GEO1016
Photogrammetry and 3D Computer Vision

Lecture
Epipolar Geometry

Liangliang Nan

Today's Agenda

- Review of Previous Lecture
  - Camera calibration
- Epipolar geometry

Review of Camera Calibration

- Camera calibration
  - Recovering K
  - Recovering R and T

$$\mathbf{P}' = \mathcal{M}\mathbf{P}_w$$
$$= \mathcal{K}\begin{bmatrix} \mathcal{R} & \mathbf{T} \end{bmatrix}\mathbf{P}_w$$

Internal (intrinsic) parameters

External (extrinsic) parameters

Review of Camera Calibration

- How many parameters to recover?
  - 5 intrinsic parameters
    - 2 for focal length
    - 2 for offset
    - 1 for skewness
  - 6 extrinsic parameters
    - 3 for rotation
    - 3 for translation

$$\mathbf{P}' = \mathcal{M}\mathbf{P}_w$$
$$= \mathcal{K}\begin{bmatrix}\mathcal{R} & \mathbf{T}\end{bmatrix}\mathbf{P}_w$$

$$\mathbf{K} = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \quad \mathrm{R} = \begin{bmatrix} \mathbf{r}_1^{\mathrm{T}} \\ \mathbf{r}_2^{\mathrm{T}} \\ \mathbf{r}_3^{\mathrm{T}} \end{bmatrix} \quad \mathrm{T} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Review of Camera Calibration

- 11 parameters to recover
- Corresponding 3D-2D point pairs
  – Each 3D-2D point pair -> 2 constraints
  – 11 unknown -> 6 point correspondence
  – Use more to handle noisy data

$$\mathbf{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = M\mathbf{P}_i = \begin{bmatrix} \frac{\mathbf{P}_i^T \mathbf{m}_1}{\mathbf{P}_i^T \mathbf{m}_3} \\ \\ \frac{\mathbf{P}_i^T \mathbf{m}_2}{\mathbf{P}_i^T \mathbf{m}_3} \end{bmatrix} \Rightarrow \begin{array}{l} \mathbf{P}_i^T \mathbf{m}_1 - u_i(\mathbf{P}_i^T \mathbf{m}_3) = 0 \\ \mathbf{P}_i^T \mathbf{m}_2 - v_i(\mathbf{P}_i^T \mathbf{m}_3) = 0 \end{array}$$

Review of Camera Calibration

- 11 parameters to recover
- Corresponding 3D-2D point pairs
- How to solve it?
  - $m = 0$ always a trivial solution
  - $k * m$ (k is non-zero) is also a solution

$$\begin{bmatrix} \mathbf{P}_1^T & 0^T & -u_1\mathbf{P}_1^T \\ 0^T & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ & \vdots & \\ \mathbf{P}_n^T & 0^T & -u_n\mathbf{P}_n^T \\ 0^T & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{bmatrix} = P\mathbf{m} = 0$$

Review of Camera Calibration

- 11 parameters to recover
- Corresponding 3D-2D point pairs
- How to solve it?
  - *m = 0* always a trivial solution
  - *k * m* (*k* is non-zero) is also a solution
  - Constrained optimization

$$\begin{bmatrix} \mathbf{P}_1^T & 0^T & -u_1\mathbf{P}_1^T \\ 0^T & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ & \vdots & \\ \mathbf{P}_n^T & 0^T & -u_n\mathbf{P}_n^T \\ 0^T & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{bmatrix} = P\mathbf{m} = 0 \quad \Rightarrow \quad \begin{aligned} &\underset{\mathbf{m}}{\text{minimize}} && \|P\mathbf{m}\|^2 \\ &\text{subject to} && \|\mathbf{m}\|^2 = 1 \end{aligned}$$

Review of Camera Calibration

- Solved using SVD

$$Pm = 0$$

SVD decomposition of P

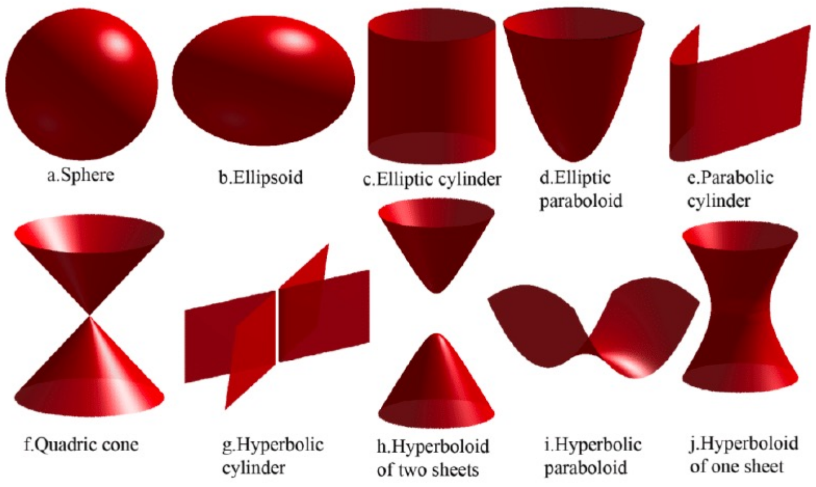
$$U_{2n\times12}\, D_{12\times12}\, V^{T}_{12\times12}$$

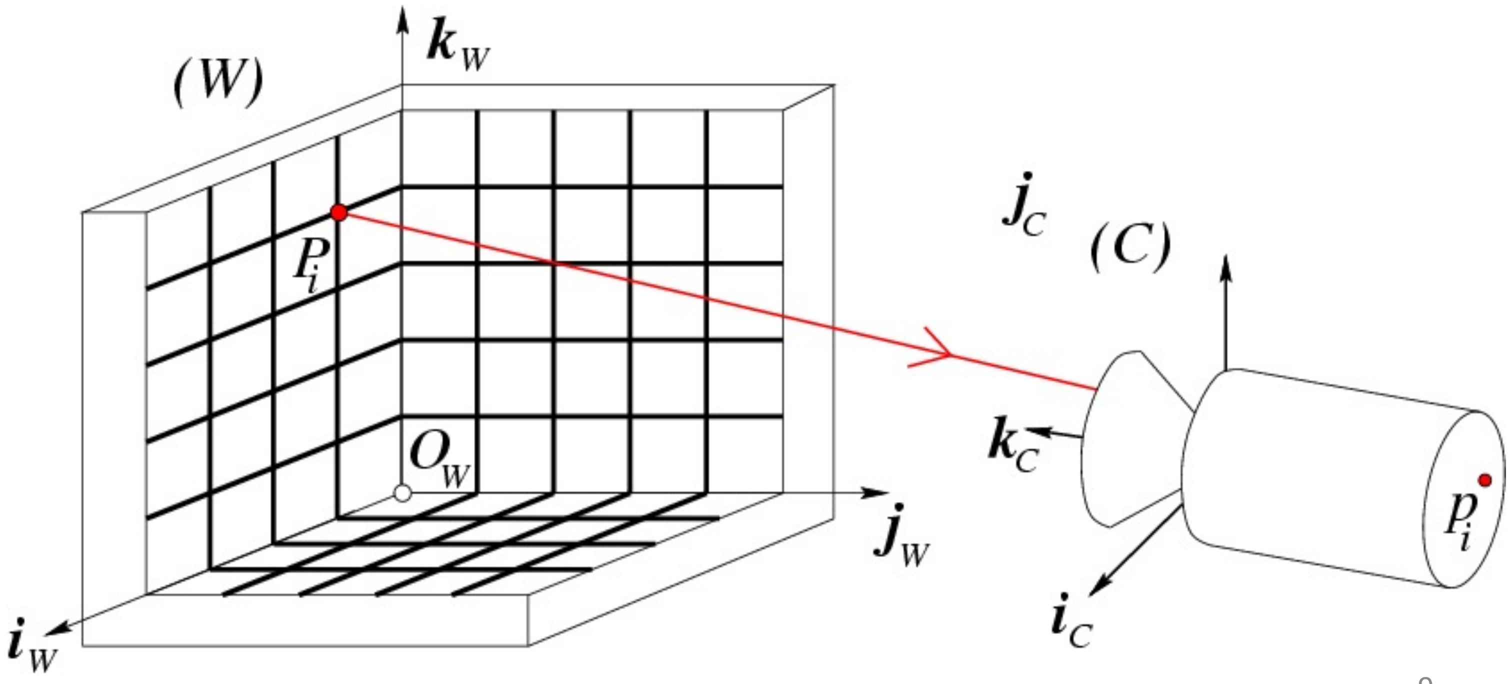
Last column of V gives ***m***

(Why? See page 593 of Hartley & Zisserman. Multiple view geometry in computer vision)

Review of Camera Calibration

- Not always solvable
  - P_is cannot lie on the same plane
  - P_is cannot lie on the intersection curve of two quadric surfaces





Quiz

Which of the following will change the camera intrinsic matrix?

(a) When zooming in.

(b) When rotating the camera around its local origin.

(c) When changing the resolution of the image.

(d) When the camera is moved.

Quiz

Which of the following will change the camera intrinsic matrix?

(a) When zooming in. [f_x, f_y]

(b) When rotating the camera around its local origin. R

(c) When changing the resolution of the image. [c_x, c_y]

(d) When the camera is moved. t

$$\mathbf{K} = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

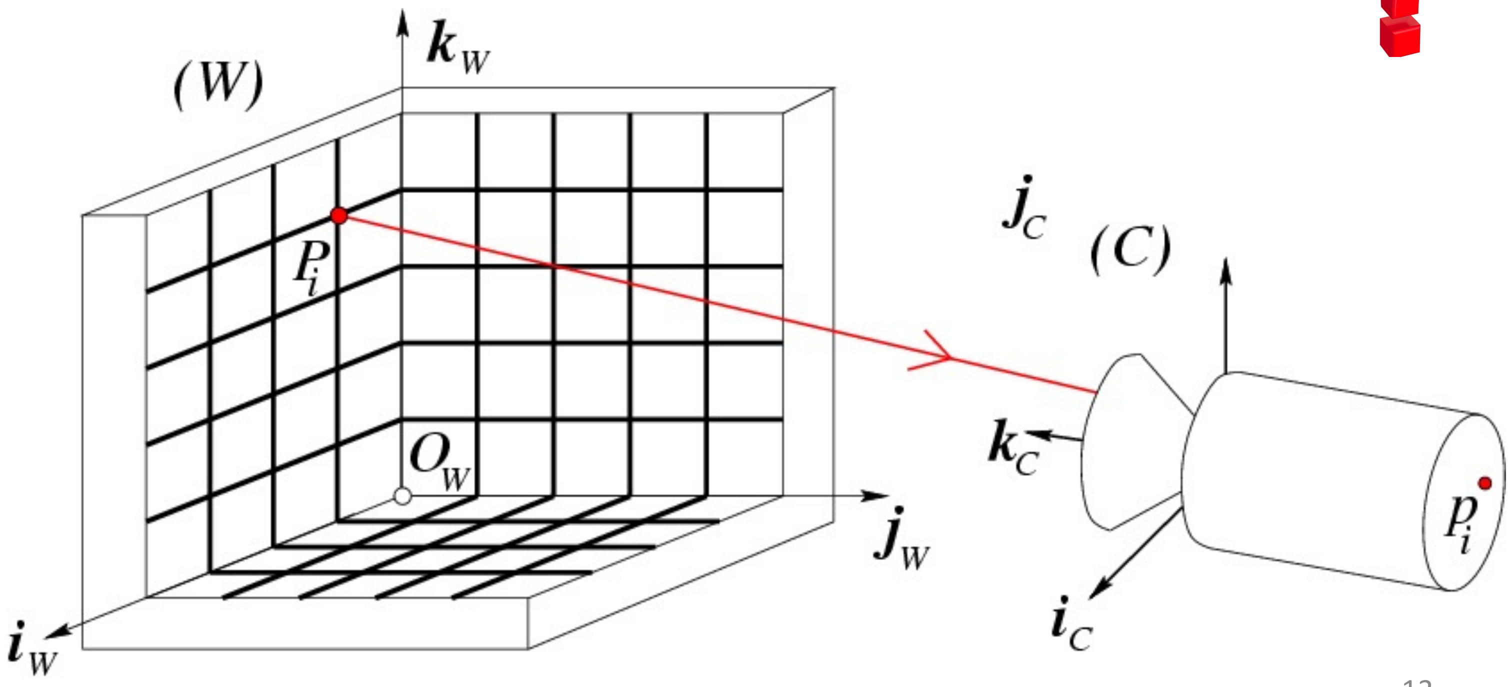
Today's Agenda

- Review of Previous Lecture
  - Camera calibration
- Epipolar Geometry

Recovering 3D Geometry

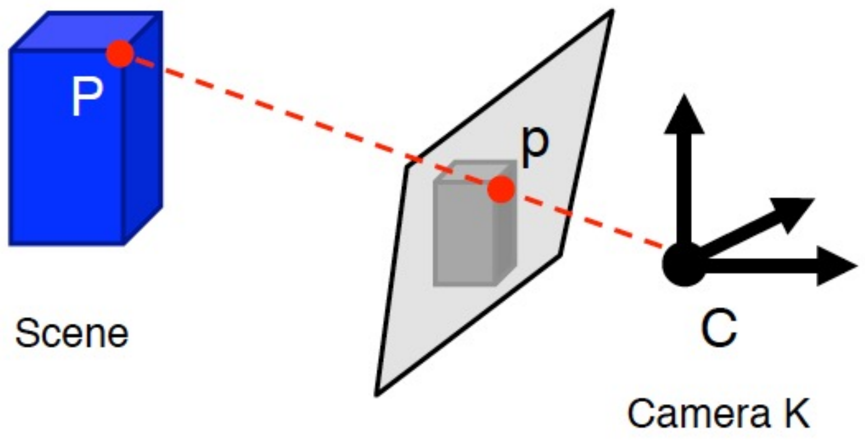
- Camera calibration from a single view
  - Camera intrinsic parameters
  - Camera orientation
  - Camera translation

Sufficient to recover some 3D geometry from a single image?



Recovering 3D Geometry

- Camera calibration from a single view
  - Camera intrinsic parameters
  - Camera orientation
  - Camera translation
- Recover 3D geometry from a single view?
  - No: due to ambiguity of 3D -> 2D mapping



Recovering 3D Geometry

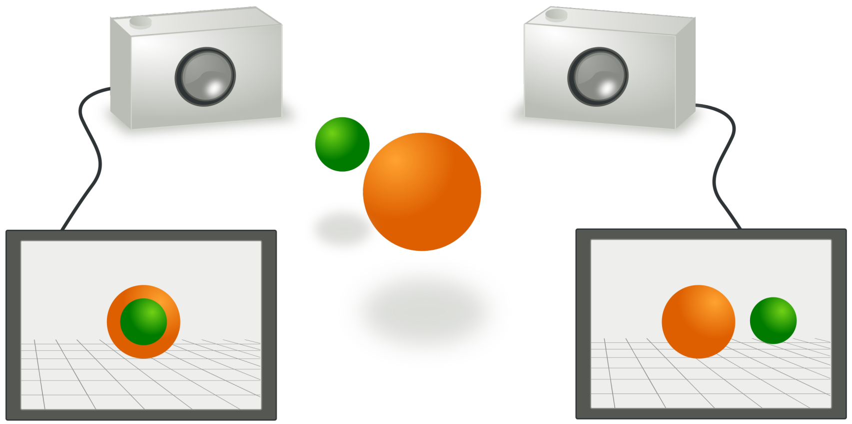
- Camera calibration from a single view
- Recover 3D geometry from a single view?
  - Ambiguity in 3D -> 2D mapping
  - Two (or more) views help

Core Problems in Recovering 3D Geometry

- **Image correspondences**: find the corresponding points in two or more images.
- **Calibration**: given corresponding points in images, recover the relation of the cameras. **Epipolar Geometry**
- **Recover scene geometry**: find coordinates of 3D point from its projections onto 2 or multiple images.

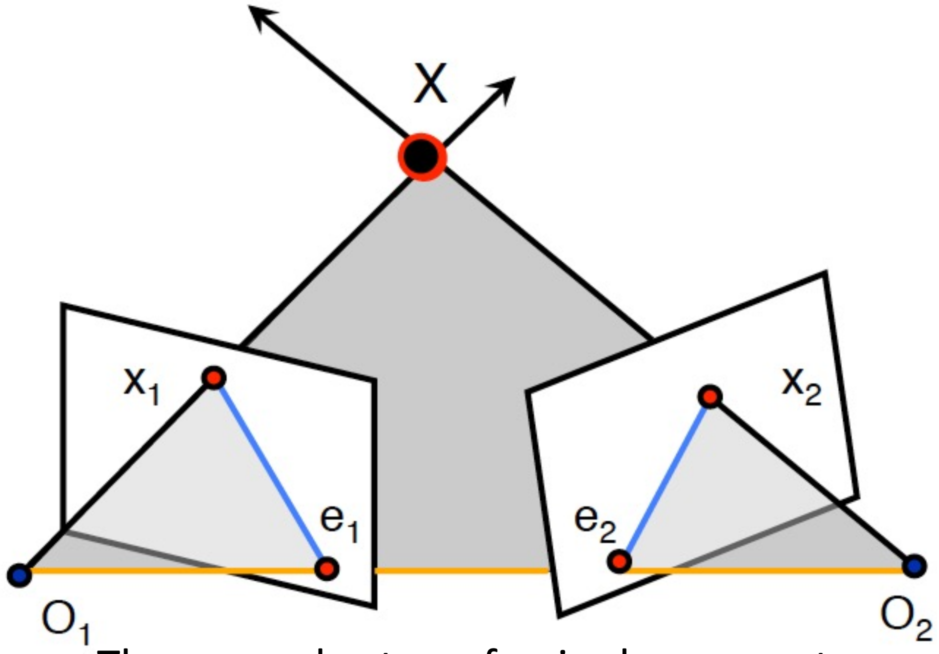
Epipolar Geometry

- The geometry of stereo vision
  - Geometric relations between the corresponding 3D points
    - Define constraints between the 3D points
  - Geometric relations between the corresponding image points
    - Define constraints between the image points



Epipolar Geometry

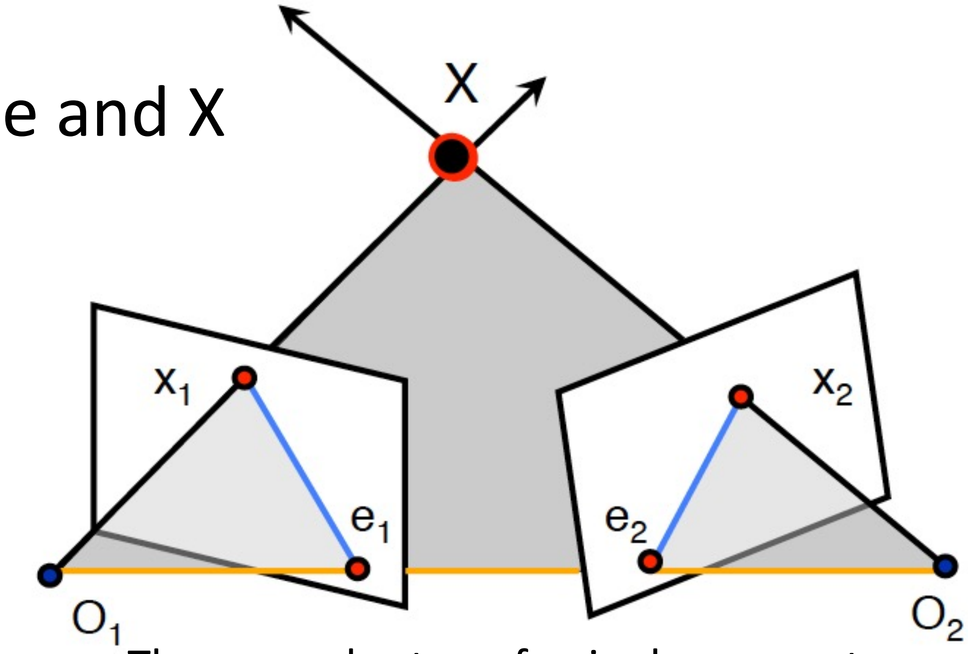
- Baseline
  – The line between the two camera centers



The general setup of epipolar geometry

Epipolar Geometry

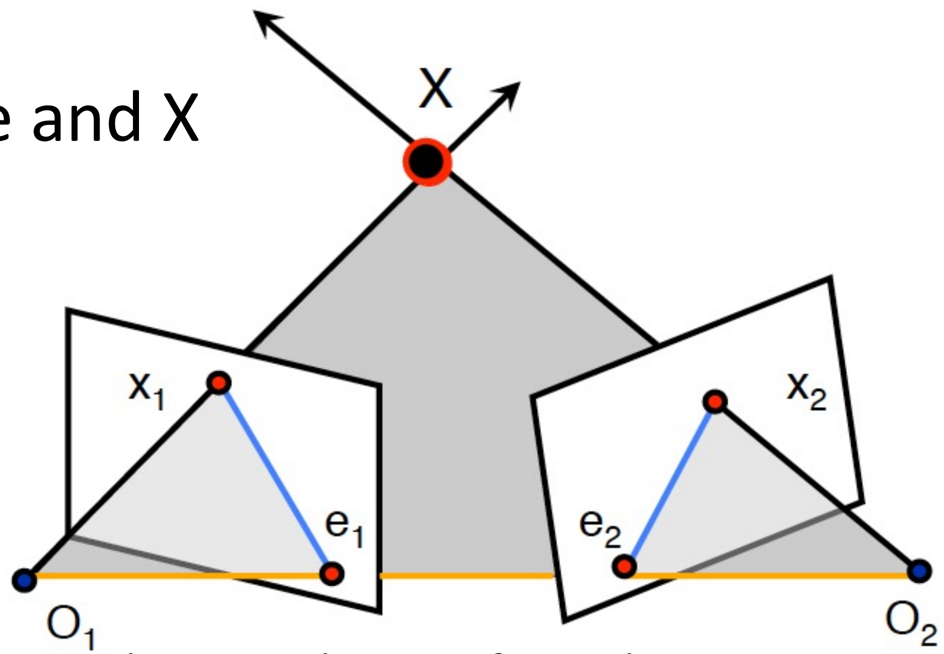
- Baseline
  - The line between the two camera centers
- Epipolar plane
  - Defined by X, O_1, and O_2; contains baseline and X



The general setup of epipolar geometry

Epipolar Geometry

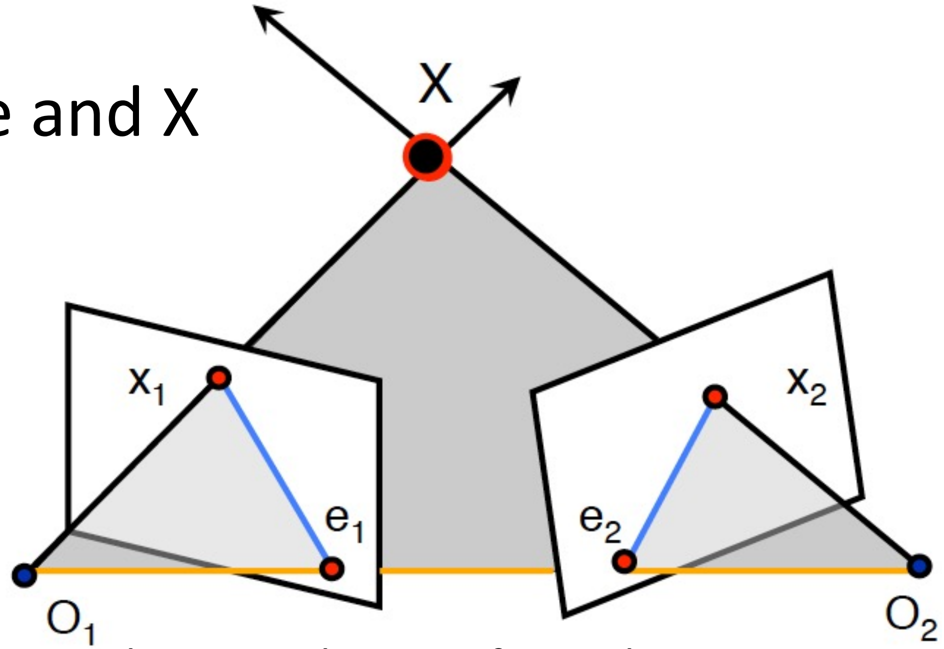
- Baseline
  - The line between the two camera centers
- Epipolar plane
  - Defined by X, O_1, and O_2; contains baseline and X
- Epipoles
  - ∩ of baseline and image plane
  - Projection of the other camera center



The general setup of epipolar geometry

Epipolar Geometry

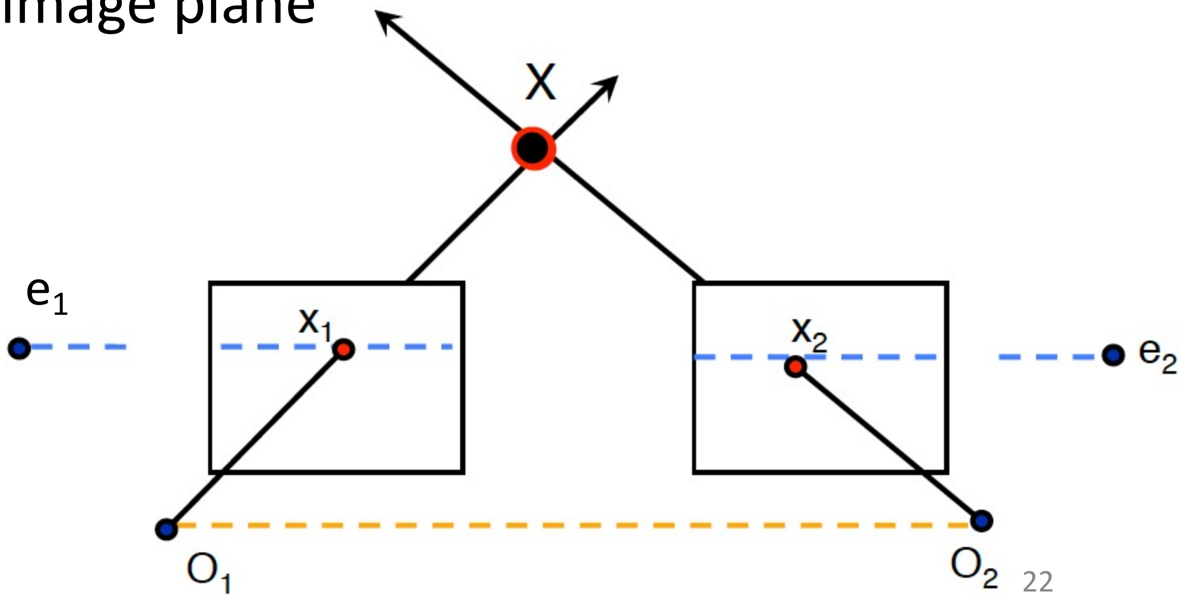
- Baseline
  - The line between the two camera centers
- Epipolar plane
  - Defined by X, O_1, and O_2; contains baseline and X
- Epipoles
  - ∩ of baseline and image plane
  - Projection of the other camera center
- Epipolar lines
  - ∩ of epipolar plane with the image plane



The general setup of epipolar geometry

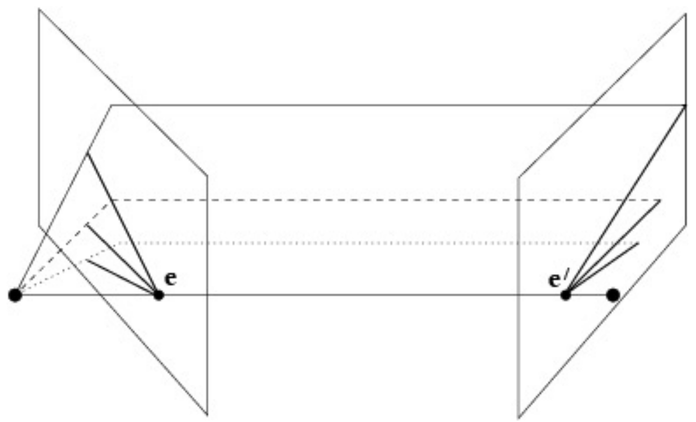
Epipolar Geometry

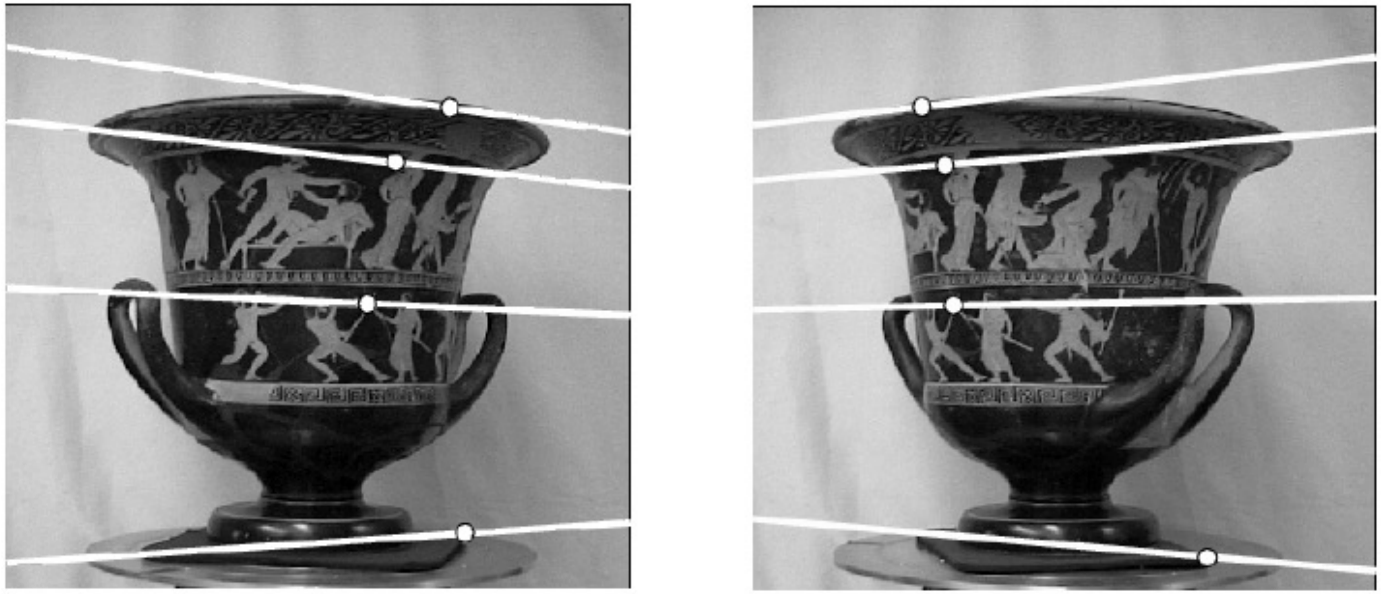
- Example
  - Parallel Image Planes (a special case)
    - Baseline intersects the image plane at infinity!
    - Epipoles are at infinity!
    - Epipolar lines are parallel to U axis of image plane



Epipolar Geometry

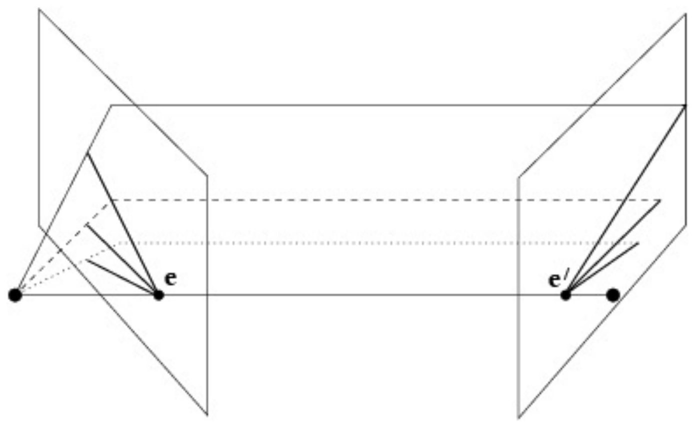
- Example
  - Converging image planes (most common case)
    - All epipolar lines intersect at the epipole

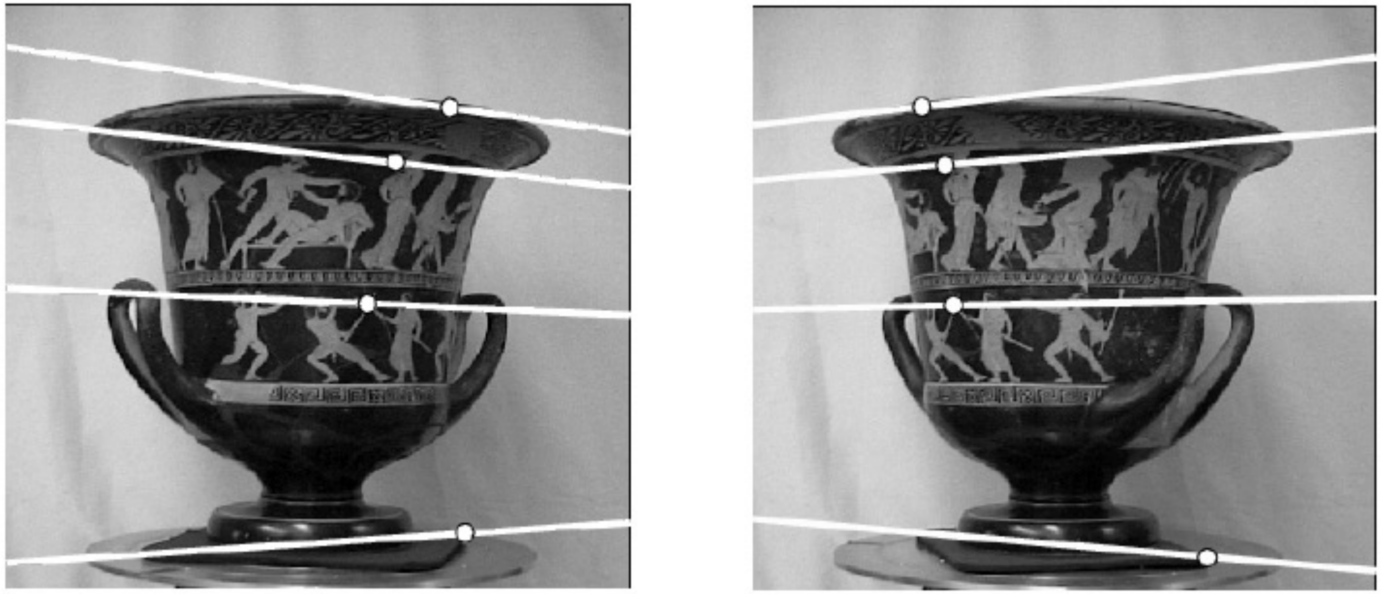




Epipolar Geometry

- The relations between different views?
  - How to use for recovering 3D geometry?
    - Unknown: 3D points
    - Known: image points; camera parameters



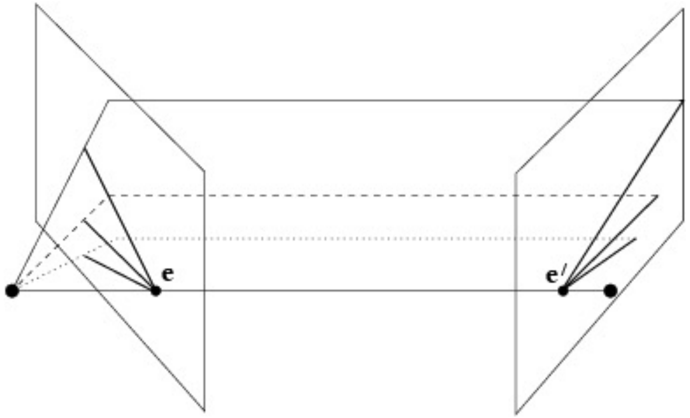


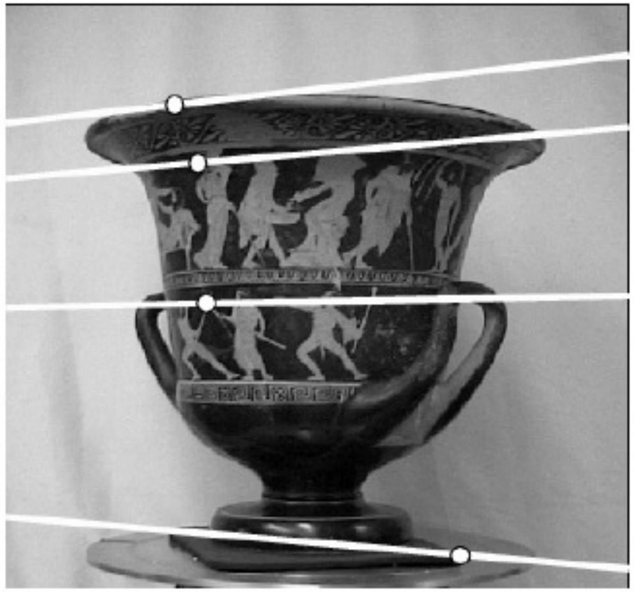
Epipolar Geometry

- Constraints between images (without knowing 3D geometry)

    O_1, O_2, image point → epipolar plane → epipolar line (no known 3D)

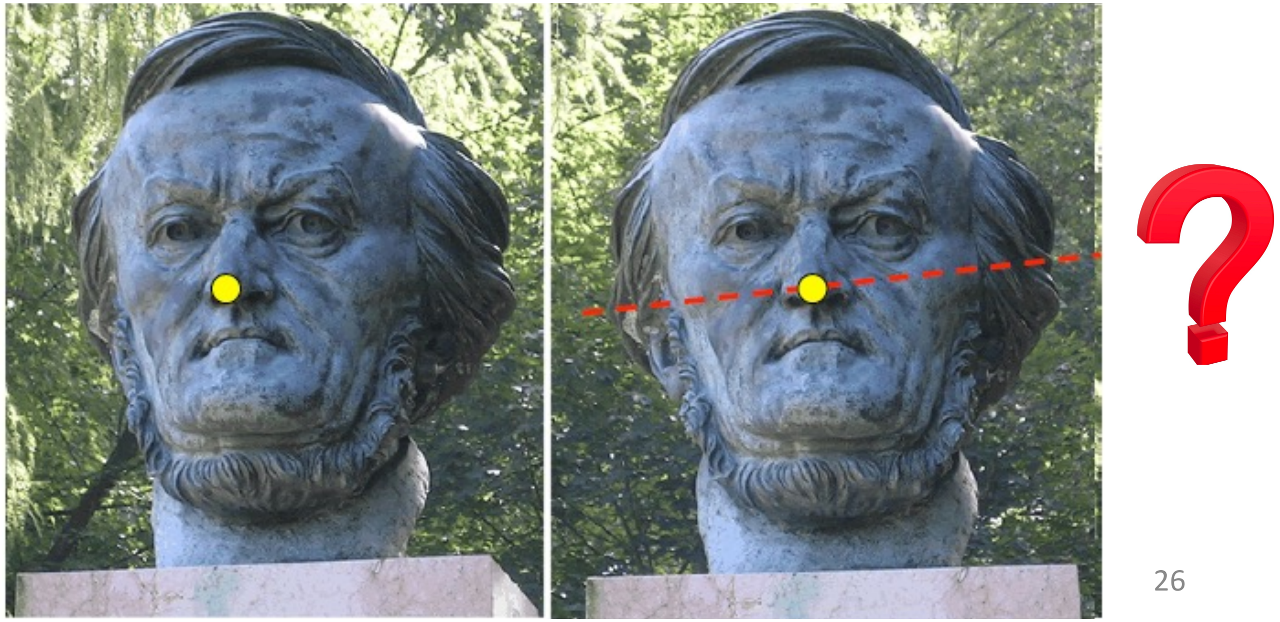
  - Epipolar lines determined by just camera centers and an image point
  - The image point on the second image must be on its Epipolar line



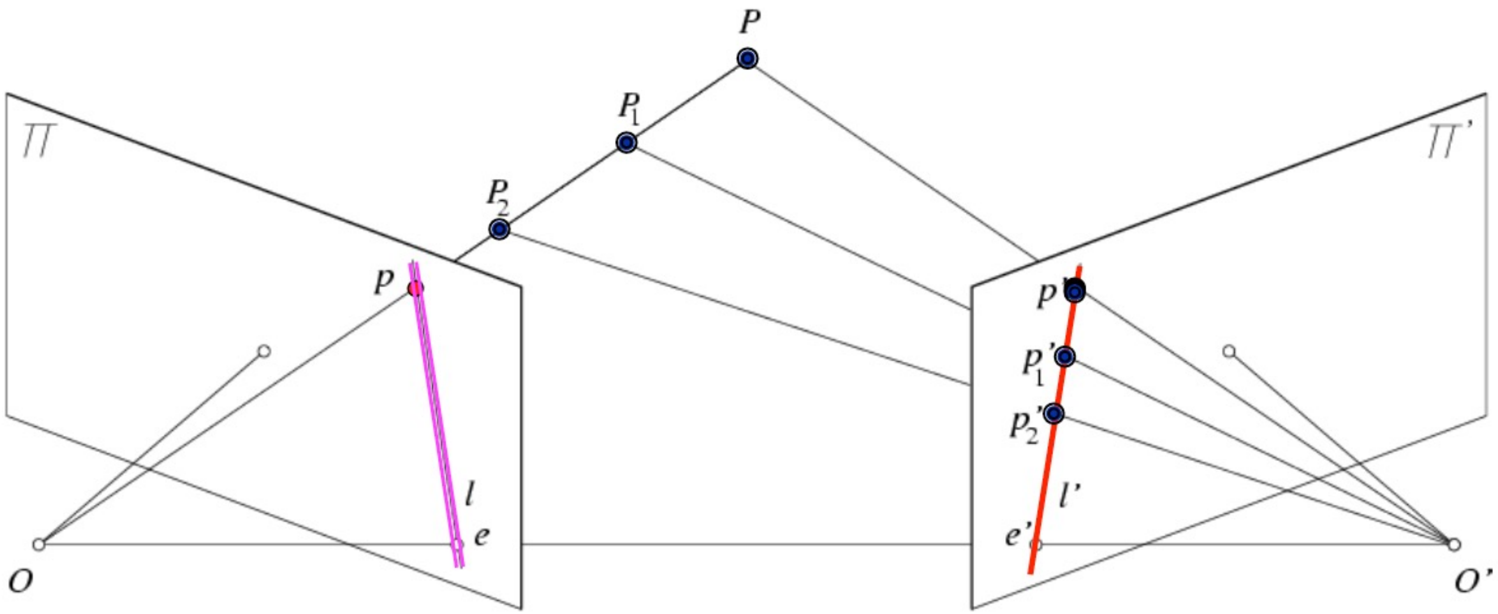
Epipolar Constraint

- Given a point on left image, where to search the corresponding point on right image?
  - Two views of the same object
  - Known camera positions and camera matrices



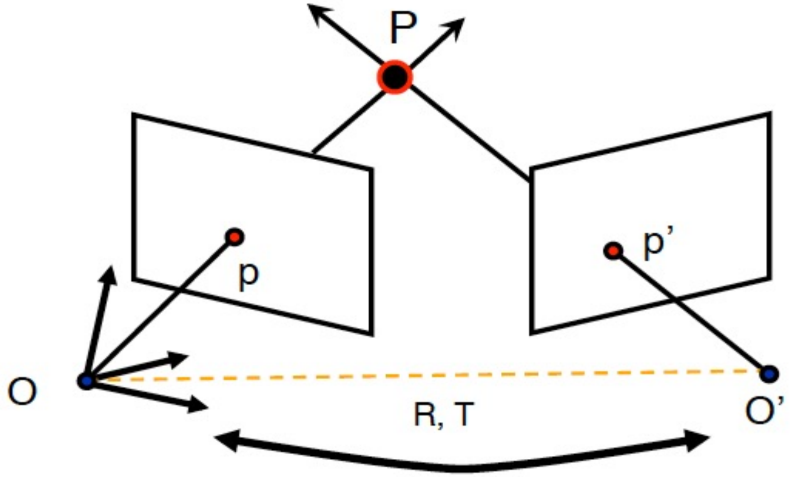
Epipolar Constraint

- Potential matches for a point in one image have to lie on the corresponding epipolar line of the other image
- Can we find the the exact location of that line?



Epipolar Constraint

- The relationship between the two image points
  - Assume the world reference system aligned with the left camera
  - The right camera has orientation *R* and offset *T*



$$M = K[I \quad 0] \qquad M' = K[R \quad T]$$

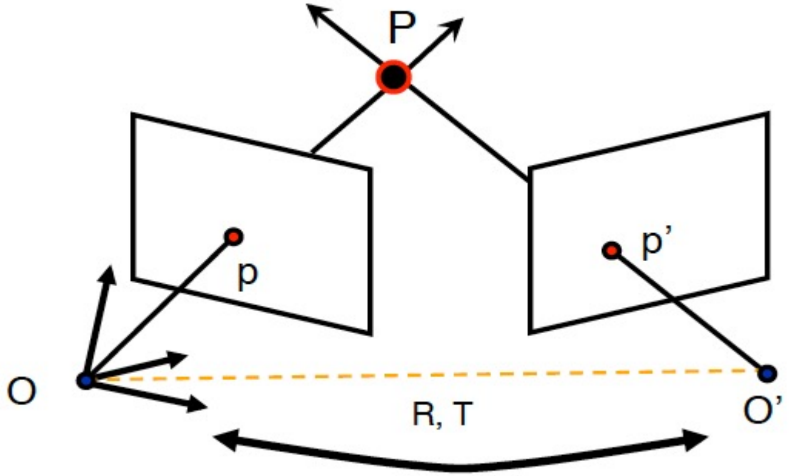
Camera projection matrices

$$P \rightarrow M\,P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \qquad P \rightarrow M'\,P = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$$

Epipolar Constraint

- The relationship between the two image points
  - Canonical cameras
    - K is identity

p′ in camera 1's coordinate system

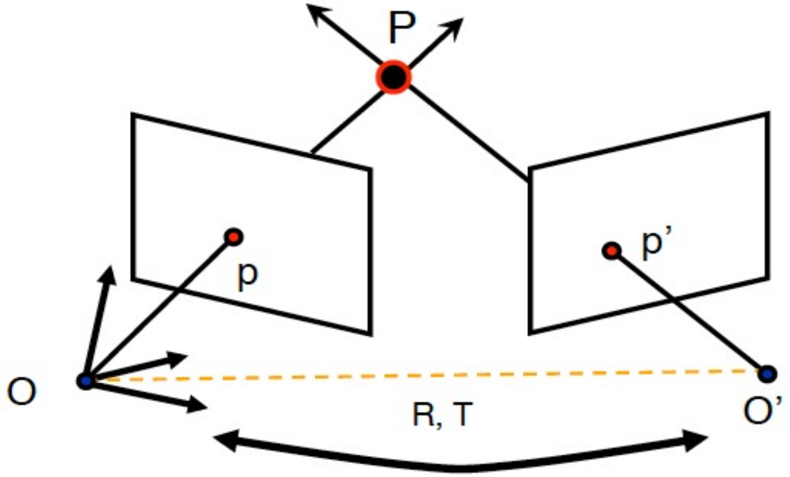


Epipolar Constraint

- The relationship between the two image points
  - Canonical cameras
    - *K* is identity

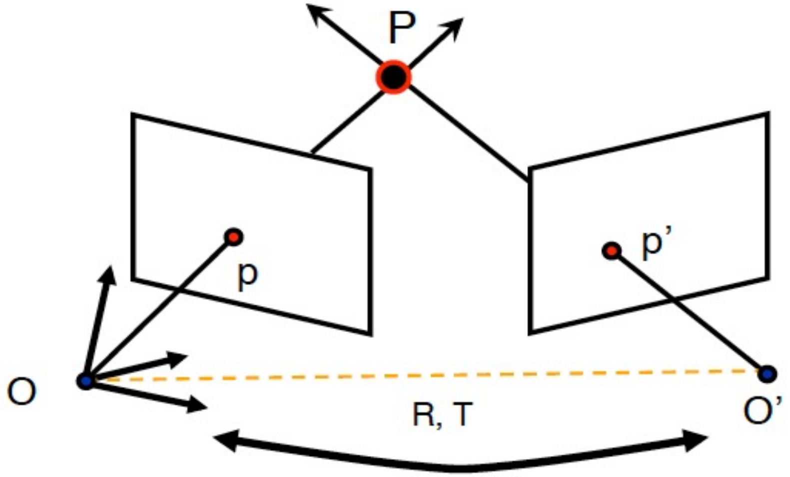
p' in camera 1's coordinate system $R^T(p' - T)$

O' in camera 1's coordinate system



Epipolar Constraint

- The relationship between the two image points
  - Canonical cameras
    - *K* is identity



p' in camera 1's coordinate system $R^T(p' - T)$

O' in camera 1's coordinate system $R^T(O' - T) = -R^T T$

Normal of the Epipolar plane

Epipolar Constraint

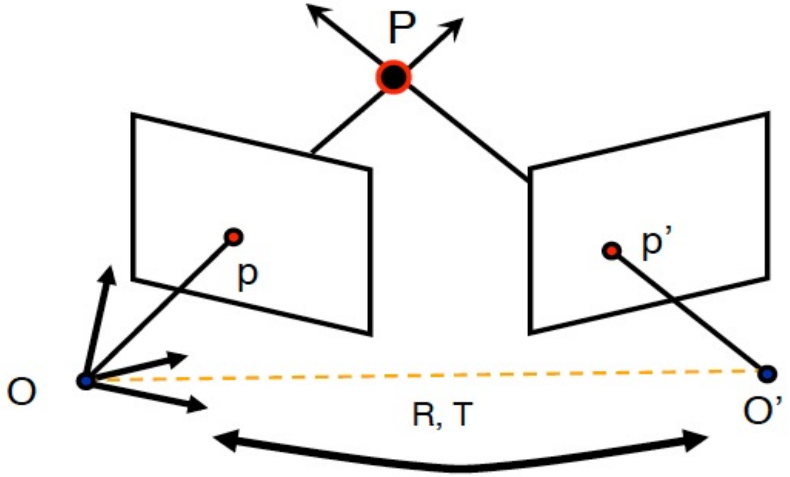
- The relationship between the two image points
  - Canonical cameras
    - *K* is identity

p' in camera 1's coordinate system $R^T(p' - T)$

O' in camera 1's coordinate system $R^T(O' - T) = -R^T T$

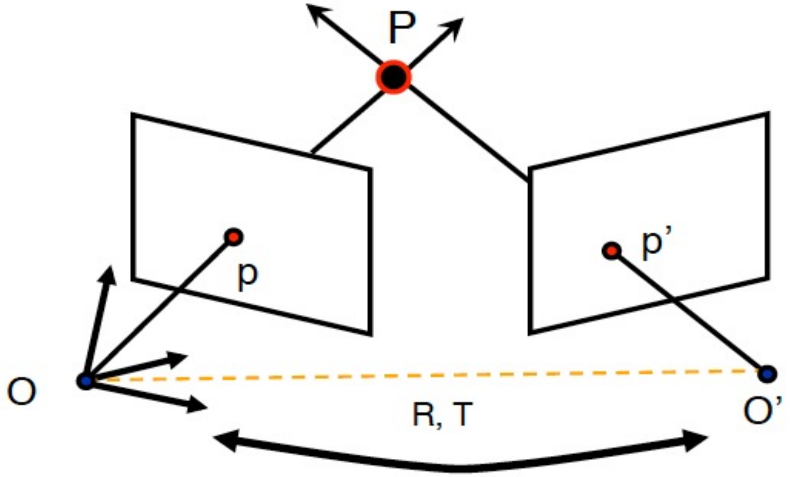
Normal of the Epipolar plane $R^T T \times [R^T(p' - T)] = R^T(T \times p')$

Op lies in the Epipolar plane



Epipolar Constraint

- The relationship between the two image points
  - Canonical cameras
    - *K* is identity



p' in camera 1's coordinate system $R^T(p' - T)$

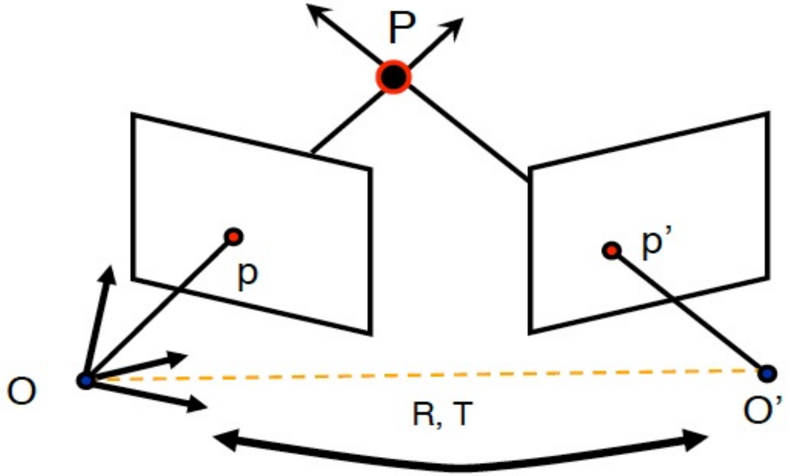
O' in camera 1's coordinate system $R^T(O' - T) = -R^T T$

Normal of the Epipolar plane $R^T T \times [R^T(p' - T)] = R^T(T \times p')$

Op lies in the Epipolar plane $[R^T(T \times p')]^T p = 0$

Epipolar Constraint

- The relationship between the two image points
  - Canonical cameras
    - *K* is identity



p' in camera 1's coordinate system $R^T(p' - T)$

O' in camera 1's coordinate system $R^T(O' - T) = -R^T T$

Normal of the Epipolar plane $R^T T \times [R^T(p' - T)] = R^T(T \times p')$

Op lies in the Epipolar plane $[R^T(T \times p')]^T p = 0 \Rightarrow (T \times p')^T R p = 0$

Epipolar Constraint

- The relationship between the two image points
  - Canonical cameras
    - *K* is identity

Cross product as matrix-vector multiplication

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -\mathbf{a}_z & \mathbf{a}_y \\ \mathbf{a}_z & 0 & -\mathbf{a}_x \\ -\mathbf{a}_y & \mathbf{a}_x & 0 \end{bmatrix} \begin{bmatrix} \mathbf{b}_x \\ \mathbf{b}_y \\ \mathbf{b}_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

$[a_\times]^T = -[a_\times]$

p' in camera 1's coordinate system $R^T(p' - T)$

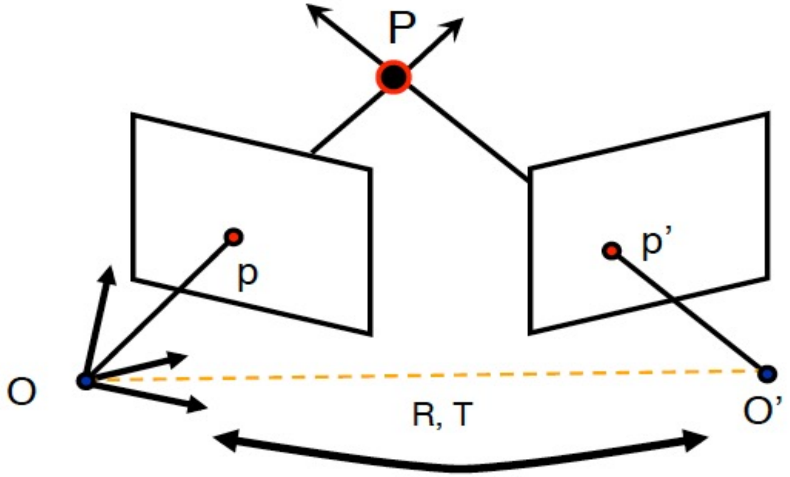
O' in camera 1's coordinate system $R^T(O' - T) = -R^T T$

Normal of the Epipolar plane $R^T T \times [R^T(p' - T)] = R^T(T \times p')$

Op lies in the Epipolar plane $[R^T(T \times p')]^T p = 0 \Rightarrow (T \times p')^T R p = 0$

Epipolar Constraint

- The relationship between the two image points
  - Canonical cameras
    - *K* is identity



p' in camera 1's coordinate system $R^T(p' - T)$

O' in camera 1's coordinate system $R^T(O' - T) = -R^T T$

Normal of the Epipolar plane $R^T T \times [R^T(p' - T)] = R^T(T \times p')$

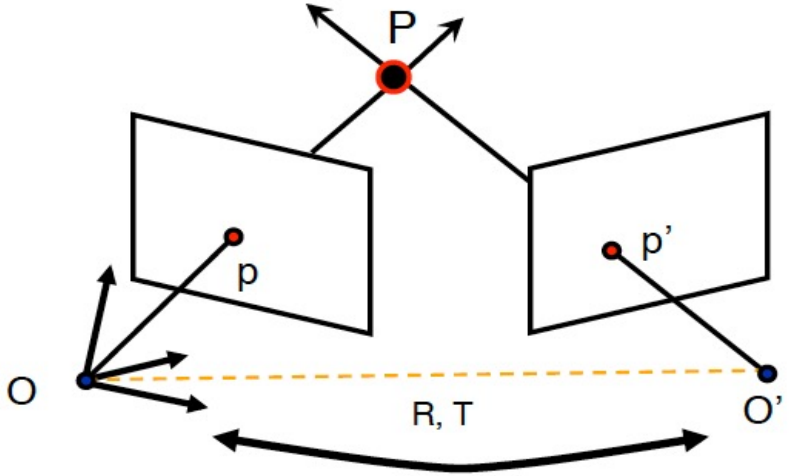
Op lies in the Epipolar plane $[R^T(T \times p')]^T p = 0 \Rightarrow (T \times p')^T R p = 0$

$T \times p' = [T_\times] p'$

$\Rightarrow ([T_\times] p')^T R p = 0$

Epipolar Constraint

- The relationship between the two image points
  - Canonical cameras
    - *K* is identity



p' in camera 1's coordinate system $R^T(p' - T)$

O' in camera 1's coordinate system $R^T(O' - T) = -R^T T$

Normal of the Epipolar plane $R^T T \times [R^T(p' - T)] = R^T(T \times p')$

Op lies in the Epipolar plane $[R^T(T \times p')]^T p = 0$ ⇒ $(T \times p')^T R p = 0$

$T \times p' = [T_\times] p'$ ⇒ $([T_\times] p')^T R p = 0$ ⇒ $p'^T [T_\times] R\, p = 0$

Epipolar Constraint

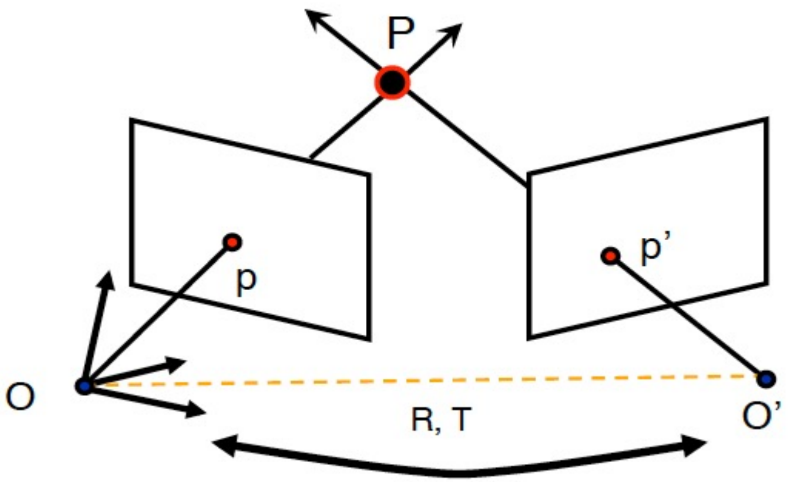
- Essential matrix
  - Establish constraints between matching image points
  - Determine relative position and orientation of two cameras
  - 5 degrees of freedom (R: 3, T: 3, but scale is not known)

$$p'^T [T_\times] R \, p = 0$$

$$E = [T_\times] R$$

$$p'^T E \, p = 0$$

Essential matrix



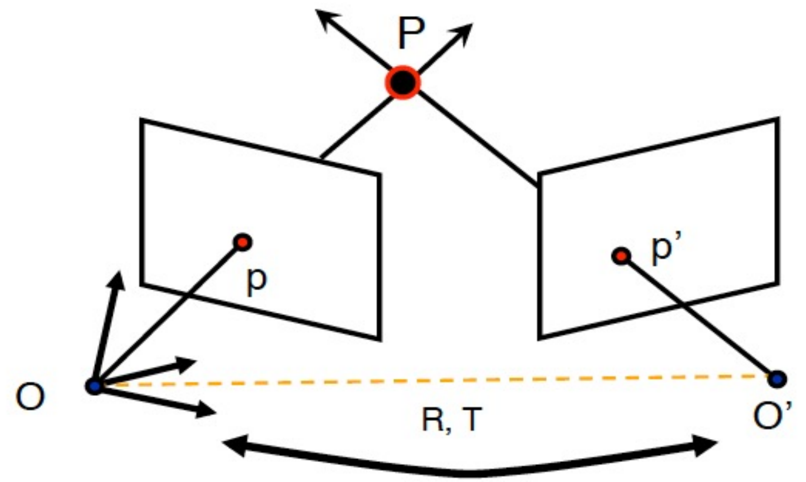
Epipolar Constraint

- How to generalize Essential matrix?
  - Canonical cameras
    - *K* is identity

$$E = [T_X]R \qquad p'^T E\, p = 0$$

$$M = [I \quad 0] \qquad K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M' = [R \quad T]$$

$$\mathrm{M} = \mathrm{K}[\mathrm{I} \quad 0] \qquad \mathrm{M'} = \mathrm{K}[\mathrm{R} \quad \mathrm{T}]$$

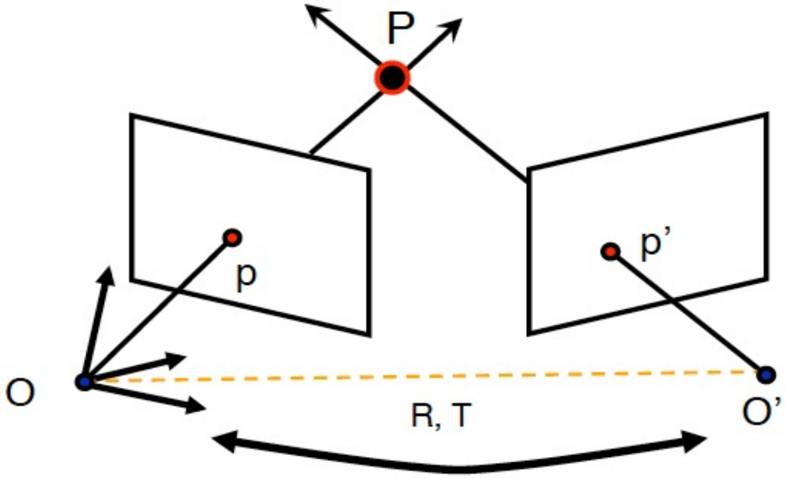


Epipolar Constraint

- How to generalize Essential matrix?

$M = [I \quad 0]$ $K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $M' = [R \quad T]$ → $p'^T E\, p = 0, \quad E = [T_X]R$

$M = K[I \quad 0]$ $M' = K[R \quad T]$?



Epipolar Constraint

- Fundamental matrix

$$M = [I \quad 0] \qquad K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M' = [R \quad T] \Rightarrow p'^T E\, p = 0, \;\; E = [T_\times]R$$

$$M = K[I \quad 0] \qquad M' = K[R \quad T] \Rightarrow p'^T F\, p = 0, \;\; F = K'^{-T}[T_\times]RK^{-1}$$

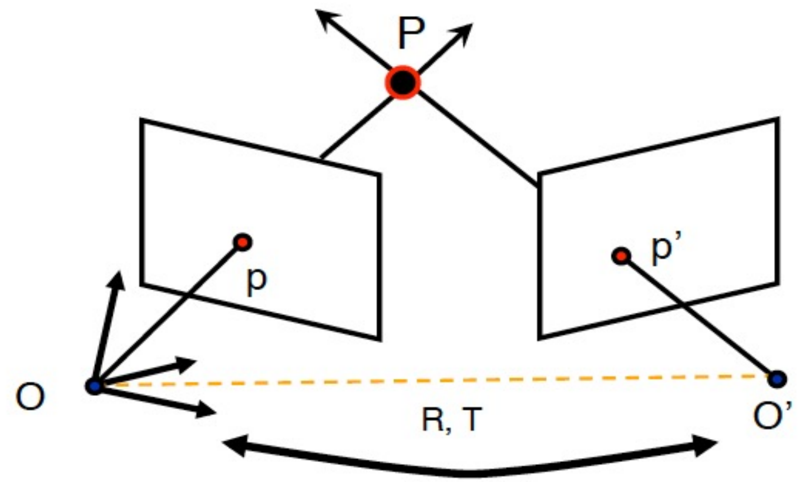
Try to derive F after the lecture

Hint for derivation

$$\begin{cases} p \rightarrow K^{-1}\, p \\ p' \rightarrow K'^{-1}\, p' \end{cases}$$

$$p'^T \boxed{K'^{-T}[T_\times]RK^{-1}} p = 0$$

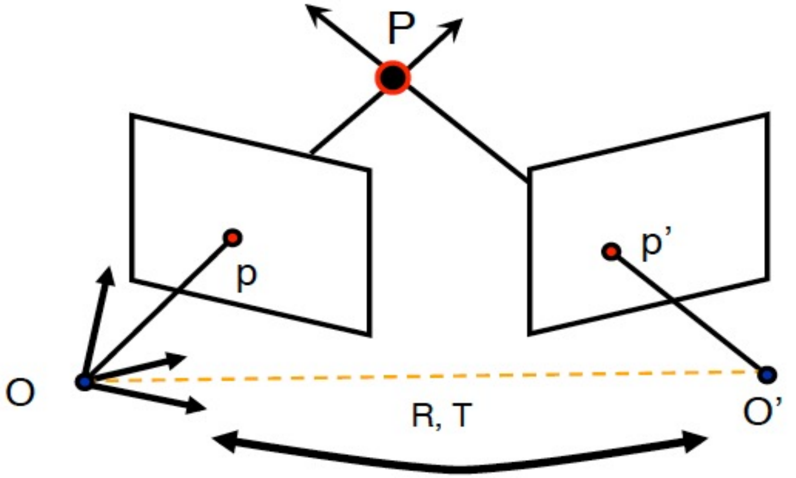
F



Epipolar Constraint

- Fundamental matrix *F* is a matrix product of camera parameters
- Encode Epipolar geometry of 2 views & camera parameters

$$p'^T F\, p = 0, \quad F = K'^{-T}[T_\times]RK^{-1}$$

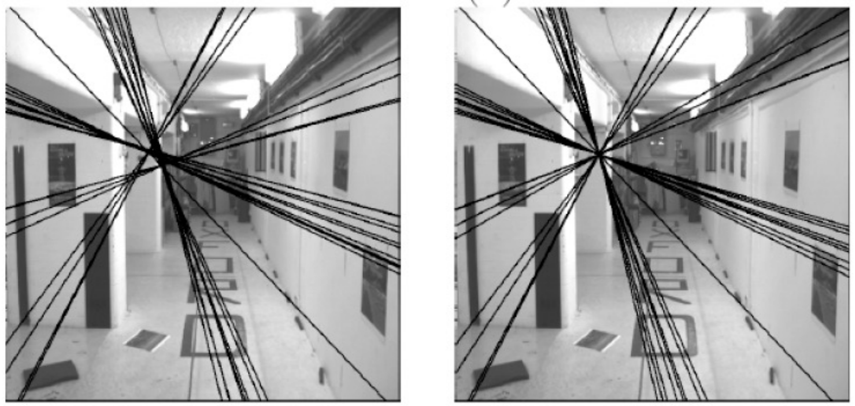


Epipolar Constraint

- Fundamental matrix *F* is a matrix product of camera parameters
- Encode Epipolar geometry of 2 views & camera parameters
- *F* has 7 degrees of freedom
  - 3 x 3
  - homogeneous (has scale ambiguity)
  - rank(*F*) = 2
    - The potential matching point is located on a line

$$p'^T F\, p = 0, \quad F = K'^{-T}[T_\times]RK^{-1}$$

Fundamental matrix has rank 2 : $\det(\mathtt{F}) = 0$.



Left : Uncorrected F – epipolar lines are not coincident.

Right : Epipolar lines from corrected F.

Epipolar Constraint

- Fundamental matrix *F* is a matrix product of camera parameters
- Encode Epipolar geometry of 2 views & camera parameters
- *F* has 7 degrees of freedom
- How is the fundamental matrix useful?

$$p'^T F\, p = 0, \quad F = K'^{-T}[T_\times]RK^{-1}$$

Epipolar Constraint

- Fundamental matrix *F* is a matrix product of camera parameters
- Encode Epipolar geometry of 2 views & camera parameters
- *F* has 7 degrees of freedom
- How is the fundamental matrix useful?
  - A 3D point's image in one image -> the Epipolar line in the other image
    - No need 3D location
    - No need camera intrinsic and extrinsic parameters

$$p'^T F\, p = 0, \quad F = K'^{-T}[T_\times]RK^{-1}$$

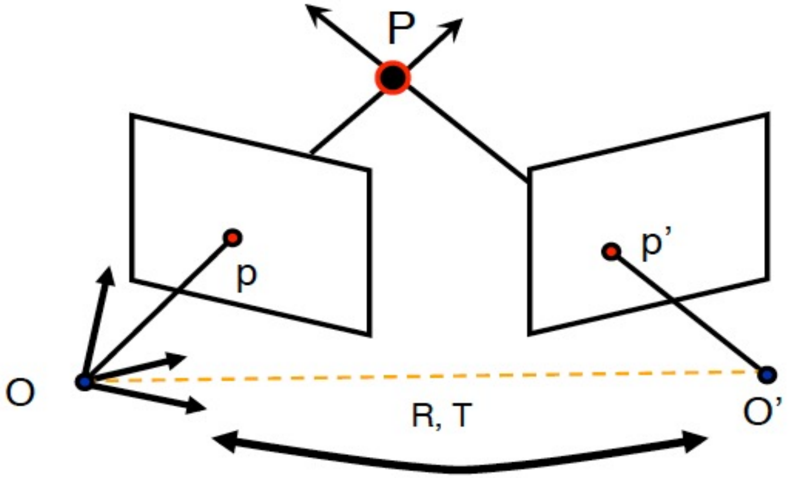
Epipolar Constraint

- Fundamental matrix *F* is a matrix product of camera parameters
- Encode Epipolar geometry of 2 views & camera parameters
- *F* has 7 degrees of freedom
- How is the fundamental matrix useful?
  - A 3D point's image in one image -> the Epipolar line in the other image
    - No need 3D location
    - No need camera intrinsic and extrinsic parameters
  - Powerful tool
    - 3D reconstruction
    - Multi-view object/scene matching

Recovering Fundamental Matrix

- How to recover F?
  - From image correspondences

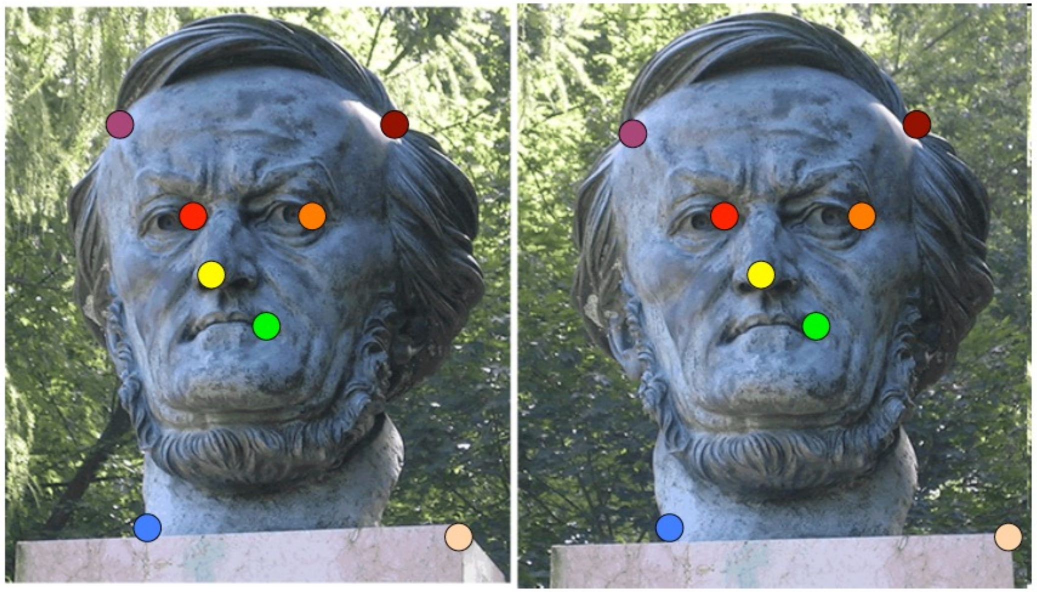
$$p'^{T} F\, p = 0, \quad F = K'^{-T}[T_{\times}]RK^{-1}$$



Recovering Fundamental Matrix

- How to recover *F*?
  - From image correspondences
  - How many point pairs needed?

$$p'^T F\, p = 0, \quad F = K'^{-T}[T_\times]RK^{-1}$$



Recovering Fundamental Matrix

- How to recover *F*?
  - From image correspondences
  - At least 8-point pairs are needed
    - Each point pair give one equation
    - The linear system is homogeneous

$$\begin{cases} p_i = (u_i, v_i, 1) \\ p'_i = (u'_i, v'_i, 1) \end{cases} + \quad p'^T F\, p = 0$$

$$\begin{bmatrix} u_i u'_i & v_i u'_i & u'_i & u_i v'_i & v_i v'_i & v'_i & u_i & v_i & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Recovering Fundamental Matrix

- How to recover *F*?
  - From image correspondences
  - At least 8-point pairs are needed
    - Each point pair give one equation
    - The linear system is homogeneous

$$\begin{cases} p_i = (u_i, v_i, 1) \\ p'_i = (u'_i, v'_i, 1) \end{cases} + \quad p'^T F\, p = 0$$

$$\begin{bmatrix} u_i u'_i & v_i u'_i & u'_i & u_i v'_i & v_i v'_i & v'_i & u_i & v_i & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

F has 7 degrees of freedom

Are 7-point pairs sufficient?

Recovering Fundamental Matrix

- 8-point algorithm

$$\begin{bmatrix} u_i u_i' & v_i u_i' & u_i' & u_i v_i' & v_i v_i' & v_i' & u_i & v_i & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

$$\begin{bmatrix} u_1 u_1' & v_1 u_1' & u_1' & u_1 v_1' & v_1 v_1' & v_1' & u_1 & v_1 & 1 \\ u_2 u_2' & v_2 u_2' & u_2' & u_2 v_2' & v_2 v_2' & v_2' & u_2 & v_2 & 1 \\ u_3 u_3' & v_3 u_3' & u_3' & u_3 v_3' & v_3 v_3' & v_3' & u_3 & v_3 & 1 \\ u_4 u_4' & v_4 u_4' & u_4' & u_4 v_4' & v_4 v_4' & v_4' & u_4 & v_4 & 1 \\ u_5 u_5' & v_5 u_5' & u_5' & u_5 v_5' & v_5 v_5' & v_5' & u_5 & v_5 & 1 \\ u_6 u_6' & v_6 u_6' & u_6' & u_6 v_6' & v_6 v_6' & v_6' & u_6 & v_6 & 1 \\ u_7 u_7' & v_7 u_7' & u_7' & u_7 v_7' & v_7 v_7' & v_7' & u_7 & v_7 & 1 \\ u_8 u_8' & v_8 u_8' & u_8' & u_8 v_8' & v_8 v_8' & v_8' & u_8 & v_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Recovering Fundamental Matrix

- 8-point algorithm
- How to solve it?

$$W\mathbf{f} = 0$$

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ u_3u_3' & v_3u_3' & u_3' & u_3v_3' & v_3v_3' & v_3' & u_3 & v_3 & 1 \\ u_4u_4' & v_4u_4' & u_4' & u_4v_4' & v_4v_4' & v_4' & u_4 & v_4 & 1 \\ u_5u_5' & v_5u_5' & u_5' & u_5v_5' & v_5v_5' & v_5' & u_5 & v_5 & 1 \\ u_6u_6' & v_6u_6' & u_6' & u_6v_6' & v_6v_6' & v_6' & u_6 & v_6 & 1 \\ u_7u_7' & v_7u_7' & u_7' & u_7v_7' & v_7v_7' & v_7' & u_7 & v_7 & 1 \\ u_8u_8' & v_8u_8' & u_8' & u_8v_8' & v_8v_8' & v_8' & u_8 & v_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Recovering Fundamental Matrix

- 8-point algorithm
- Solved using SVD

$$W\mathbf{f} = 0$$

Just the idea on how to recover F.
Details in the lecture note.

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ u_3u_3' & v_3u_3' & u_3' & u_3v_3' & v_3v_3' & v_3' & u_3 & v_3 & 1 \\ u_4u_4' & v_4u_4' & u_4' & u_4v_4' & v_4v_4' & v_4' & u_4 & v_4 & 1 \\ u_5u_5' & v_5u_5' & u_5' & u_5v_5' & v_5v_5' & v_5' & u_5 & v_5 & 1 \\ u_6u_6' & v_6u_6' & u_6' & u_6v_6' & v_6v_6' & v_6' & u_6 & v_6 & 1 \\ u_7u_7' & v_7u_7' & u_7' & u_7v_7' & v_7v_7' & v_7' & u_7 & v_7 & 1 \\ u_8u_8' & v_8u_8' & u_8' & u_8v_8' & v_8v_8' & v_8' & u_8 & v_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Next Lecture

- Image Matching
  - Find corresponding image points
- Triangulation
- Structure from Motion
  - Go beyond two views
  - Simultaneously determine 3D structure & camera parameters