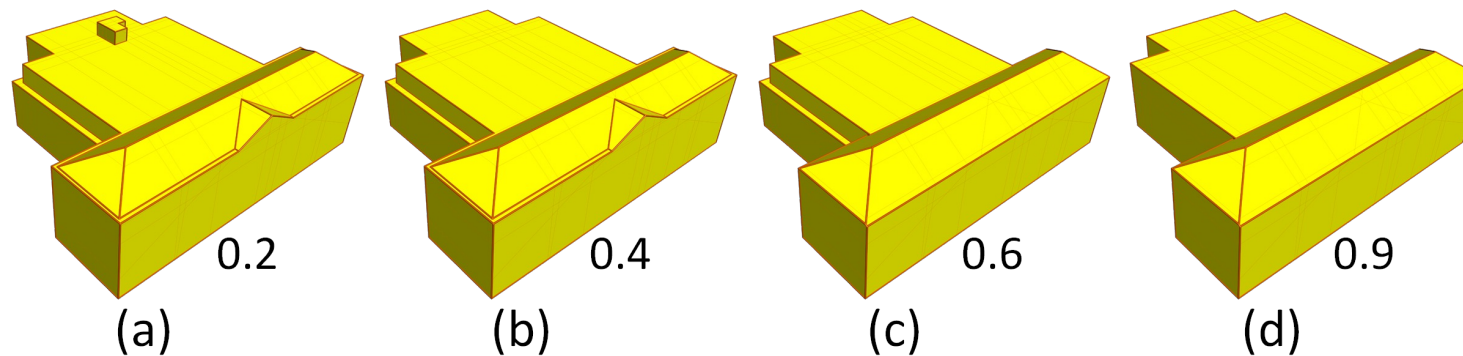 - Model complexity
    - Penalize sharp corners

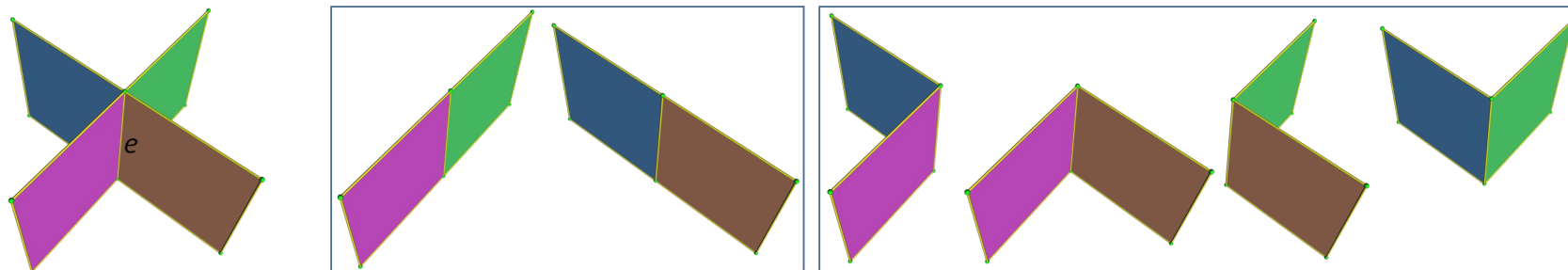(a)   (b)   (c)

Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
  - Model complexity
    - Penalize sharp corners

(a) 0.2   (b) 0.4   (c) 0.6   (d) 0.9

Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
  - Model complexity
    - Penalize sharp corners
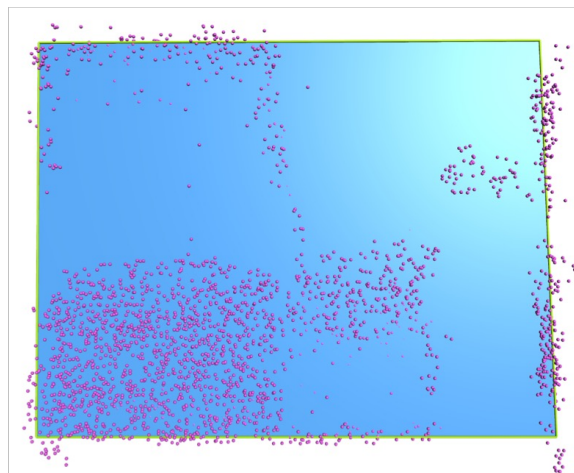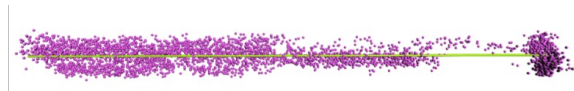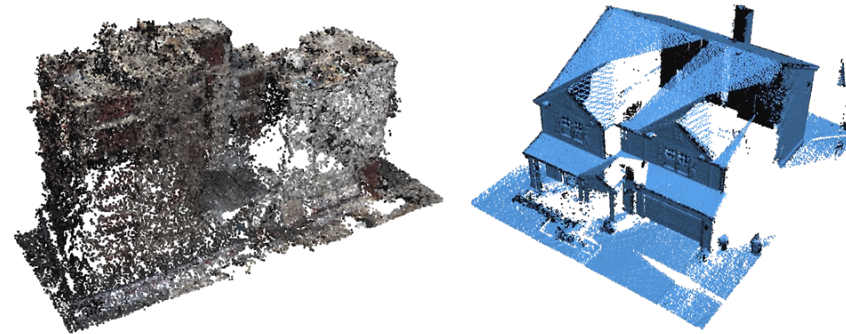
$$E_m = \frac{1}{|E|} \sum_{i=1}^{|E|} corner(e_i)$$

Intersecting two faces
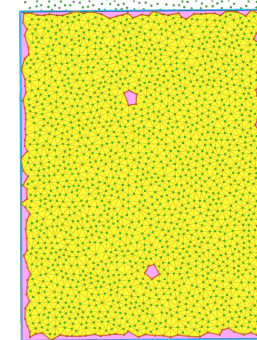
Polygonal Surface Reconstruction

- Objective Function
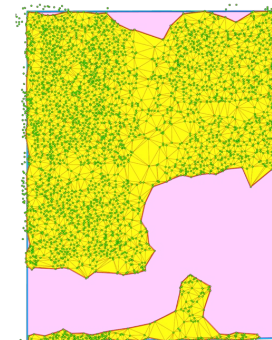 - Data fitting
  - Model complexity
  - Point coverage

Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
  - Model complexity
  - Point coverage

$$E_c = \frac{1}{area(M)} \sum_{i=1}^{N} x_i \cdot (area(f_i) - area(M_i^\alpha)).$$

0.93   0.65
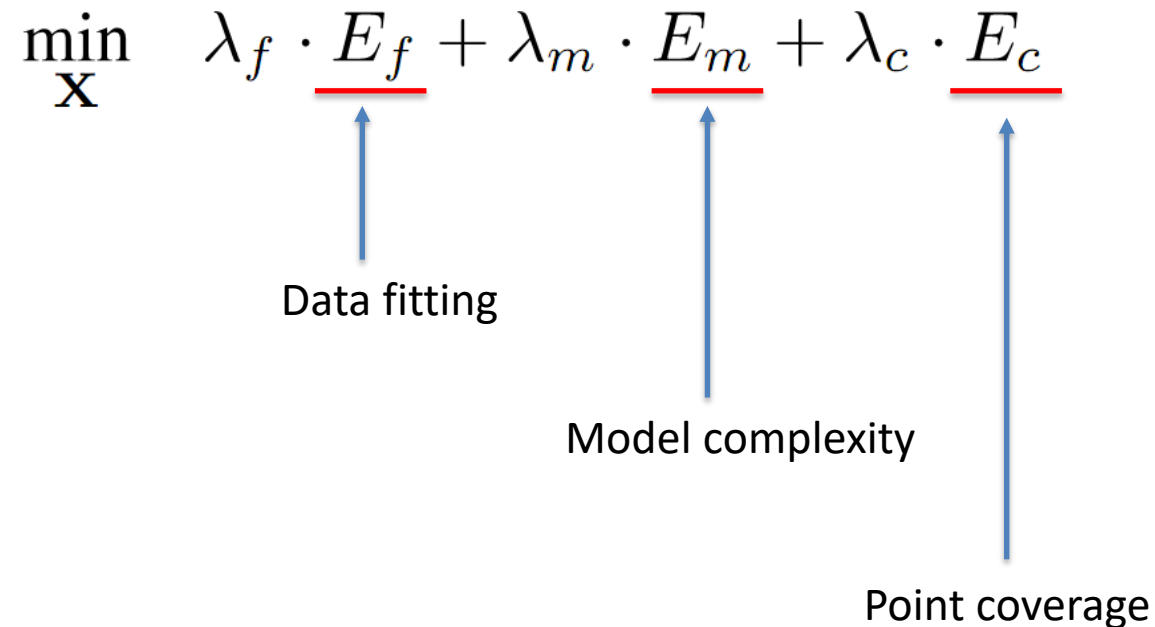
Polygonal Surface Reconstruction

- Face Selection
  - Linear integer program

$$\min_{\mathbf{X}} \quad \lambda_f \cdot \underline{E_f} + \lambda_m \cdot \underline{E_m} + \lambda_c \cdot \underline{E_c}$$
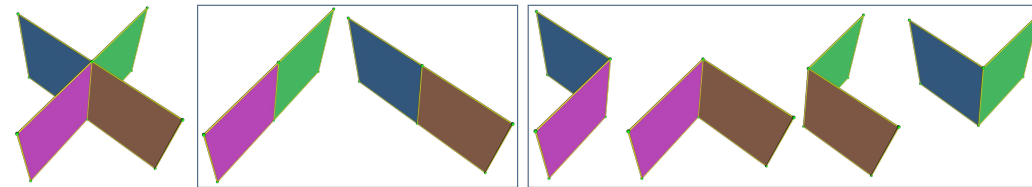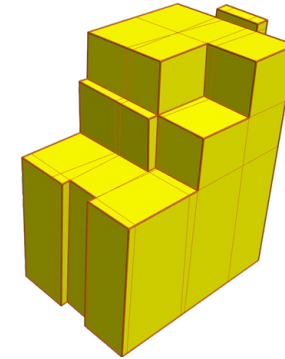
Data fitting

Model complexity

Point coverage

Polygonal Surface Reconstruction

- ## Face Selection

  - ### Linear integer program

    - #### Constraints

      - ##### Watertight
      - ##### Manifold

$$\min_{\mathbf{X}} \quad \lambda_f \cdot E_f + \lambda_m \cdot E_m + \lambda_c \cdot E_c$$

$$\text{s.t.} \quad \begin{cases} \sum_{j \in \mathcal{N}(e_i)} x_j = 2 \quad \text{or} \quad 0, \quad 1 \le i \le |E| \\ x_i \in \{0, 1\}, \qquad\qquad\qquad 1 \le i \le N \end{cases}$$
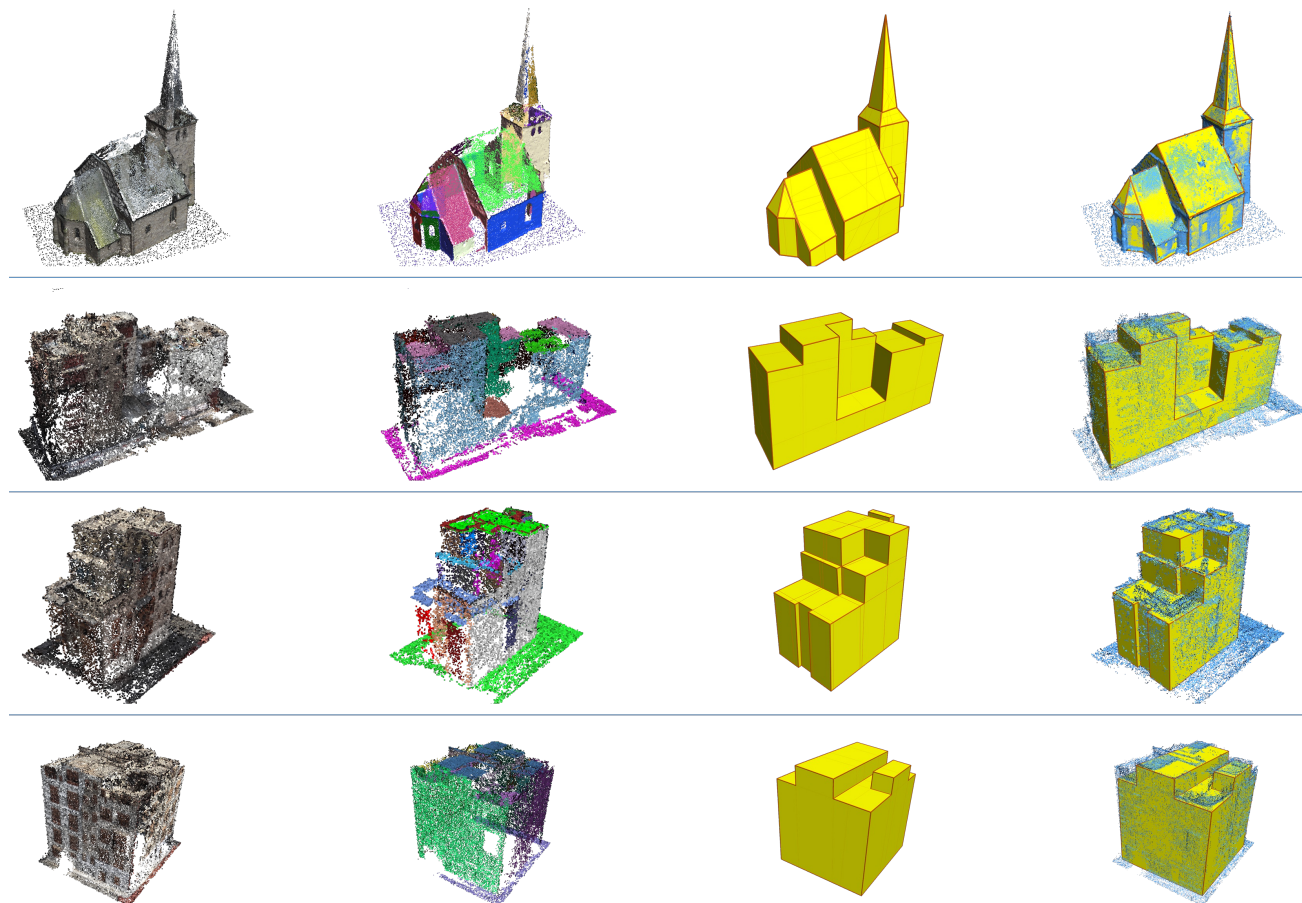
Polygonal Surface Reconstruction

- Face Selection
  - Linear integer program
    - Constraints
    - Solvers (SCIP, CBC, GLPK, Gurobi…)

$$\min_{\mathbf{X}} \quad \lambda_f \cdot E_f + \lambda_m \cdot E_m + \lambda_c \cdot E_c$$

$$\text{s.t.} \quad \begin{cases} \displaystyle\sum_{j \in \mathcal{N}(e_i)} x_j = 2 \quad \text{or} \quad 0, \quad 1 \leq i \leq |E| \\ x_i \in \{0, 1\}, \quad\quad\quad\quad\quad 1 \leq i \leq N \end{cases}$$
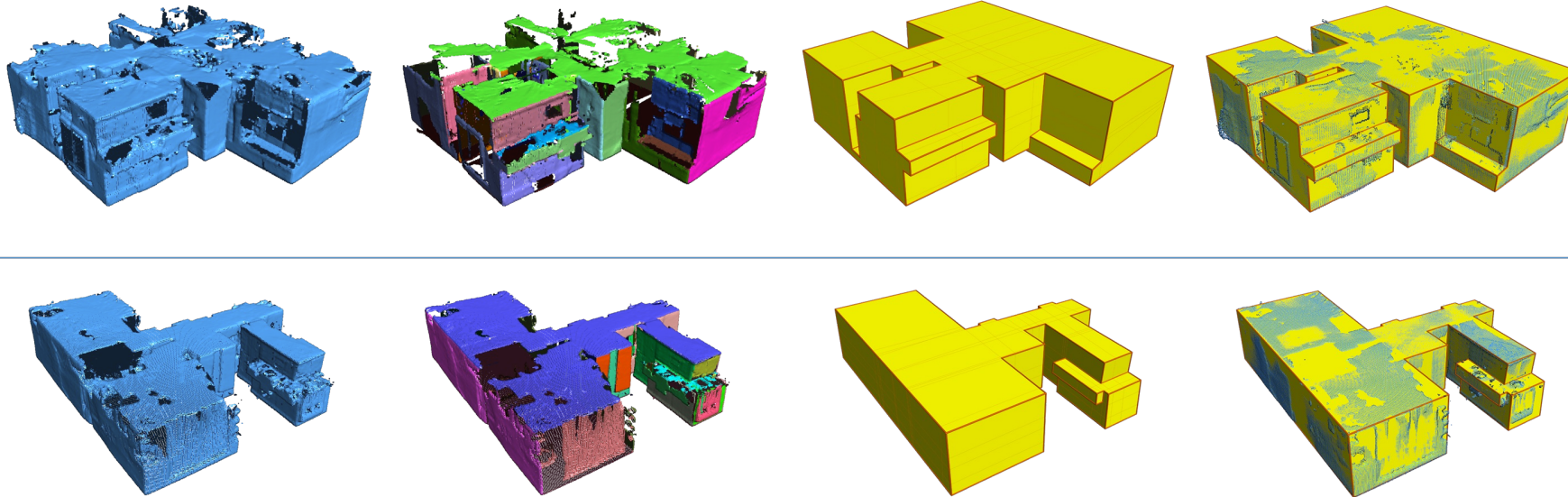
Polygonal Surface Reconstruction

- Reconstruction Results
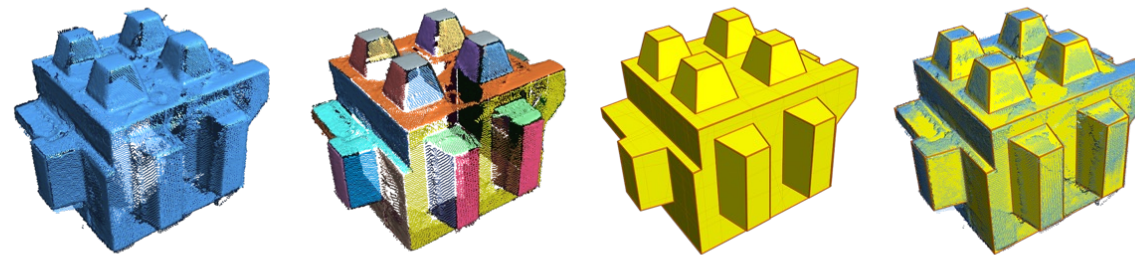
Polygonal Surface Reconstruction
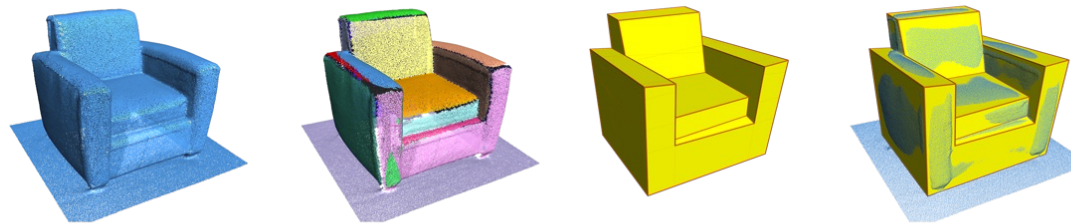
- Reconstruction Results

Piecewise Planar Reconstruction
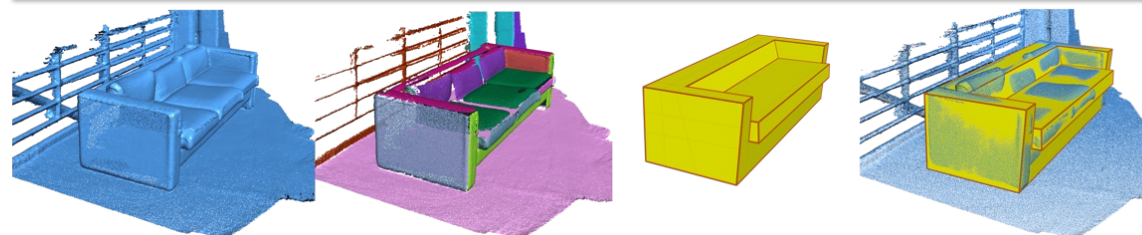
- Reconstruction Results

Packing foam box

Chair

Sofa

Piecewise Planar Reconstruction

- Robustness to noise
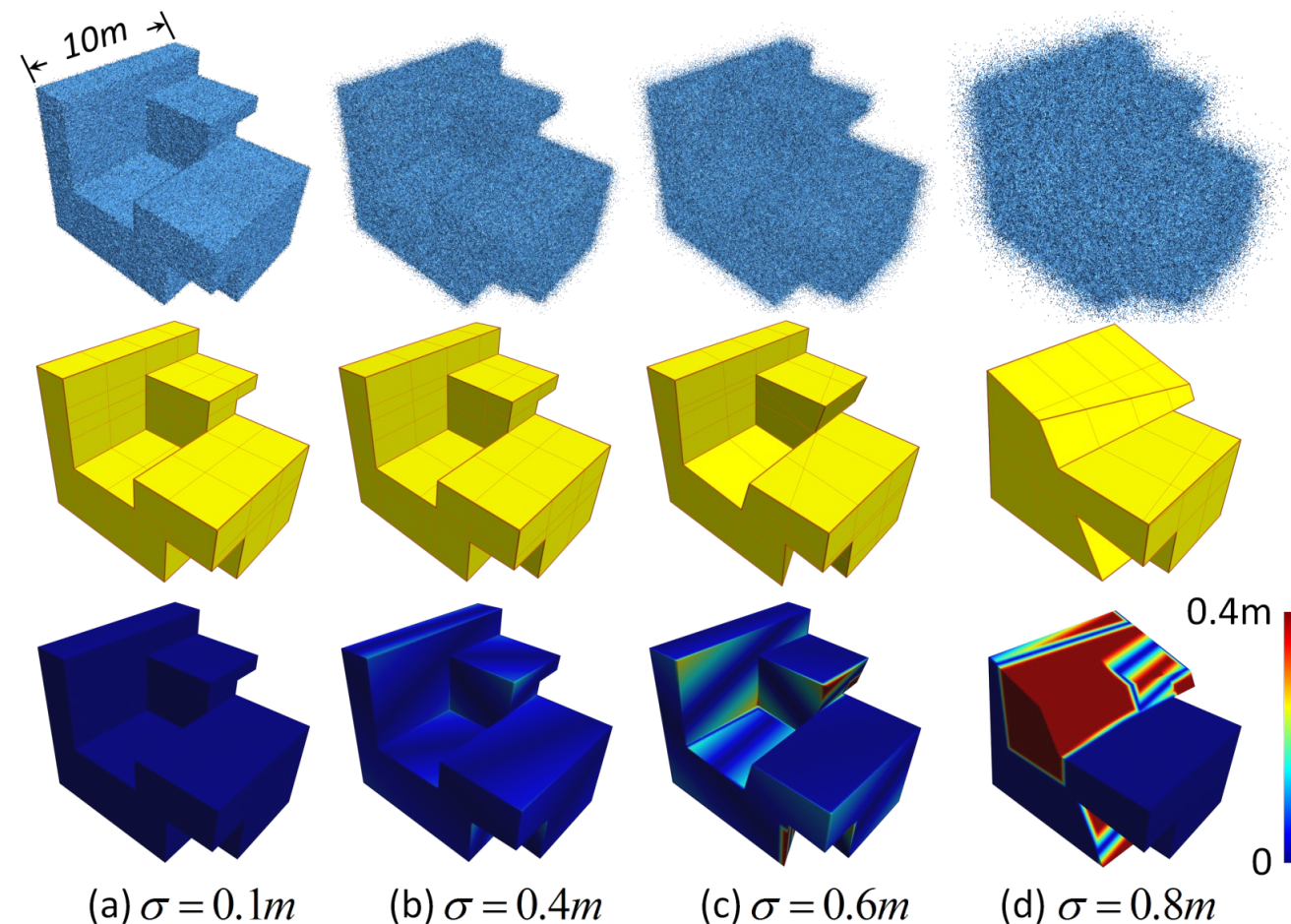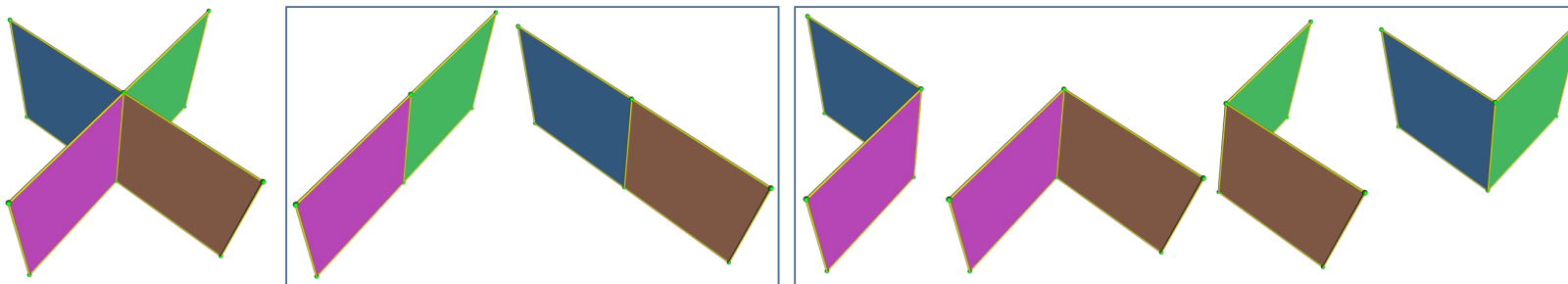
(a) $\sigma = 0.1m$ (b) $\sigma = 0.4m$ (c) $\sigma = 0.6m$ (d) $\sigma = 0.8m$

Limitations

- Open surfaces

$$\text{s.t.} \begin{cases} \boxed{\sum_{j \in \mathcal{N}(e_i)} x_j = 2 \quad \text{or} \quad 0, \quad 1 \leq i \leq |E|} \\ x_i \in \{0, 1\}, \qquad\qquad\qquad 1 \leq i \leq N \end{cases}$$

Limitations
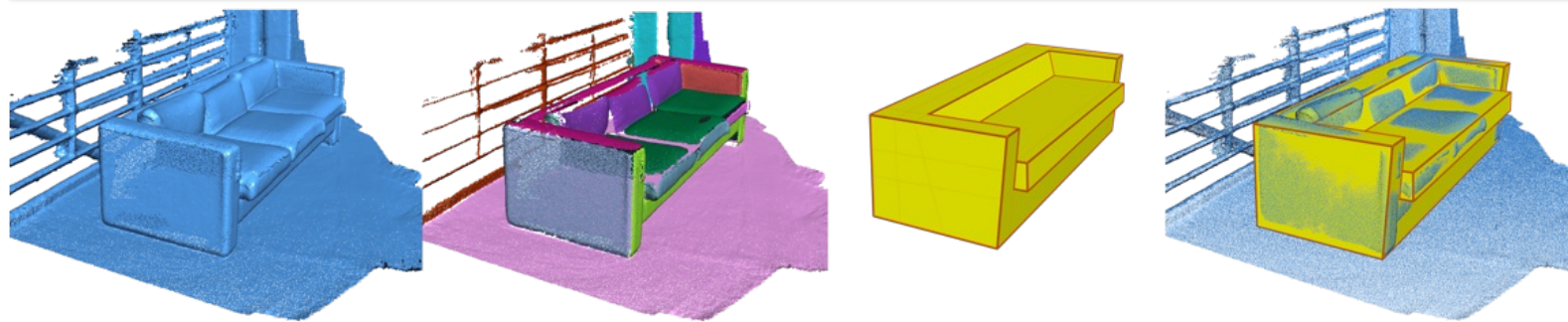
- Open surfaces

$$\text{s.t.} \quad \begin{cases} \boxed{\displaystyle\sum_{j \in \mathcal{N}(e_i)} x_j = 2 \quad \text{or} \quad 0, \quad 1 \le i \le |E|} \\ x_i \in \{0, 1\}, \qquad\qquad\qquad 1 \le i \le N \end{cases}$$

Limitations

- Open surfaces
- Finer surface details
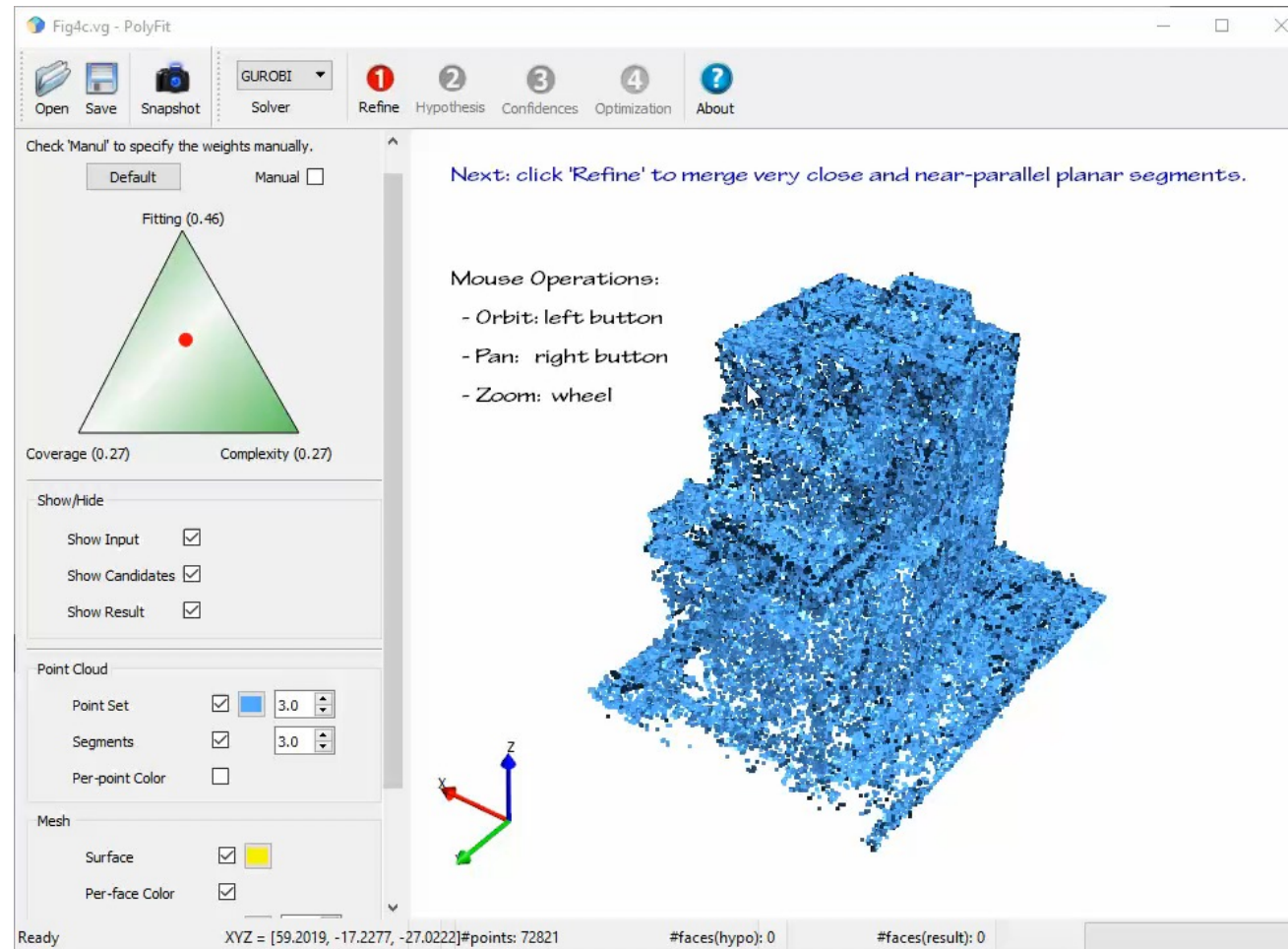 - Fence
  - Façade decorations
  - Door handle
  - …

Limitations

- Open surfaces
- Finer surface details
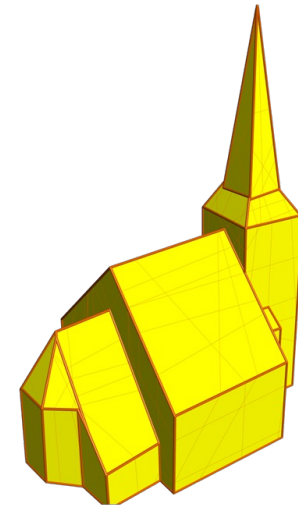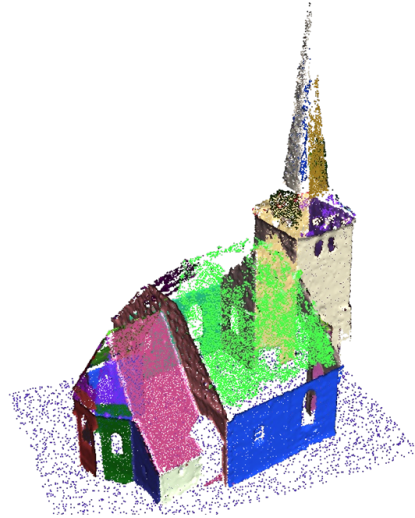- Complexity of the algorithm
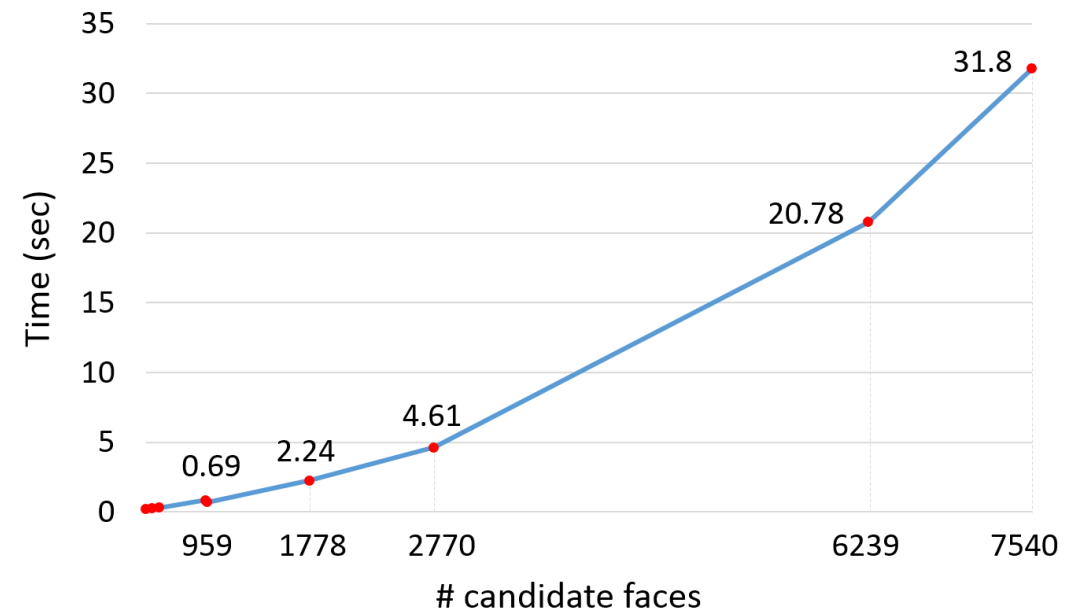
Polygonal Surface Reconstruction

- Demo

Source Code (in C++) https://github.com/LiangliangNan/PolyFit

Questions?

Next

- Feedback on the assignments
- Example questions (for the final exam)
- Q & A