Lecture
Camera Calibration

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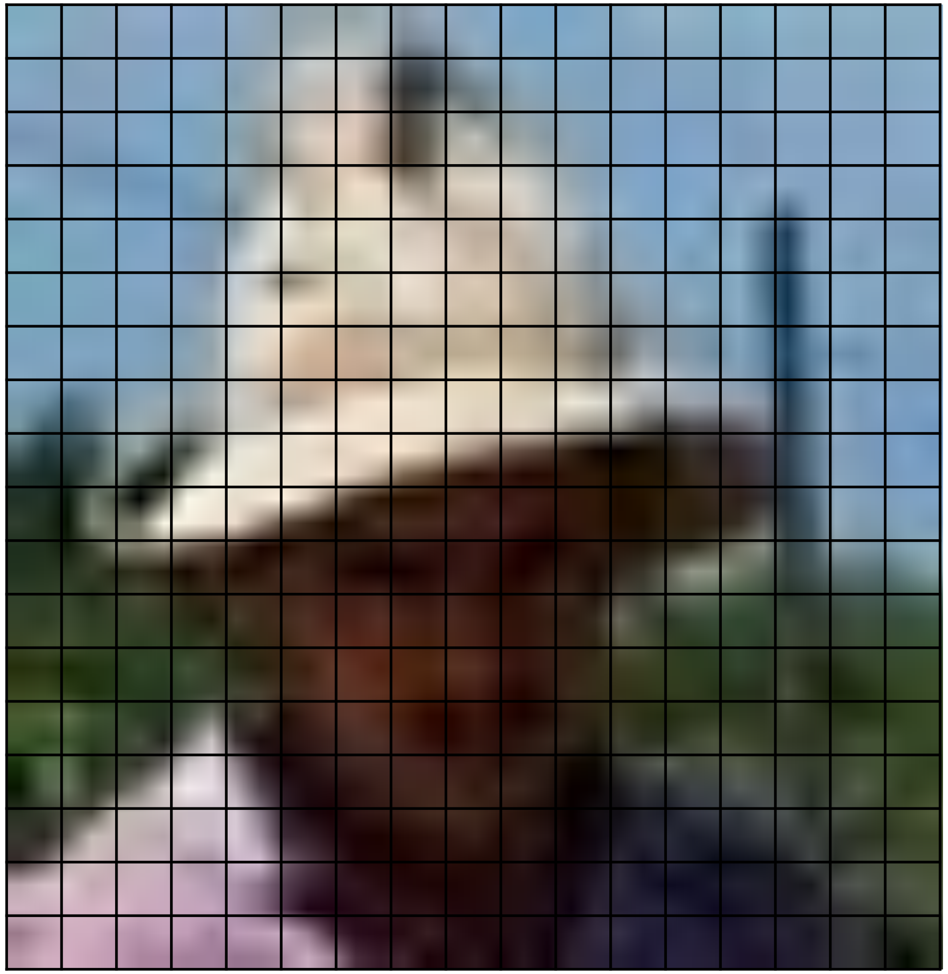
Today’s Agenda

- Review: Camera models
- Camera calibration
- A1: Camera calibration

Images

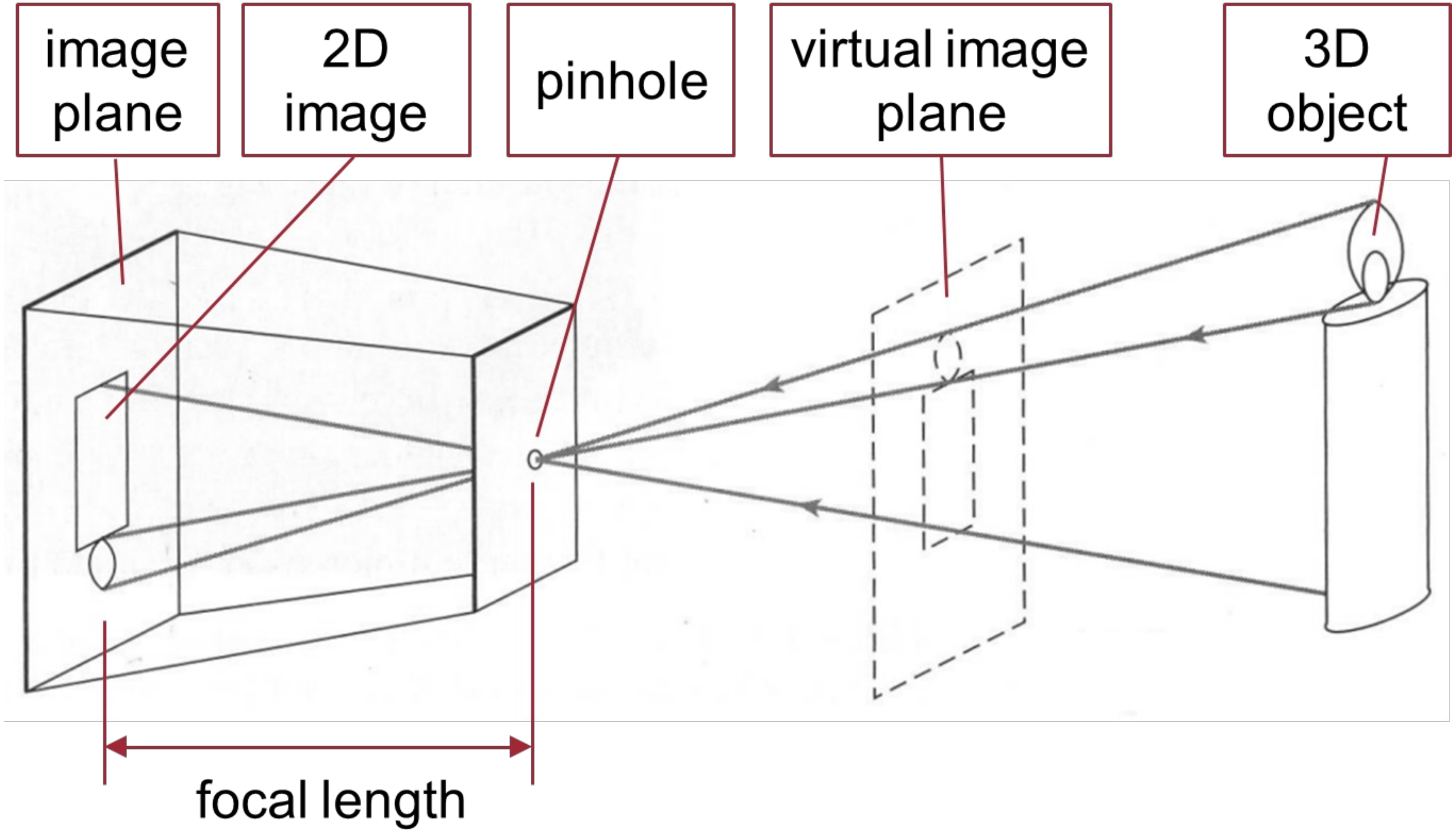


A color image: R, G, B channels

$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}$$

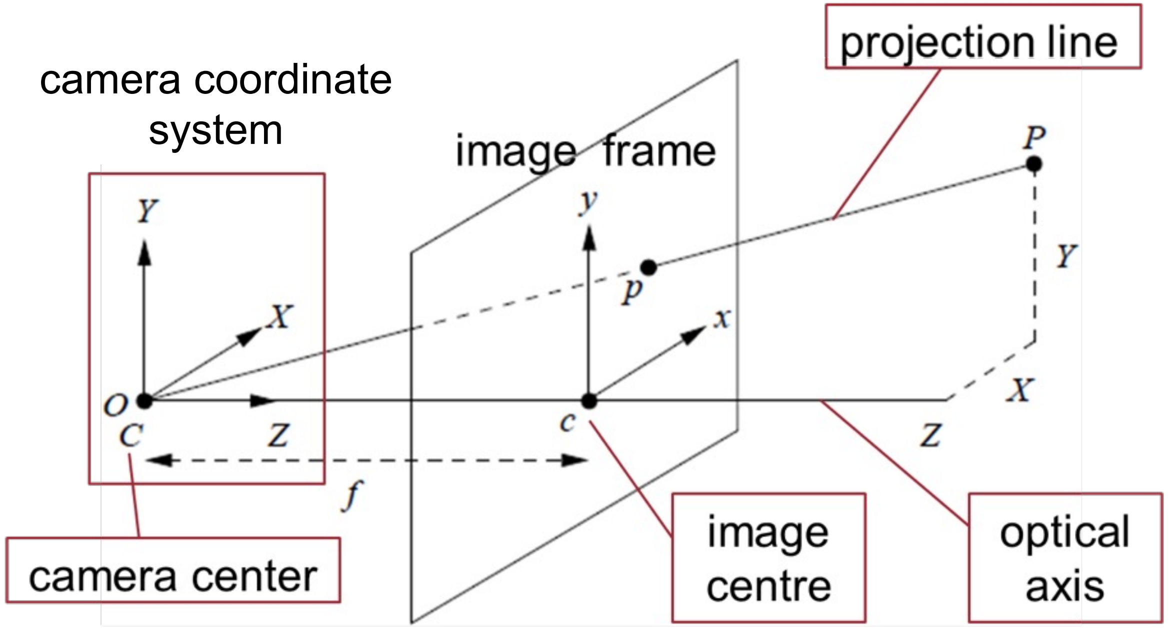
"vector-valued" function

Pinhole camera model



Pinhole camera model

- 3D point $\boldsymbol{P} = (X, Y, Z)^T$ projected to 2D image $\boldsymbol{p} = (x, y)^T$



$$x = f\frac{X}{Z}, \quad y = f\frac{Y}{Z}$$

Perspective projection model

- Camera sensor's pixels not exactly square $x = kf\frac{X}{Z}, \quad y = lf\frac{Y}{Z}$

Perspective projection model

- Camera sensor's pixels not exactly square $x = kf\frac{X}{Z}, \quad y = lf\frac{Y}{Z}$
- Image center or principal point c may not be at origin

$$x = f_x\frac{X}{Z} + c_x, \quad y = f_y\frac{Y}{Z} + c_y$$

Perspective projection model

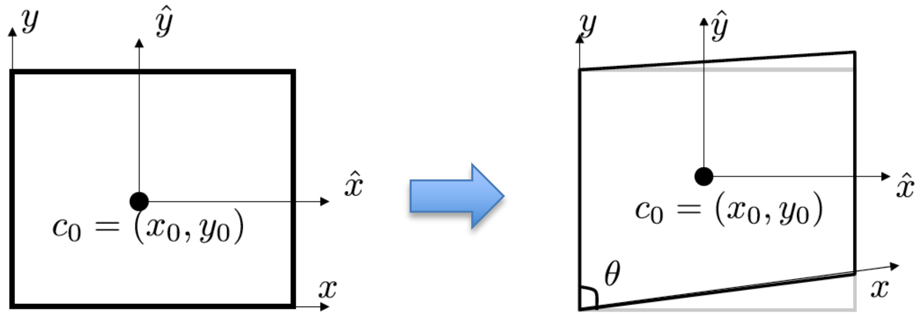
- Camera sensor's pixels not exactly square $x = kf\frac{X}{Z}, \quad y = lf\frac{Y}{Z}$
- Image center or principal point c may not be at origin

$$x = f_x \frac{X}{Z} + c_x, \quad y = f_y \frac{Y}{Z} + c_y$$

- Skew: image frame may not be exactly rectangular

$$x = f_x \frac{X}{Z} - f_x \cot\theta \frac{Y}{Z} + c_x, \quad y = \frac{f_y}{\sin\theta}\frac{Y}{Z} + c_y$$

θ: the skew angle between x and y axes of the image frame



Perspective projection model

$$x = f_x \frac{X}{Z} - f_x \cot\theta \frac{Y}{Z} + c_x, \quad y = \frac{f_y}{\sin\theta}\frac{Y}{Z} + c_y$$

- Intrinsic parameters
- Intrinsic parameter matrix

$$\tilde{\mathrm{x}} = \frac{1}{Z}\mathbf{K}\mathbf{X}, \quad \mathbf{K} = \begin{bmatrix} f_x & -f_x \cot\theta & c_x \\ 0 & \frac{f_y}{\sin\theta} & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

Attention:
Notation change

$$\mathbf{K} = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

Intrinsic parameter matrix

Perspective projection model

- Camera motion
  - World frame may not align with the frame
  - Camera can move and rotate
  - Extrinsic parameters

$$^{C}\mathbf{X} = {}^{C}_{W}\mathbf{R}\,{}^{W}\mathbf{X} + {}^{C}_{W}\mathbf{T}$$

- R: rotation matrix of the world coordinate system defined in the camera coordinate system
- T: the position of world coordinate system's origin in camera coordinate system

  (Note: T is often mistakenly considered the position of the camera in the world coordinate system)

Perspective projection model

- The complete transformation

$$\mathbf{p} = M\mathbf{P}$$
$$= K \begin{bmatrix} R & \mathbf{t} \end{bmatrix} \mathbf{P}$$

Internal (intrinsic) parameters (K)

External (extrinsic) parameters ($[R \ \mathbf{t}]$)

Today's Agenda

- Review: Camera models
- Camera calibration
- A1: Camera calibration

General Idea

- Why is camera calibration necessary?
  - Given 3D scene, knowing the precise 3D to 2D projection requires
    - Intrinsic and extrinsic parameters
  - Reconstructing 3D geometry from images also requires these parameters

$$\mathbf{p} = M\mathbf{P}$$

$$= K \begin{bmatrix} R & \mathbf{t} \end{bmatrix} \mathbf{P}$$

Internal (intrinsic) parameters

External (extrinsic) parameters

General Idea

- Why is camera calibration necessary?
- What information do we have?
  - Images only

General Idea

- Why is camera calibration necessary?
- What information do we have?
- Camera calibration
  - Recovering *K*
  - Recovering *R* and **t**

$$\begin{aligned}\mathbf{p} &= M\mathbf{P} \\ &= K\begin{bmatrix} R & \mathbf{t} \end{bmatrix}\mathbf{P}\end{aligned}$$

Internal (intrinsic) parameters

External (extrinsic) parameters

General Idea

- How many parameters to recover?

$$\mathbf{p} = M\mathbf{P}$$
$$= K \begin{bmatrix} R & \mathbf{t} \end{bmatrix} \mathbf{P}$$

Internal (intrinsic) parameters (K)

External (extrinsic) parameters ($[R \;\; \mathbf{t}]$)

General Idea

- How many parameters to recover?
  - How many intrinsic parameters?

$$K = \begin{bmatrix} f_x & s & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{p} &= M\mathbf{P} \\ &= \boxed{K}\,[R \quad \mathbf{t}]\,\mathbf{P} \end{aligned}$$

Internal (intrinsic) parameters

General Idea

- How many parameters to recover?
  - How many intrinsic parameters?
  - How many extrinsic parameters?

$$R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix}, \quad \mathbf{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$\mathbf{p} = M\mathbf{P} = K \begin{bmatrix} R & \mathbf{t} \end{bmatrix} \mathbf{P}$$

External (extrinsic) parameters

General Idea

- How many parameters to recover: 11
  - 5 intrinsic parameters
    - 2 for focal lengths
    - 2 for offset (image center, or principle point)
    - 1 for skewness
  - 6 extrinsic parameters
    - 3 for rotation
    - 3 for translation

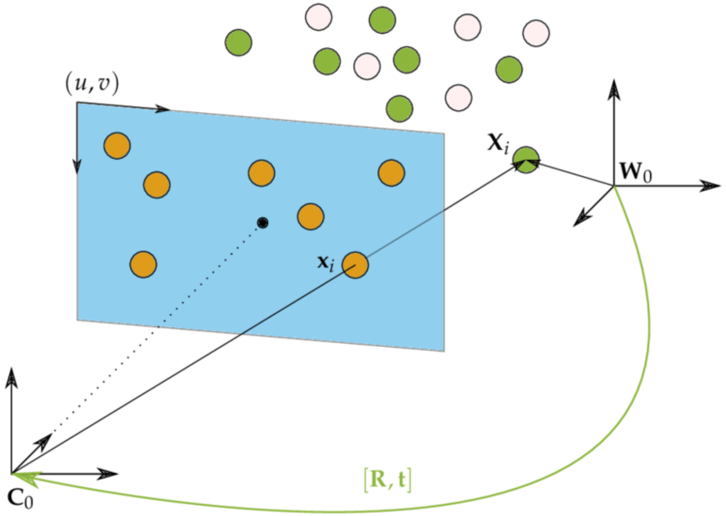
$$K = \begin{bmatrix} f_x & s & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix}, \quad \mathbf{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

General Idea

- What information to use?

  - Corresponding 3D-2D point pairs

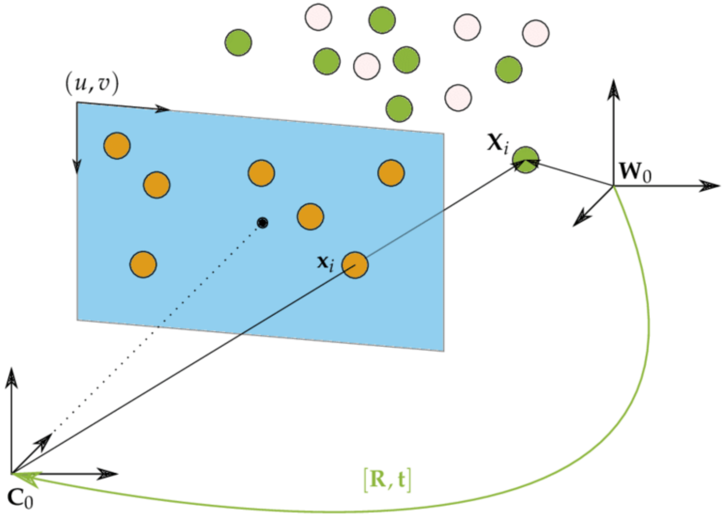
$$\mathbf{p} = M\mathbf{P}$$
$$= K\begin{bmatrix} R & \mathbf{t} \end{bmatrix}\mathbf{P}$$



General Idea

- What information to use?
  - Corresponding 3D-2D point pairs
    - How many pairs do we need?

$$\mathbf{p} = M\mathbf{P}$$
$$= K\left[R \quad \mathbf{t}\right]\mathbf{P}$$



General Idea

- What information to use?
  - Corresponding 3D-2D point pairs
    - How many pairs do we need?
      - How much information each pair of corresponding point can provide?

$$\mathbf{p} = M\mathbf{P} \Rightarrow \mathbf{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = M\mathbf{P}_i = \begin{bmatrix} \frac{\mathbf{P}_i^T \mathbf{m}_1}{\mathbf{P}_i^T \mathbf{m}_3} \\ \\ \frac{\mathbf{P}_i^T \mathbf{m}_2}{\mathbf{P}_i^T \mathbf{m}_3} \end{bmatrix} \Rightarrow \begin{matrix} \mathbf{P}_i^T \mathbf{m}_1 - u_i(\mathbf{P}_i^T \mathbf{m}_3) = 0 \\ \mathbf{P}_i^T \mathbf{m}_2 - v_i(\mathbf{P}_i^T \mathbf{m}_3) = 0 \end{matrix}$$

$\mathbf{m}_1$, $\mathbf{m}_2$, $\mathbf{m}_3$: the three rows of the projection matrix M

General Idea

- What information to use?
  - Corresponding 3D-2D point pairs
    - How many pairs do we need?
      - Each 3D-2D point pair -> 2 equations
      - 11 unknown -> 6 point correspondence
      - Use more to handle noisy data

$$\mathbf{p} = M\mathbf{P} \Rightarrow \mathbf{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = M\mathbf{P}_i = \begin{bmatrix} \frac{\mathbf{P}_i^T \mathbf{m}_1}{\mathbf{P}_i^T \mathbf{m}_3} \\ \\ \frac{\mathbf{P}_i^T \mathbf{m}_2}{\mathbf{P}_i^T \mathbf{m}_3} \end{bmatrix} \Rightarrow \begin{array}{l} \mathbf{P}_i^T \mathbf{m}_1 - u_i(\mathbf{P}_i^T \mathbf{m}_3) = 0 \\ \mathbf{P}_i^T \mathbf{m}_2 - v_i(\mathbf{P}_i^T \mathbf{m}_3) = 0 \end{array}$$

$\mathbf{m}_1$, $\mathbf{m}_2$, $\mathbf{m}_3$: the three rows of the projection matrix M

General Idea

$$\mathbf{P}_i^T\mathbf{m}_1 - u_i(\mathbf{P}_i^T\mathbf{m}_3) = 0$$
$$\mathbf{P}_i^T\mathbf{m}_2 - v_i(\mathbf{P}_i^T\mathbf{m}_3) = 0$$

Constraints from one pair

$$\begin{gathered}\mathbf{P}_1^T\mathbf{m}_1 - u_1(\mathbf{P}_1^T\mathbf{m}_3) = 0\\ \mathbf{P}_1^T\mathbf{m}_2 - v_1(\mathbf{P}_1^T\mathbf{m}_3) = 0\\ \vdots\\ \mathbf{P}_n^T\mathbf{m}_1 - u_n(\mathbf{P}_n^T\mathbf{m}_3) = 0\\ \mathbf{P}_n^T\mathbf{m}_2 - v_n(\mathbf{P}_n^T\mathbf{m}_3) = 0\end{gathered}$$

Equations from n pairs

$$\begin{bmatrix}\mathbf{P}_1^T & 0^T & -u_1\mathbf{P}_1^T\\ 0^T & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T\\ & \vdots & \\ \mathbf{P}_n^T & 0^T & -u_n\mathbf{P}_n^T\\ 0^T & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T\end{bmatrix}\begin{bmatrix}\mathbf{m}_1\\ \mathbf{m}_2\\ \mathbf{m}_3\end{bmatrix} = P\mathbf{m} = 0$$

12 x 1

2n x 12

What is the dimension of the *P* matrix?
What is the dimension of **m**?

Details: the derivation of the linear system

- The equations $\mathbf{p} = M\mathbf{P}$ $\quad [X, Y, Z]^T \rightarrow [u, v]^T$

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$su = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$
$$sv = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$
$$s = m_{31}X + m_{32}Y + m_{33}Z + m_{34}$$

$$u = \frac{m_{11}X + m_{12}Y + m_{13}Z + m_{14}}{m_{31}X + m_{32}Y + m_{33}Z + m_{34}}$$
$$v = \frac{m_{21}X + m_{22}Y + m_{23}Z + m_{24}}{m_{31}X + m_{32}Y + m_{33}Z + m_{34}}$$

Details: the derivation of the linear system

- The equations

$$u = \frac{m_{11}X + m_{12}Y + m_{13}Z + m_{14}}{m_{31}X + m_{32}Y + m_{33}Z + m_{34}}$$
$$v = \frac{m_{21}X + m_{22}Y + m_{23}Z + m_{24}}{m_{31}X + m_{32}Y + m_{33}Z + m_{34}}$$

$\Rightarrow$

$$(m_{31}X + m_{32}Y + m_{33}Z + m_{34})\,u = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$
$$(m_{31}X + m_{32}Y + m_{33}Z + m_{34})\,v = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$

$\Rightarrow$

$$m_{11}X + m_{12}Y + m_{13}Z + m_{14} - m_{31}uX - m_{32}uY - m_{33}uZ - m_{34}u = 0$$
$$m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v = 0$$

Details: the derivation of the linear system

- The equations

For every pair of 3D-2D corresponding points

$$m_{11}X + m_{12}Y + m_{13}Z + m_{14} - m_{31}uX - m_{32}uY - m_{33}uZ - m_{34}u = 0$$
$$m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v = 0$$

Given n pairs of 3D-2D corresponding points

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

General Idea

- How to solve it?
  - It is a homogeneous linear system
  - It is overdetermined

$$\begin{bmatrix} \mathbf{P}_1^T & 0^T & -u_1\mathbf{P}_1^T \\ 0^T & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ & \vdots & \\ \mathbf{P}_n^T & 0^T & -u_n\mathbf{P}_n^T \\ 0^T & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} = P\mathbf{m} = 0$$

General Idea

- How to solve it?
  - **m** = *0* always a trivial solution
  - if **m** ≠ 0 is a solution, then any *k* * **m** is also a solution

$$\begin{bmatrix} \mathbf{P}_1^T & 0^T & -u_1\mathbf{P}_1^T \\ 0^T & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ & \vdots & \\ \mathbf{P}_n^T & 0^T & -u_n\mathbf{P}_n^T \\ 0^T & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} = P\mathbf{m} = 0$$

General Idea

- How to solve it?
  - **m** = *0* always a trivial solution
  - if **m** ≠ 0 is a solution, then any *k* * **m** is also a solution
  - Constrained optimization

$$\begin{bmatrix} \mathbf{P}_1^T & 0^T & -u_1\mathbf{P}_1^T \\ 0^T & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ & \vdots & \\ \mathbf{P}_n^T & 0^T & -u_n\mathbf{P}_n^T \\ 0^T & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} = P\mathbf{m} = 0$$

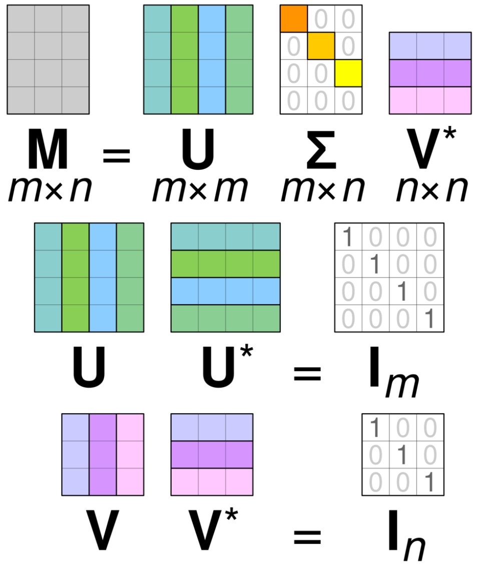
$$\begin{aligned} \underset{\mathbf{m}}{\text{minimize}} \quad & \|P\mathbf{m}\|^2 \\ \text{subject to} \quad & \|\mathbf{m}\|^2 = 1 \end{aligned}$$

SVD

- **S**ingular **V**alue **D**ecomposition
  - Generalization of the eigen-decomposition of a square matrix to any m by n matrix

$$A = U \Sigma V^{-1} \qquad \Sigma = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_N \end{bmatrix}$$

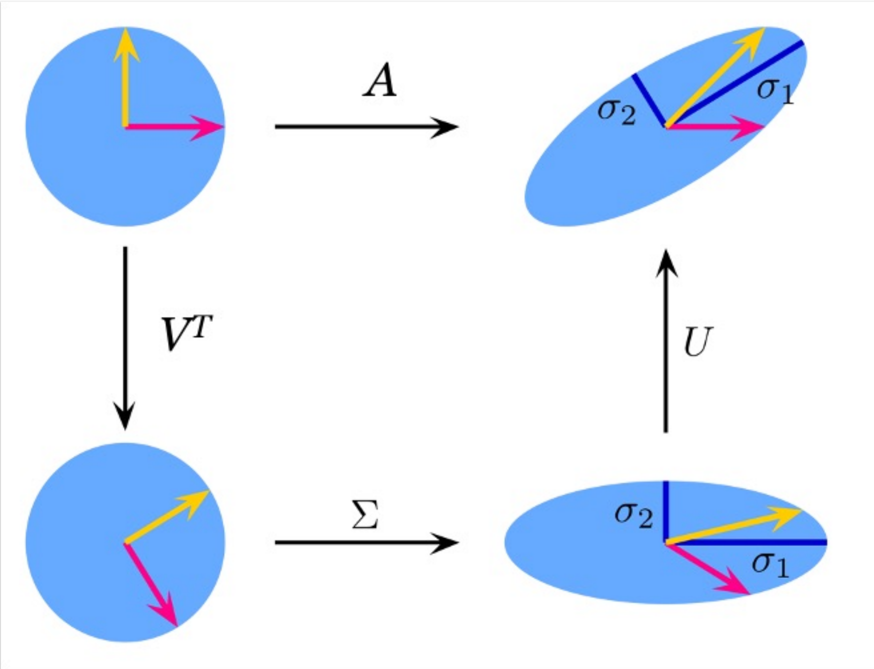
U, V = orthogonal matrix



SVD

- Geometric meaning

$$A = U \Sigma V^T$$



Example (square matrix)

$$\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} -.40 & .916 \\ .916 & .40 \end{bmatrix} \cdot \begin{bmatrix} 5.39 & 0 \\ 0 & 3.154 \end{bmatrix} \cdot \begin{bmatrix} -.05 & .999 \\ .999 & .05 \end{bmatrix}$$

A	U	Σ	V^T
Transformation	Rotation	Scaling	Rotation

Calibration: solve for projection matrix

$$P\mathbf{m} = 0 \quad \Rightarrow \quad \underset{\mathbf{m}}{\text{minimize}} \quad \|P\mathbf{m}\|^2 \quad \text{subject to} \quad \|\mathbf{m}\|^2 = 1$$

SVD of *P*

$$U_{2n\mathrm{x}2n} \; D_{2n\mathrm{x}12} \; V_{12\mathrm{x}12}$$

Last column of V gives **m**

(Why? See page 593 of Hartley & Zisserman. Multiple View Geometry in Computer Vision)

Least-squares solution of homogeneous equations

This problem is solvable as follows. Let $\mathtt{A} = \mathtt{UDV}^\mathsf{T}$. The problem then requires us to minimize $\|\mathtt{UDV}^\mathsf{T}\mathbf{x}\|$. However, $\|\mathtt{UDV}^\mathsf{T}\mathbf{x}\| = \|\mathtt{DV}^\mathsf{T}\mathbf{x}\|$ and $\|\mathbf{x}\| = \|\mathtt{V}^\mathsf{T}\mathbf{x}\|$. Thus, we need to minimize $\|\mathtt{DV}^\mathsf{T}\mathbf{x}\|$ subject to the condition $\|\mathtt{V}^\mathsf{T}\mathbf{x}\| = 1$. We write $\mathbf{y} = \mathtt{V}^\mathsf{T}\mathbf{x}$, and the problem is: minimize $\|\mathtt{D}\mathbf{y}\|$ subject to $\|\mathbf{y}\| = 1$. Now, $\mathtt{D}$ is a diagonal matrix with its diagonal entries in descending order. It follows that the solution to this problem is $\mathbf{y} = (0, 0, \ldots, 0, 1)^\mathsf{T}$ having one non-zero entry, 1 in the last position. Finally $\mathbf{x} = \mathtt{V}\mathbf{y}$ is simply the last column of $\mathtt{V}$. The method is summarized in algorithm A5.4.

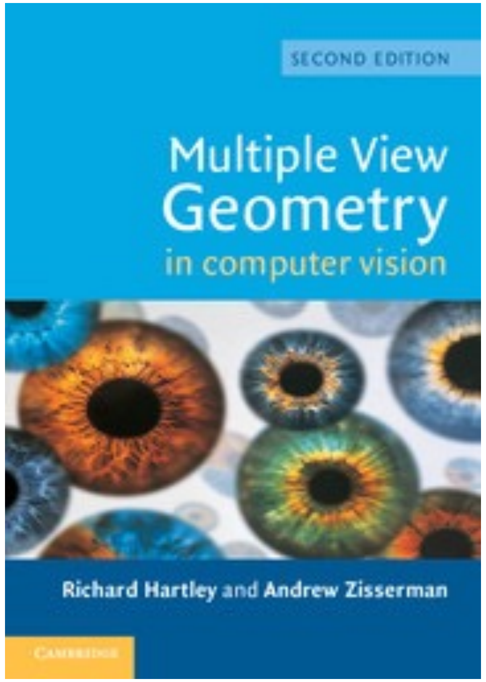
Objective

Given a matrix $\mathtt{A}$ with at least as many rows as columns, find $\mathbf{x}$ that minimizes $\|\mathtt{A}\mathbf{x}\|$ subject to $\|\mathbf{x}\| = 1$.

Solution

$\mathbf{x}$ is the last column of $\mathtt{V}$, where $\mathtt{A} = \mathtt{UDV}^\mathsf{T}$ is the SVD of $\mathtt{A}$.

Algorithm A5.4. *Least-squares solution of a homogeneous system of linear equations.*



Page 593 of Hartley & Zisserman. Multiple View Geometry in Computer Vision

Camera parameters from project matrix

$$M = K\begin{bmatrix}R & \mathbf{t}\end{bmatrix}$$

$$K = \begin{bmatrix} f_x & s & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_0 \\ 0 & \frac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix}, \ \mathbf{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$M = \begin{bmatrix} \alpha\mathbf{r}_1^T - \alpha\cot\theta\mathbf{r}_2^T + u_0\mathbf{r}_3^T & \alpha t_x - \alpha\cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta}\mathbf{r}_2^T + v_0\mathbf{r}_3^T & \frac{\beta}{\sin\theta}t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{bmatrix}$$

SVD-solved projection matrix

SVD-solved projection matrix is known up to scale, i.e., $\rho\mathcal{M} = M$ ← The true values of project matrix

$$\mathcal{M} = \frac{1}{\rho}M = \frac{1}{\rho}\begin{bmatrix} \alpha\mathbf{r}_1^T - \alpha\cot\theta\mathbf{r}_2^T + u_0\mathbf{r}_3^T & \alpha t_x - \alpha\cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta}\mathbf{r}_2^T + v_0\mathbf{r}_3^T & \frac{\beta}{\sin\theta}t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{bmatrix}$$

Camera parameters from project matrix

$$\mathcal{M} = \frac{1}{\rho}\begin{bmatrix} \alpha\mathbf{r}_1^T - \alpha\cot\theta\mathbf{r}_2^T + u_0\mathbf{r}_3^T & \alpha t_x - \alpha\cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta}\mathbf{r}_2^T + v_0\mathbf{r}_3^T & \frac{\beta}{\sin\theta}t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{bmatrix}$$

$$\text{denote } \mathcal{M} = \begin{bmatrix} A & \mathbf{b} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1^T & b_1 \\ \mathbf{a}_2^T & b_2 \\ \mathbf{a}_3^T & b_3 \end{bmatrix}$$

$$\frac{1}{\rho}\begin{bmatrix} \alpha\mathbf{r}_1^T - \alpha\cot\theta\mathbf{r}_2^T + u_0\mathbf{r}_3^T & \alpha t_x - \alpha\cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta}\mathbf{r}_2^T + v_0\mathbf{r}_3^T & \frac{\beta}{\sin\theta}t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1^T & b_1 \\ \mathbf{a}_2^T & b_2 \\ \mathbf{a}_3^T & b_3 \end{bmatrix}$$

Solving for the intrinsic and extrinsic parameters

Camera parameters from project matrix

Intrinsic parameters:

$$\rho = \pm \frac{1}{\|\mathbf{a_3}\|}$$

$$u_0 = \rho^2(\mathbf{a_1} \cdot \mathbf{a_3})$$

$$v_0 = \rho^2(\mathbf{a_2} \cdot \mathbf{a_3})$$

$$\cos\theta = -\frac{(\mathbf{a_1} \times \mathbf{a_3}) \cdot (\mathbf{a_2} \times \mathbf{a_3})}{\|\mathbf{a_1} \times \mathbf{a_3}\| \cdot \|\mathbf{a_2} \times \mathbf{a_3}\|}$$

$$\alpha = \rho^2 \|\mathbf{a_1} \times \mathbf{a_3}\| \sin\theta$$

$$\beta = \rho^2 \|\mathbf{a_2} \times \mathbf{a_3}\| \sin\theta$$

Extrinsic parameters:

$$\mathbf{r_1} = \frac{\mathbf{a_2} \times \mathbf{a_3}}{\|\mathbf{a_2} \times \mathbf{a_3}\|}$$

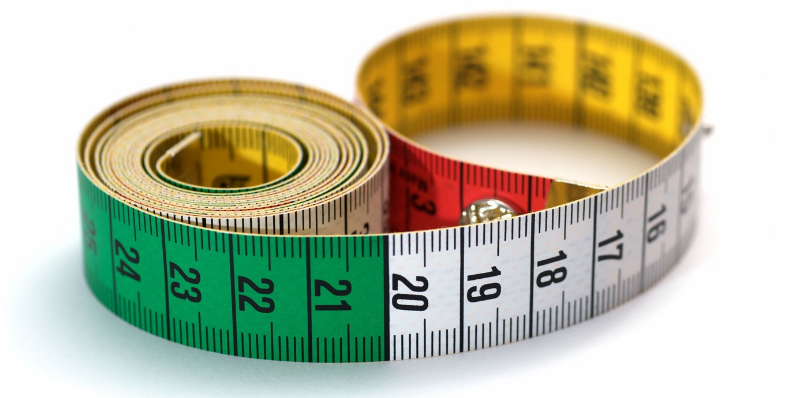
$$\mathbf{r_2} = \mathbf{r_3} \times \mathbf{r_1}$$

$$\mathbf{r_3} = \rho \mathbf{a_3}$$

$$\mathbf{t} = \rho K^{-1} \mathbf{b}$$

Find 3D-2D corresponding points

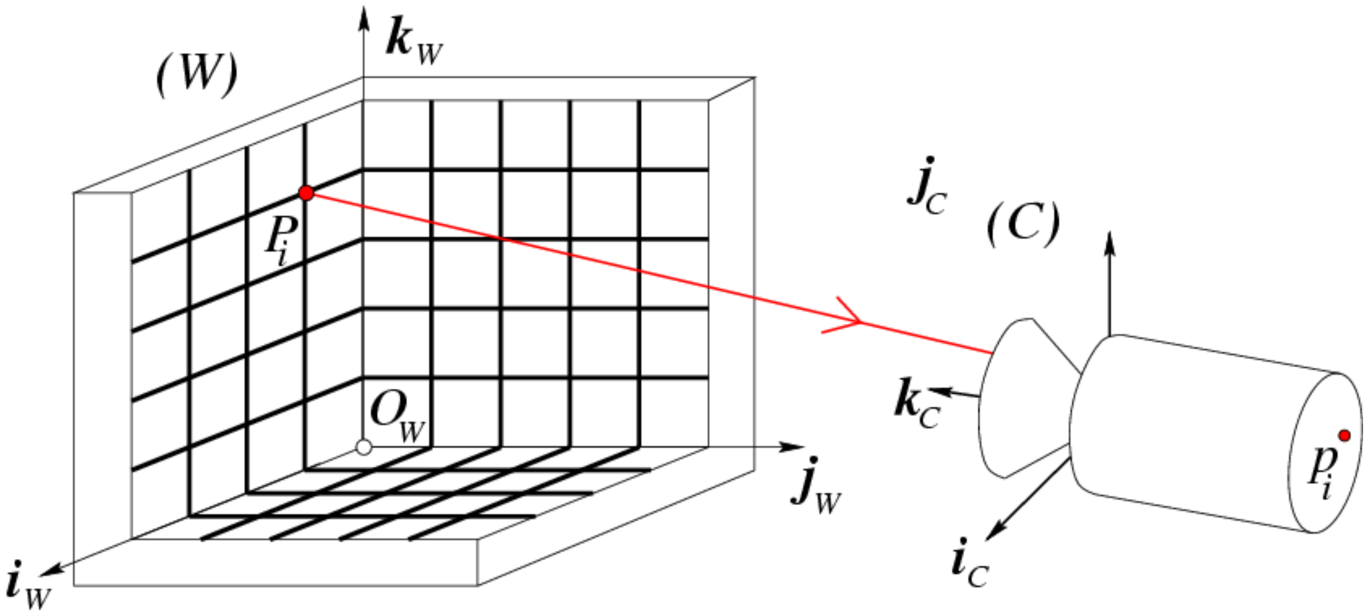
- At least 6 3D-2D point pairs
  - 3D points with known 3D coordinates
  - Corresponding image points with known 2D coordinates



tape measure

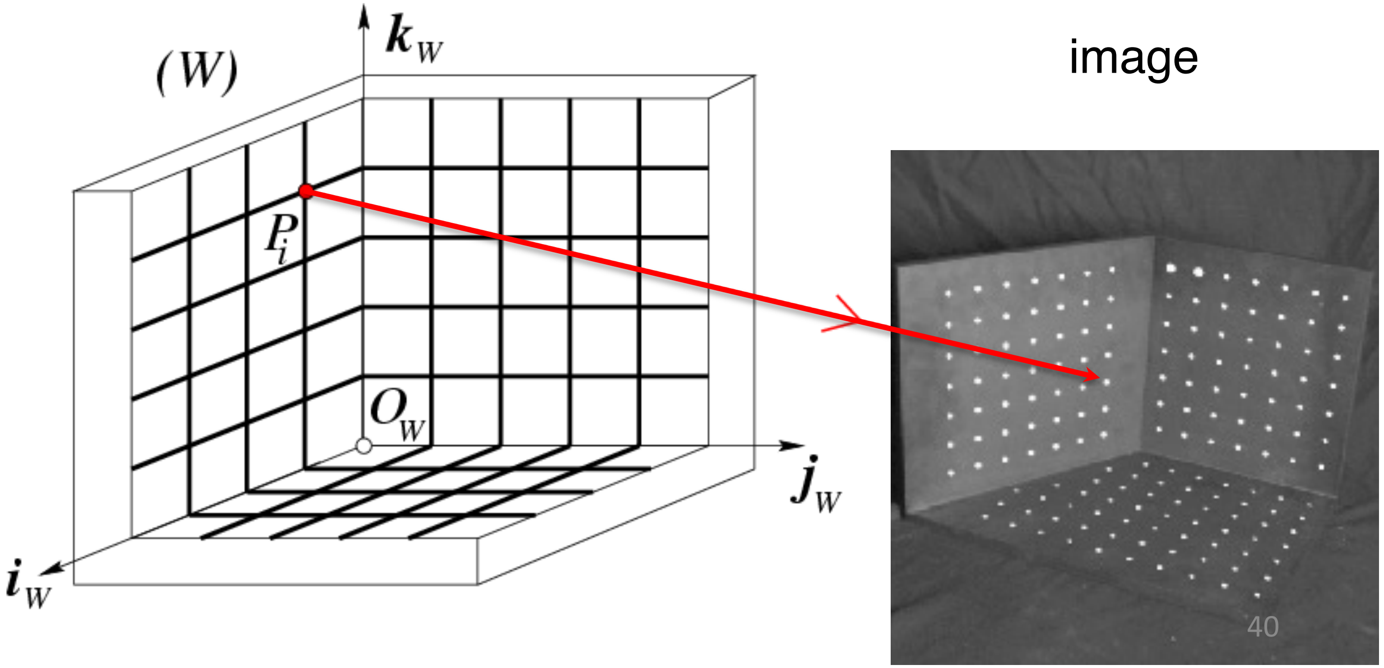
Find 3D-2D corresponding points

- Calibration rig - a special apparatus
  - $P_1, \ldots P_n$ with known positions in $[O_w, i_w, j_w, k_w]$



Find 3D-2D corresponding points

- Calibration rig - a special apparatus
  - P_1, … P_n with known positions in $[O_w, i_w, j_w, k_w]$
  - p_1, … p_n known positions in the image



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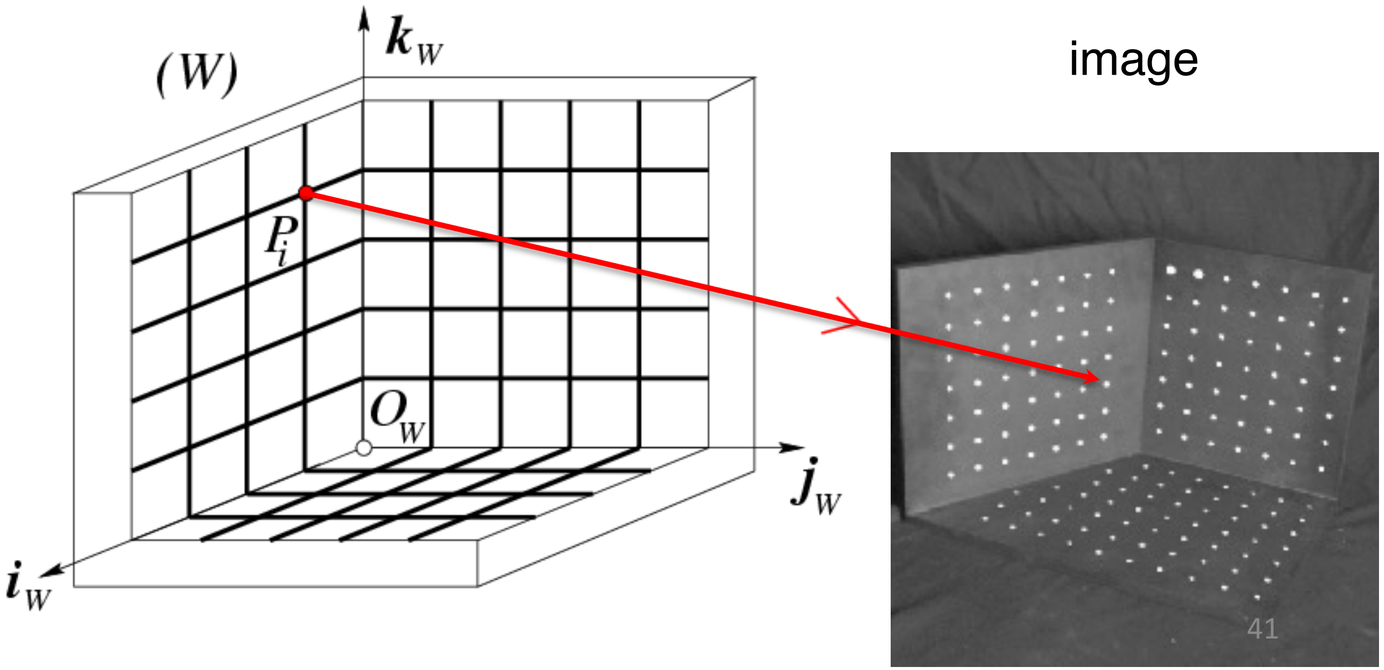
- Calibration rig - a special apparatus
  - $P_1, \dots P_n$ with known positions in $[O_W, i_W, j_W, k_W]$
  - $p_1, \dots p_n$ known positions in the image
  - At least 6 pairs



Find 3D-2D corresponding points

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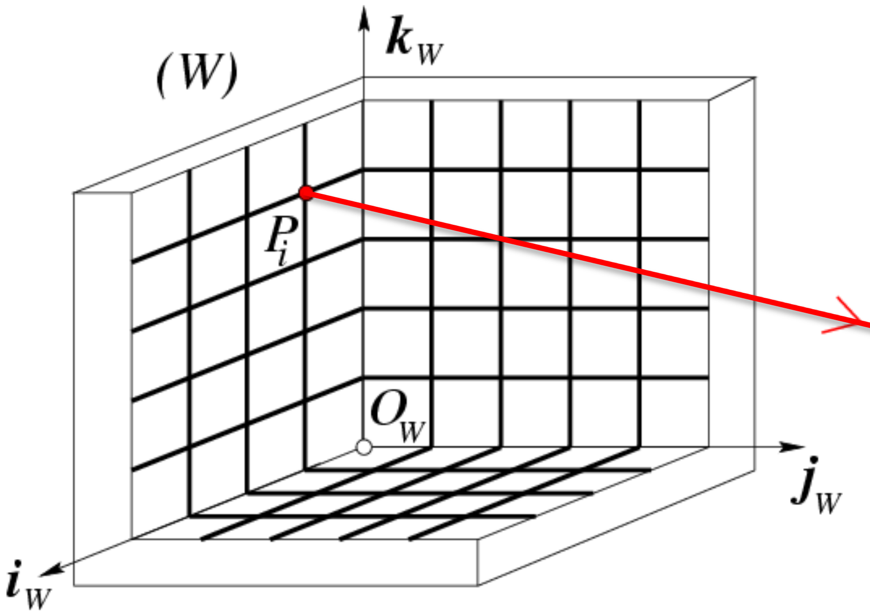
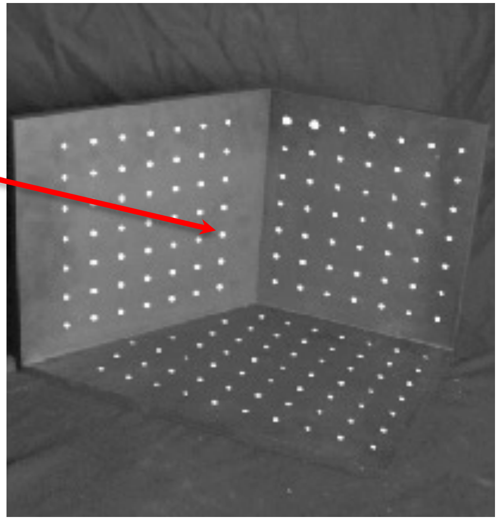
  - $p_1, \ldots p_n$ known positions in the image
  - At least 6 pairs
- Goal
  - Intrinsic parameters
  - Extrinsic parameters

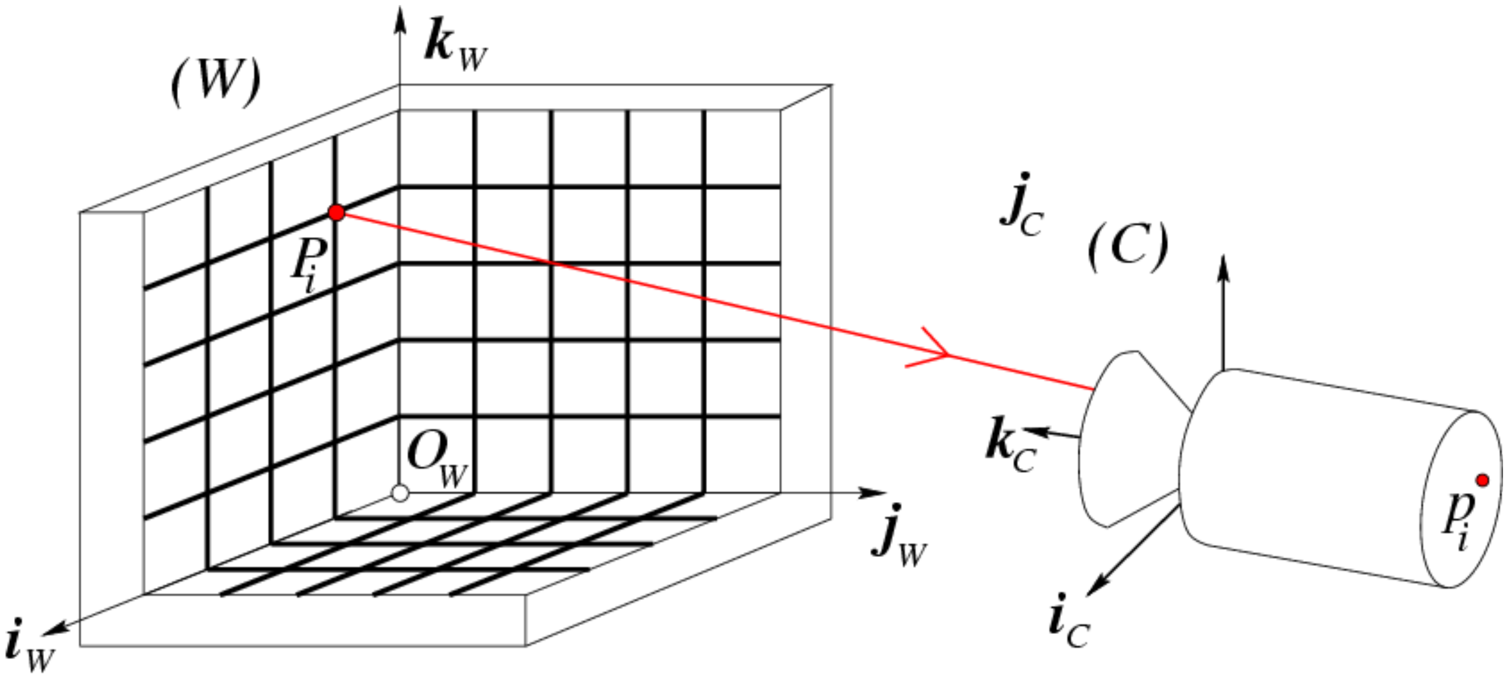


image



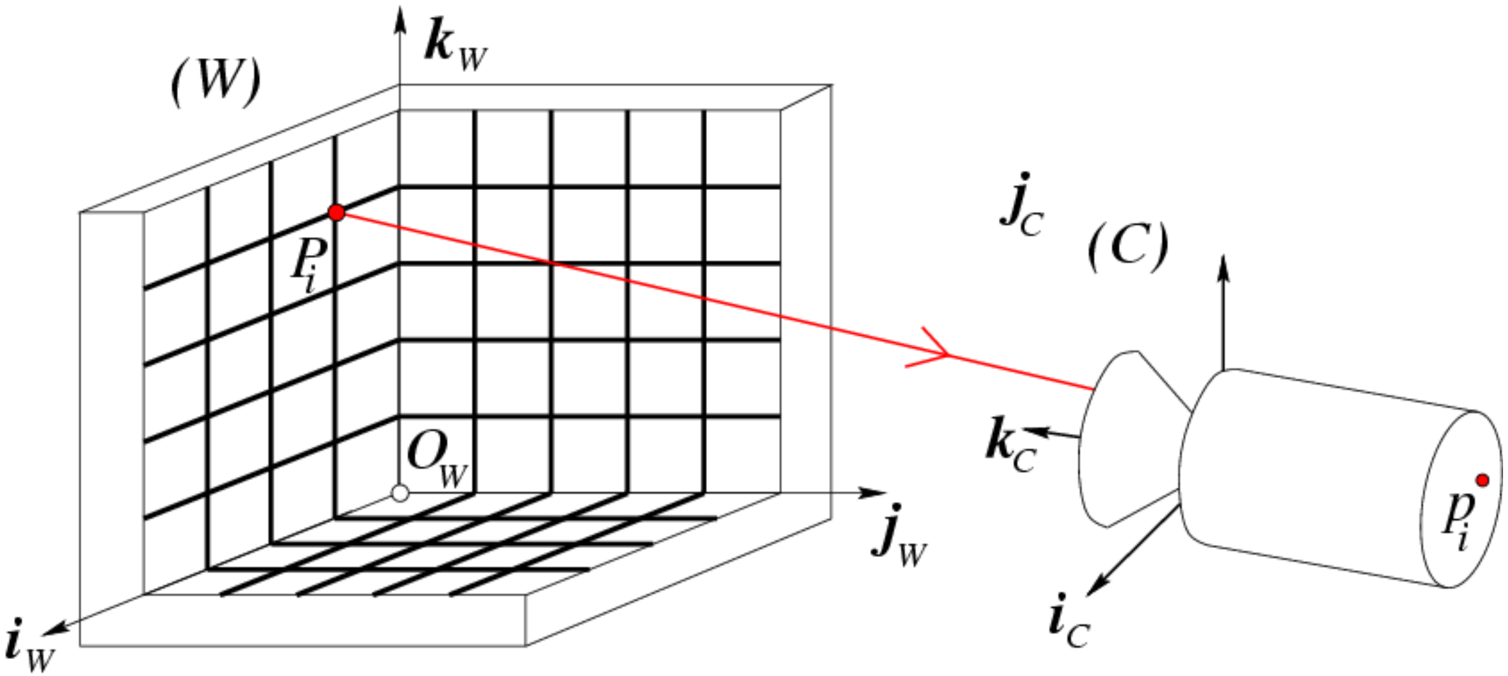
Calibration

- Always solvable?



Calibration

- Always solvable?
  - $\{P_i\}$ cannot lie on the same plane
  - $\{P_i\}$ cannot lie on the intersection curve of two quadric surfaces



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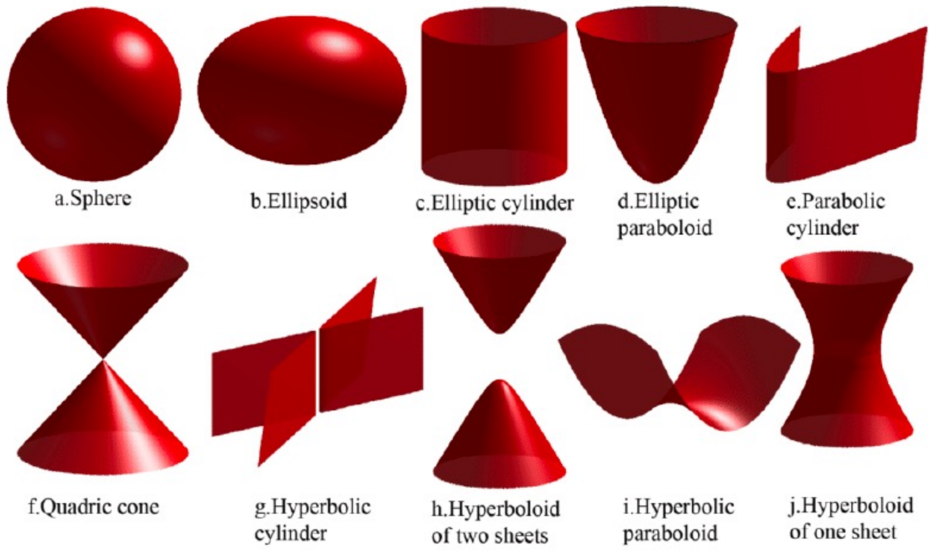
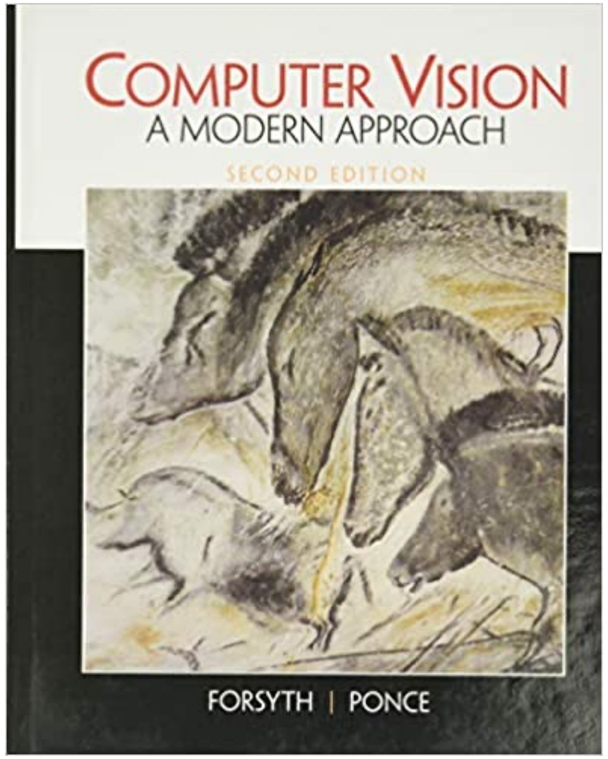
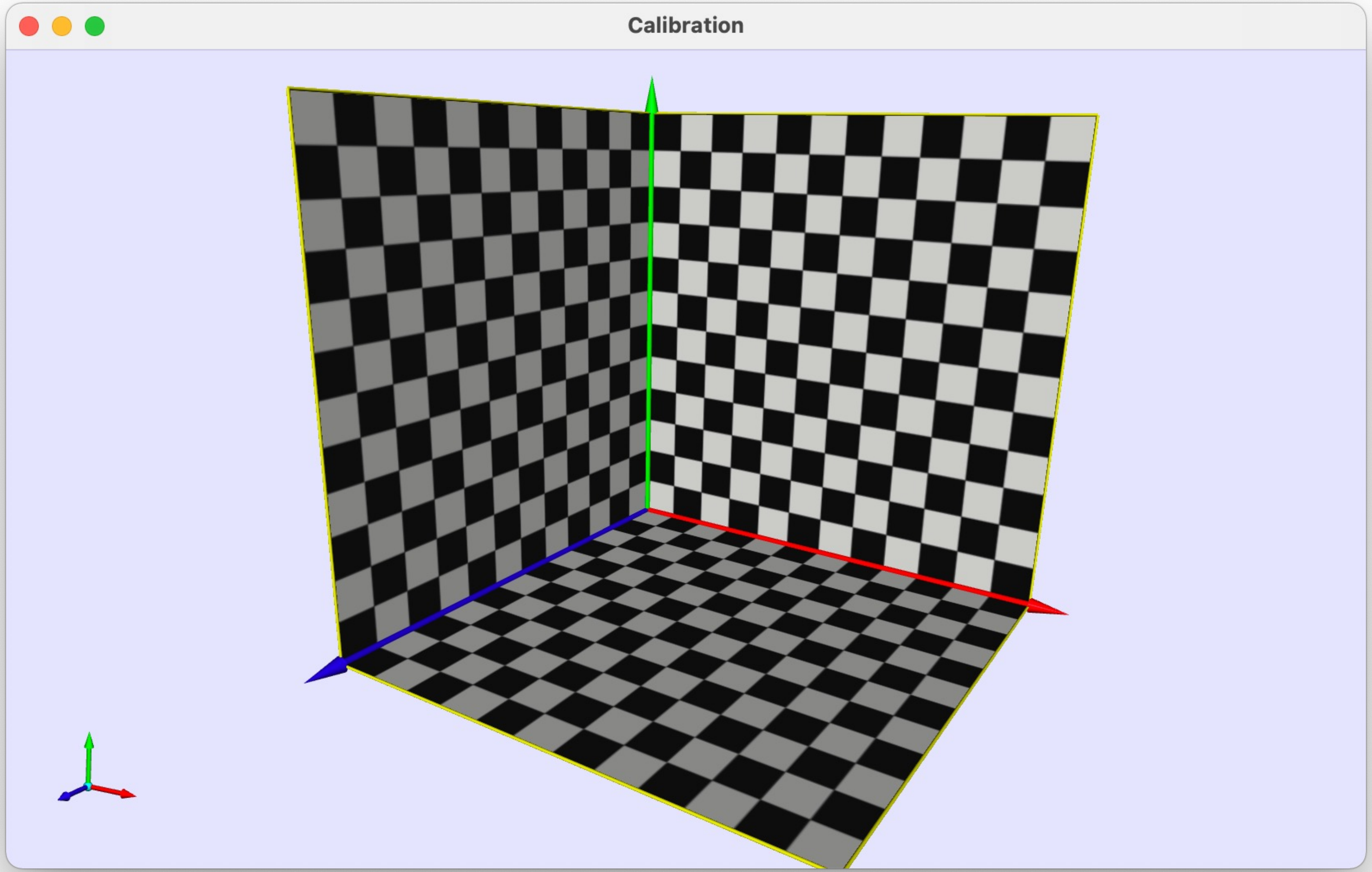




See Section 1.3 of Forsyth & Ponce. Computer Vision: A Modern Approach

A1: Camera calibration



Next Lecture

- Epipolar Geometry