TUDelft 3Dgeoinfo

GEO1016
Photogrammetry and 3D Computer Vision

Lecture 7
Surface Reconstruction

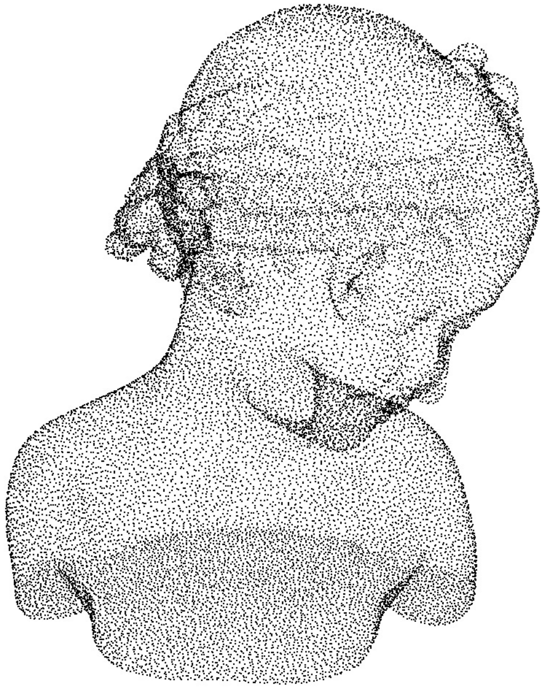
Liangliang Nan

Today's Agenda

- Introduction
- Smooth object reconstruction
  - The pioneer work of Hoppe *et al*. (1992)
  - Poisson reconstruction [Kazhdan *et al*. 2006]
  - Piecewise smooth reconstruction
- Piecewise planar object reconstruction [Nan and Wonka. 2017]

Introduction

- Data sources
  - Laser scanning

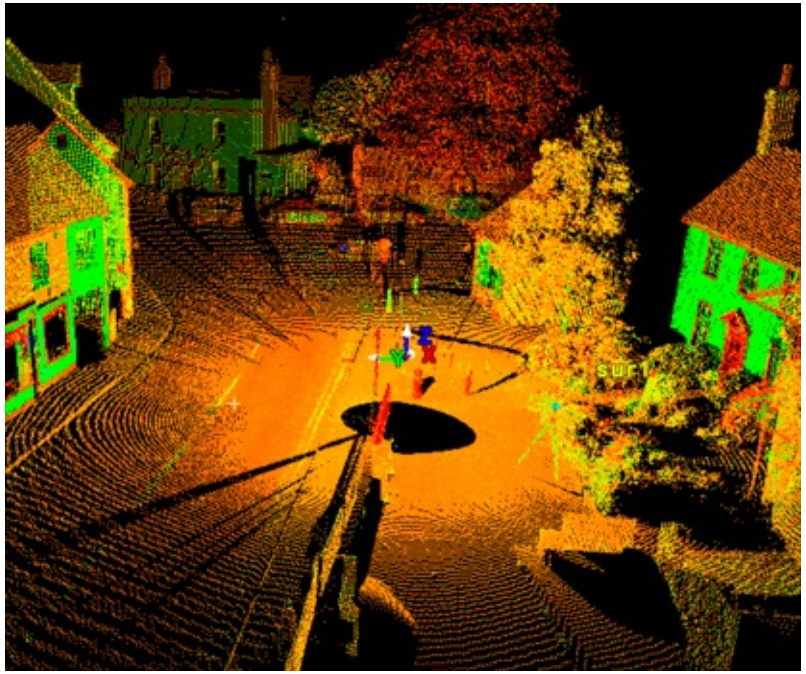


Introduction

- Data sources
  - Laser scanning

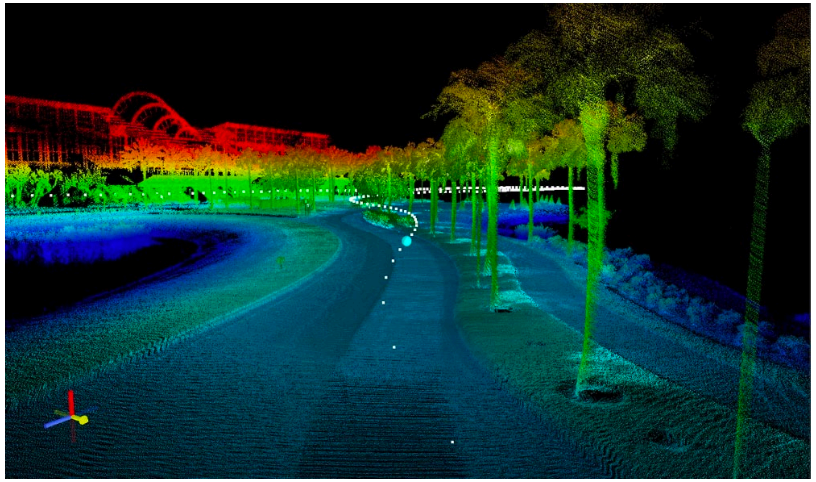
Introduction

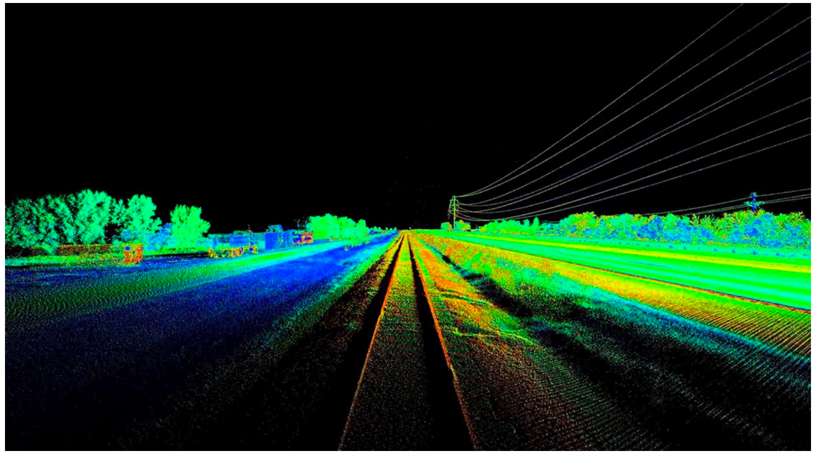
- Data sources
  - Laser scanning



Introduction

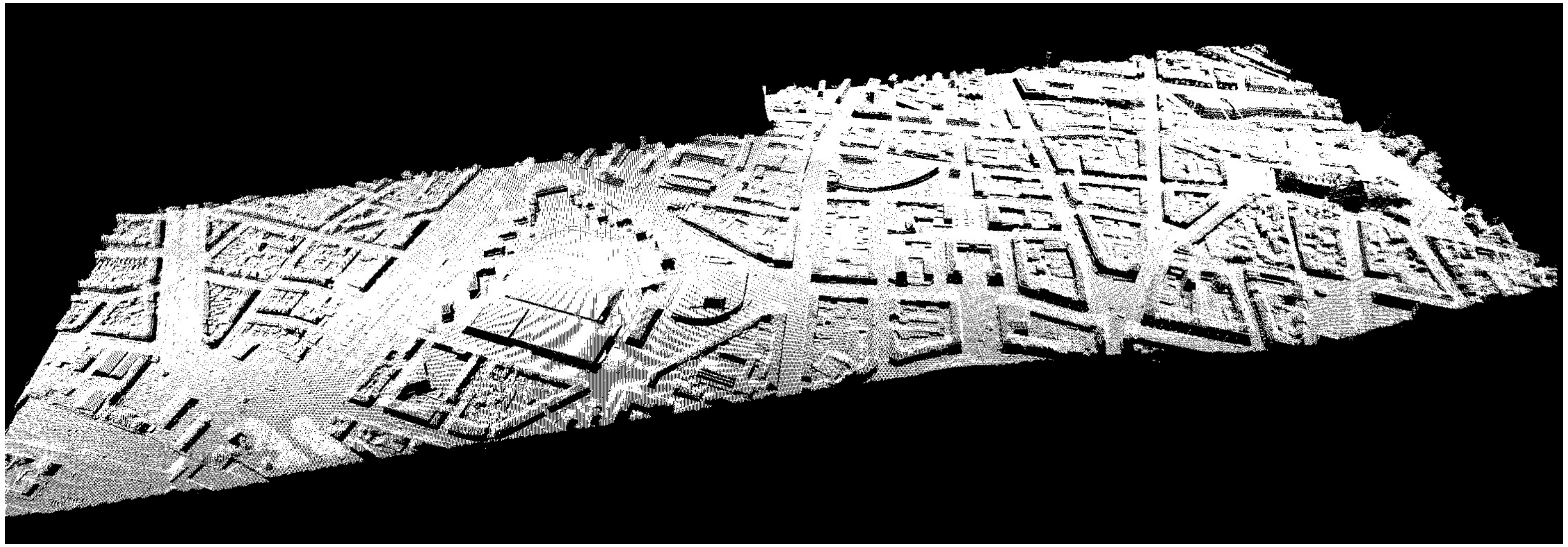
- Data sources
  - Laser scanning





Introduction

- Data sources
  - Laser scanning

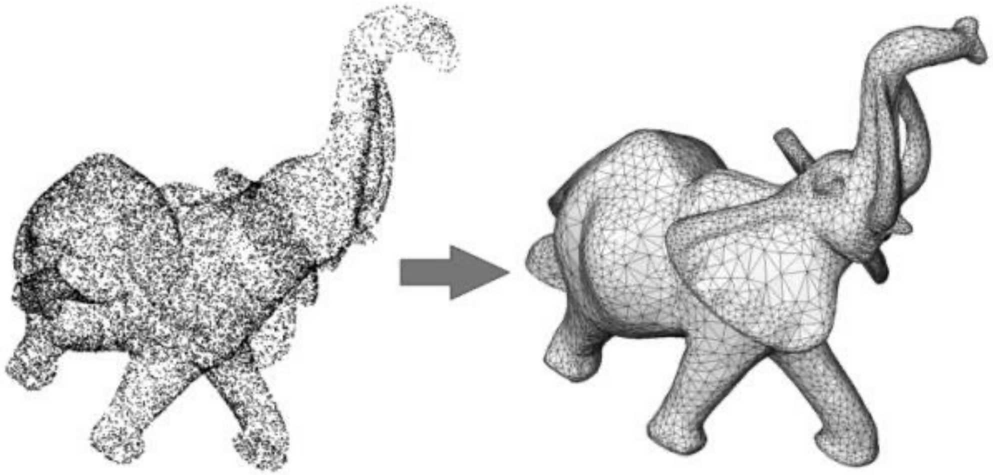


Introduction

- Data sources
  - Laser scanning
  - Structure from Motion (SfM) and Multi-view stereo (MVS)

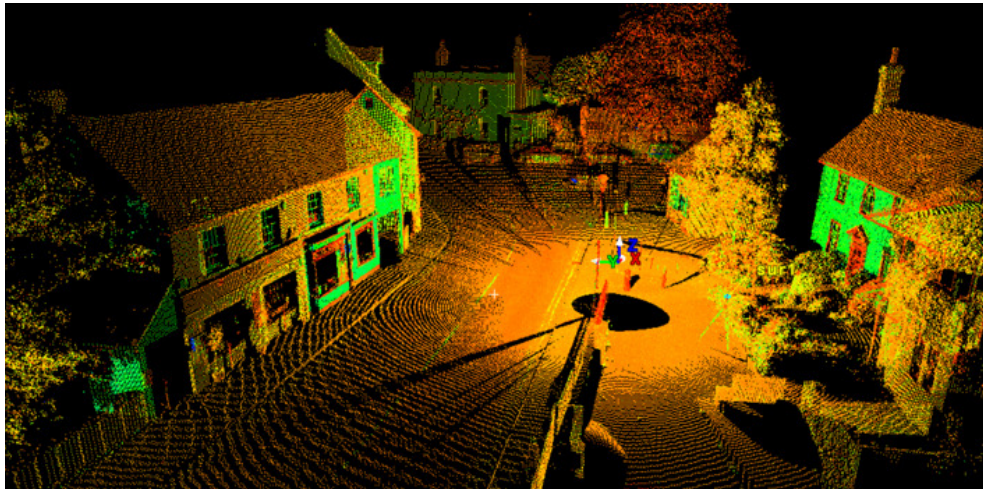
Introduction

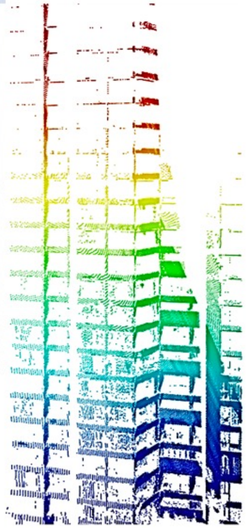
- Surface reconstruction
  - Input: point set *P* sampled over a surface *S*
    - Non-uniform sampling
    - With holes
    - With uncertainty (noise)
  - Output: surface approximating *S* in terms of topology and geometry
    - Desired
      - Watertight
      - Intersection free



Introduction

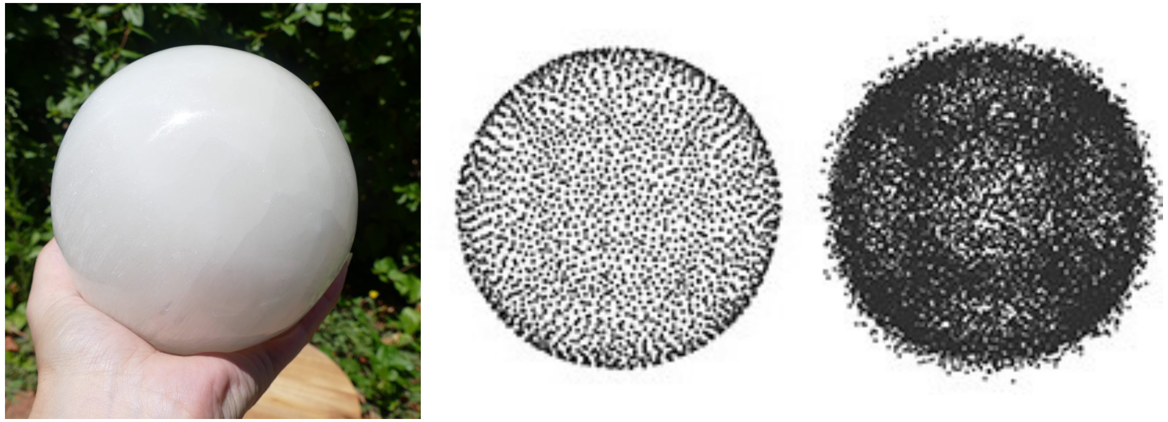
- Challenges
  - The point samples may not be uniformly distributed
    - Oblique scanning angles
    - Laser energy attenuation

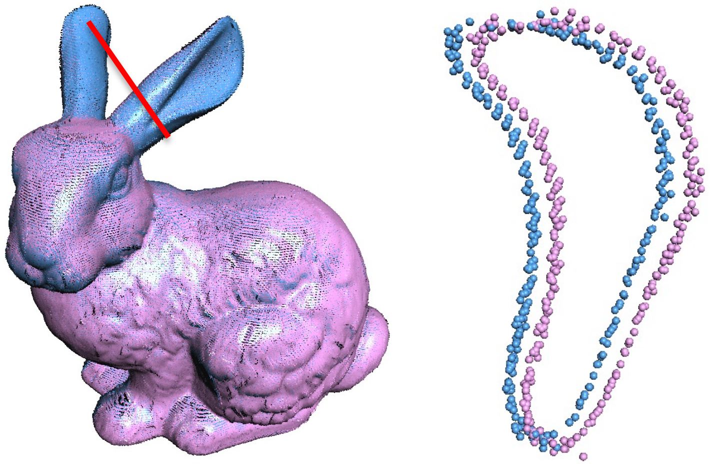




Introduction

- Challenges
  - The point samples may not be uniformly distributed
  - The positions and normals are generally noisy
    - Sampling inaccuracy
    - Scan misregistration



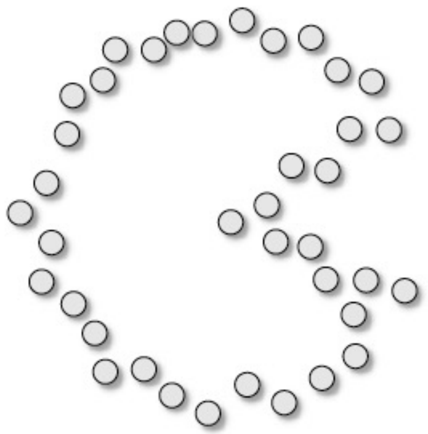


Introduction

- Challenges
  - The point samples may not be uniformly distributed
  - The positions and normals are generally noisy
  - Missing data
    - Material properties, inaccessibility, occlusion, etc.

Introduction

- Challenges
  - The point samples may not be uniformly distributed
  - The positions and normals are generally noisy
  - Missing data
  - Ill-posed problem

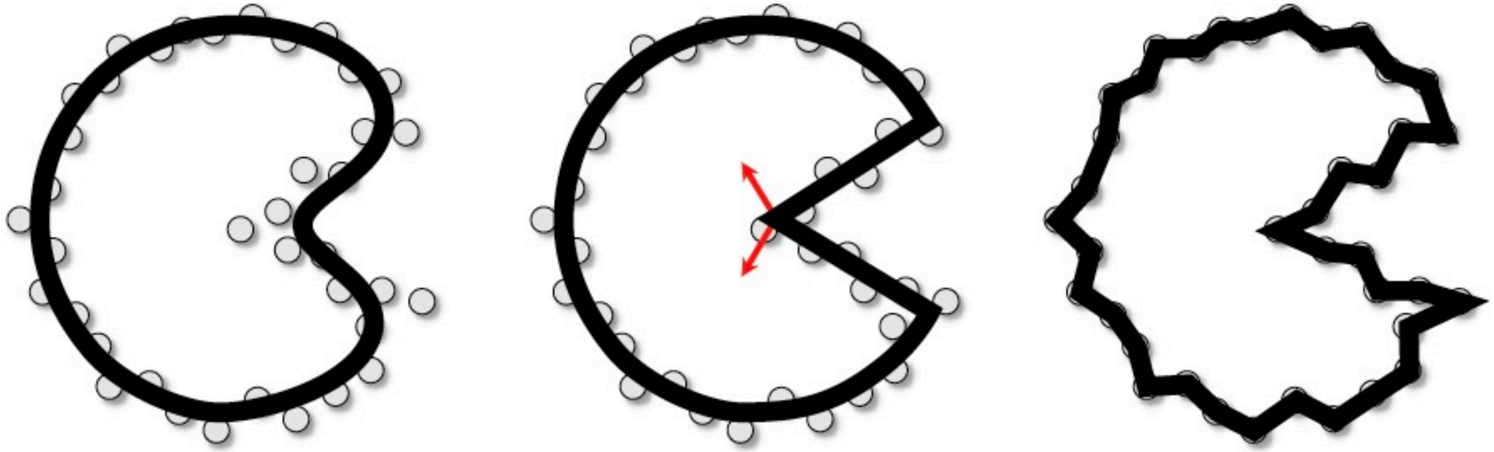


Many candidate surfaces for the reconstruction problem!

Introduction

- Challenges
  - The point samples may not be uniformly distributed
  - The positions and normals are generally noisy
  - Missing data
  - Ill-posed problem

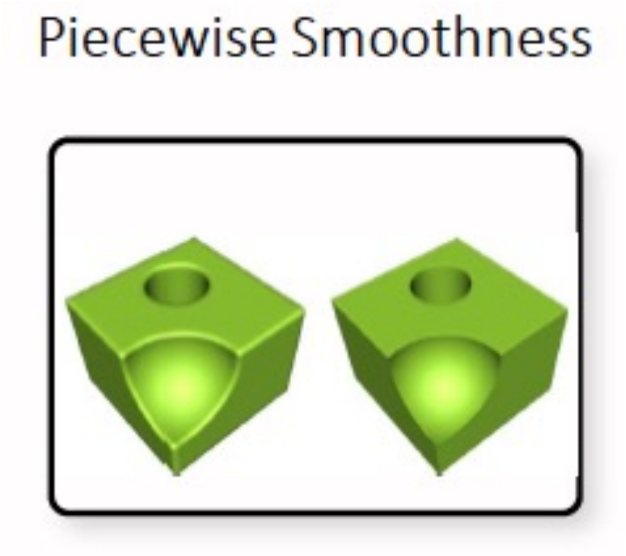
How to pick?



Many candidate surfaces for the reconstruction problem!

General Ideas

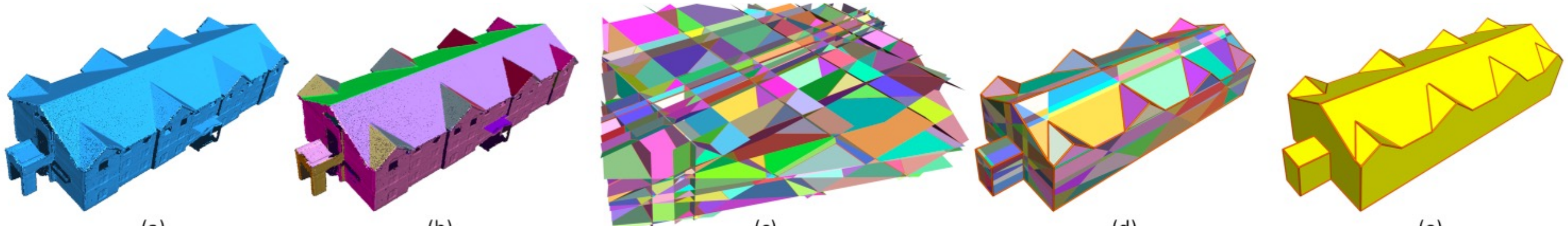
- Surface Smoothness Priors



General Ideas

- Domain-Specific Priors

[Verdie et al, 2015]

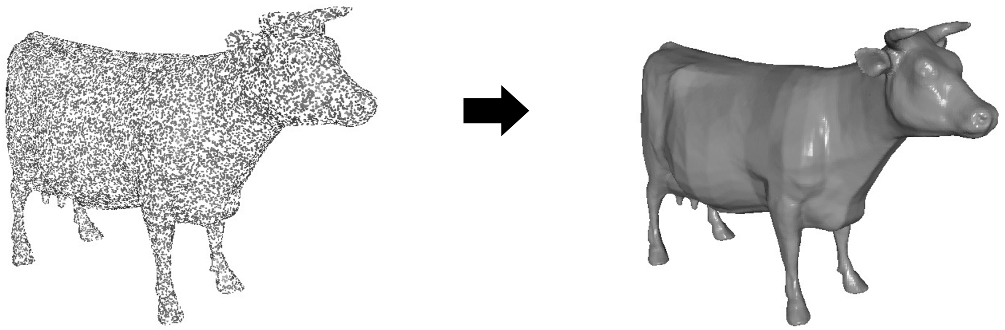


(a) (b) (c) (d) (e)

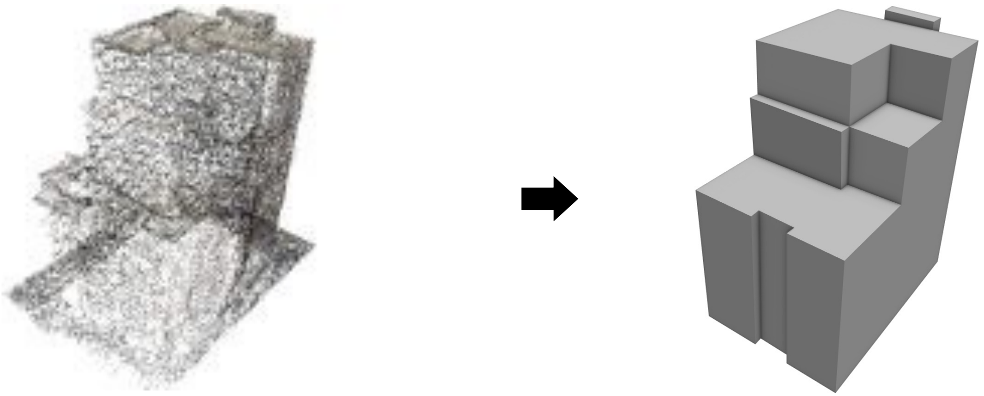
[Nan and Wonka 2017]

Introduction

- Smooth surface reconstruction



- Piecewise-planar object reconstruction

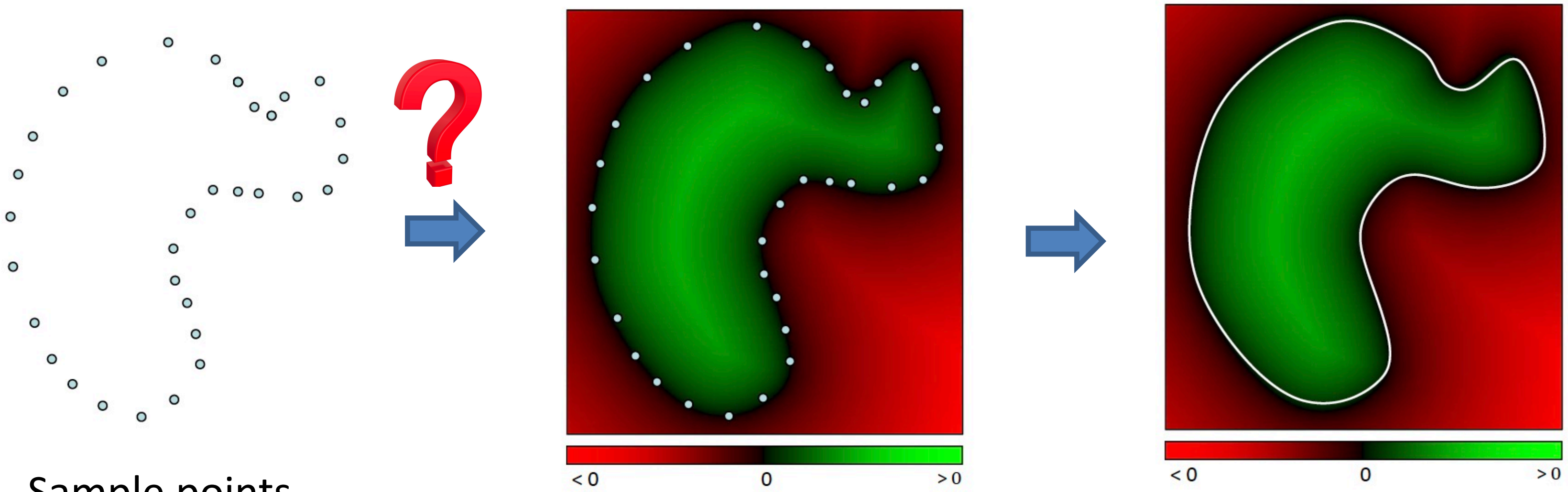


Today's Agenda

- Introduction
- Smooth object reconstruction
  - The pioneer work of Hoppe *et al*. (1992)
  - Poisson reconstruction [Kazhdan *et al*. 06]
  - Piecewise smooth reconstruction
- Piecewise planar object reconstruction [Nan and Wonka. 2017]

The pioneer work of Hoppe *et al.* (1992)

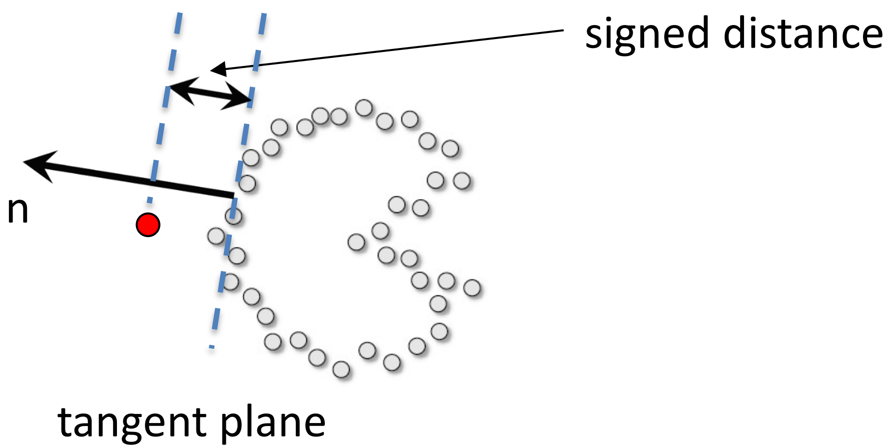
- Two main steps
  - Estimate signed geometric distance to the unknown surface
  - Extract the zero-set of the distance field using a contouring algorithm



Sample points
F(*q*)
contouring

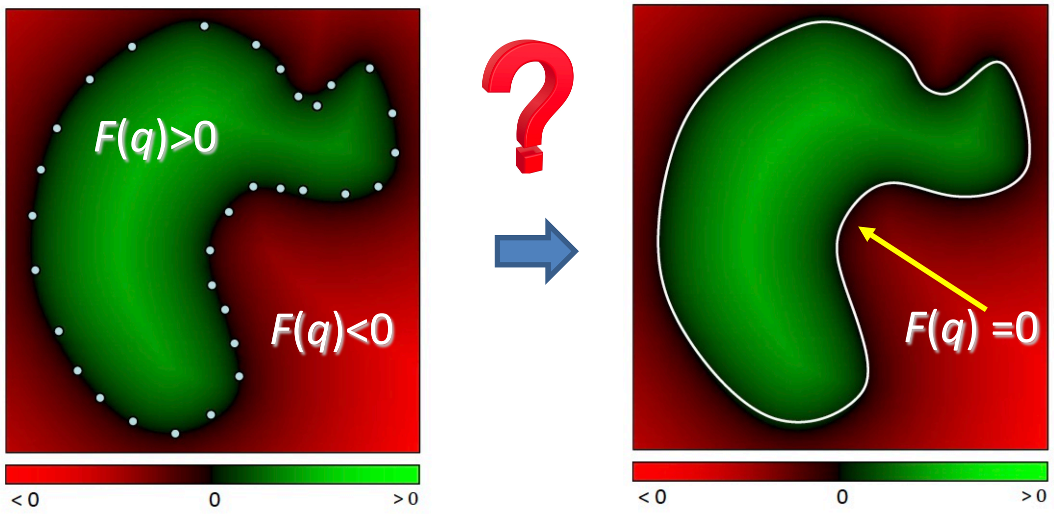
The pioneer work of Hoppe *et al.* (1992)

- Define a signed distance function (SDF)
  - Associate an oriented plane (tangent plane) with each of the data points
    - Tangent plane is a local linear approximation to the surface.
    - Used to define signed distance function to surface.



The pioneer work of Hoppe *et al.* (1992)

- Contour tracing
  - Extract 0-set iso-surface from the scalar field
    - Marching cubes



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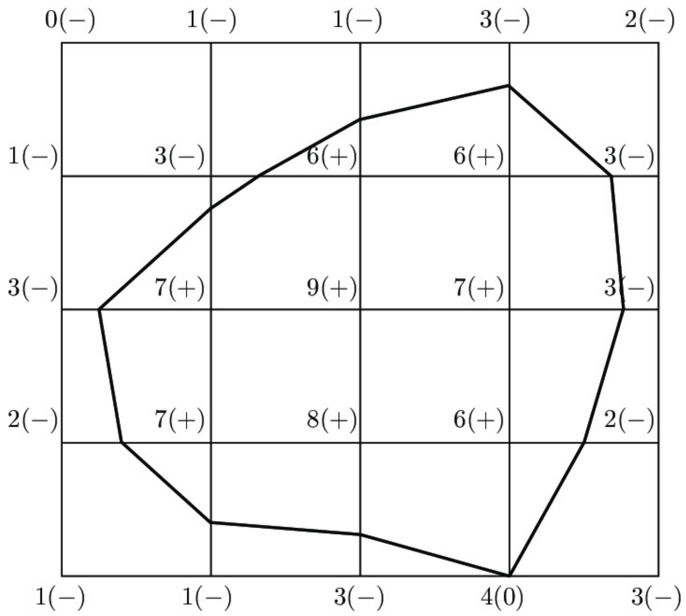
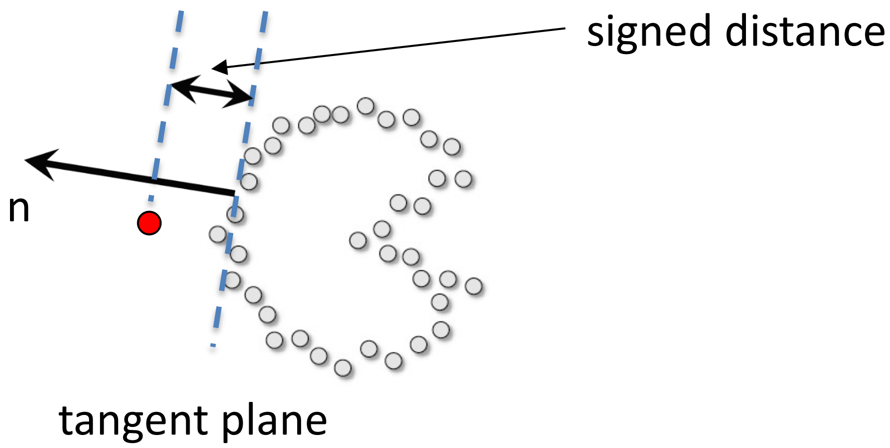
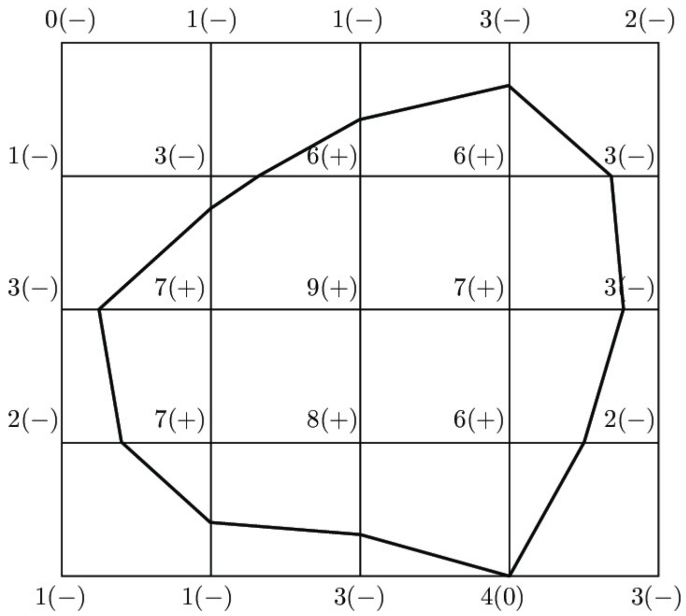


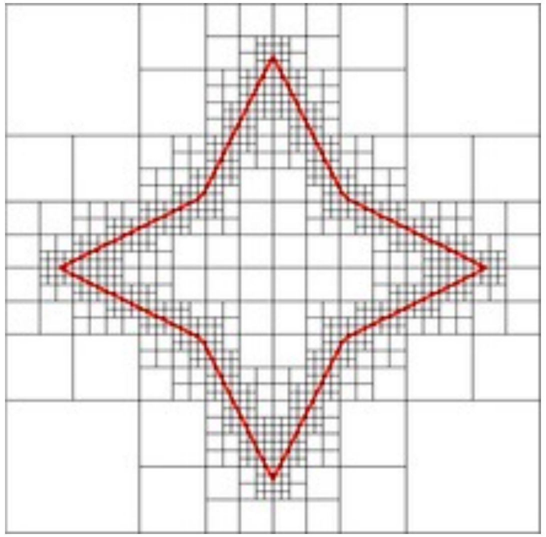
Marching squares (2D version of marching cubes)



The pioneer work of Hoppe *et al.* (1992)

- Contour tracing
  - Extract 0-set iso-surface from the scalar field
    - Marching cubes
    - Irregular grid





Marching squares (2D version of marching cubes)

The pioneer work of Hoppe *et al*. (1992)

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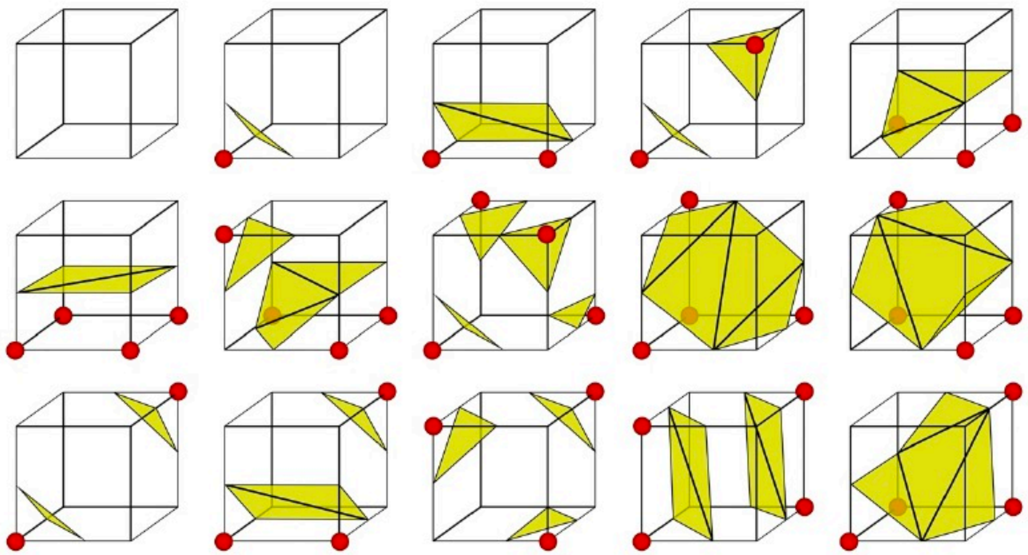
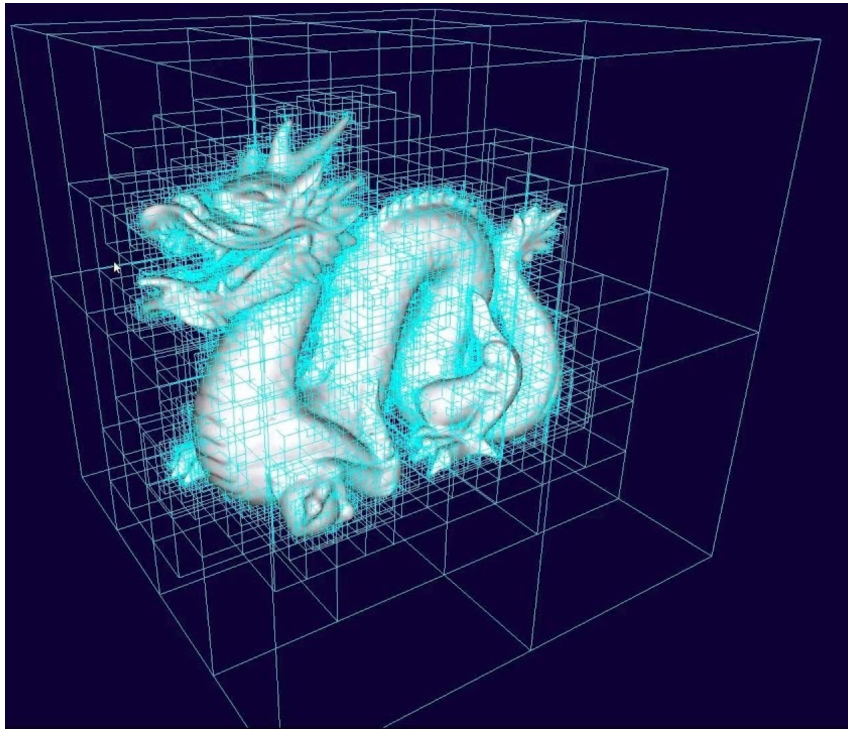
  - Extract 0-set iso-surface from the scalar field
  - Marching cubes

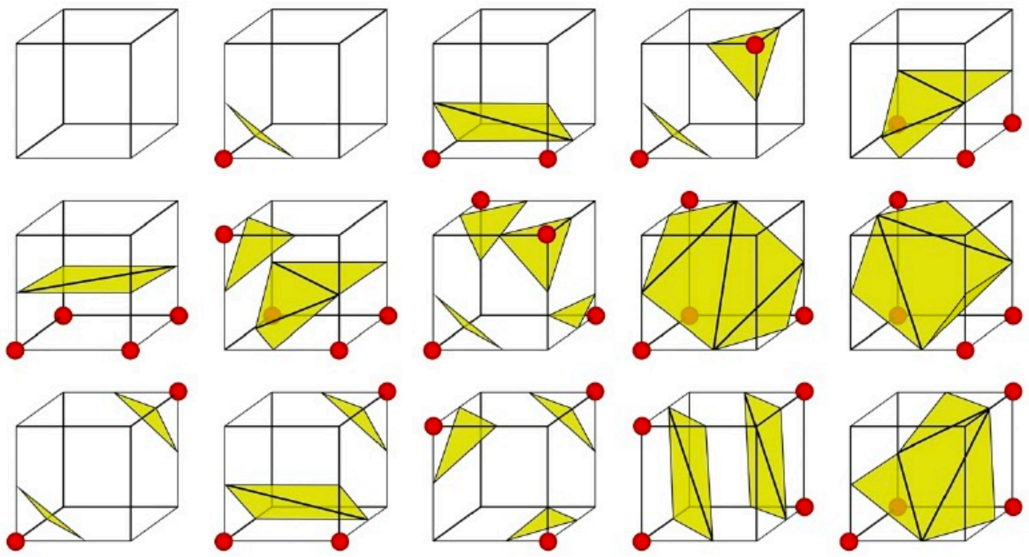


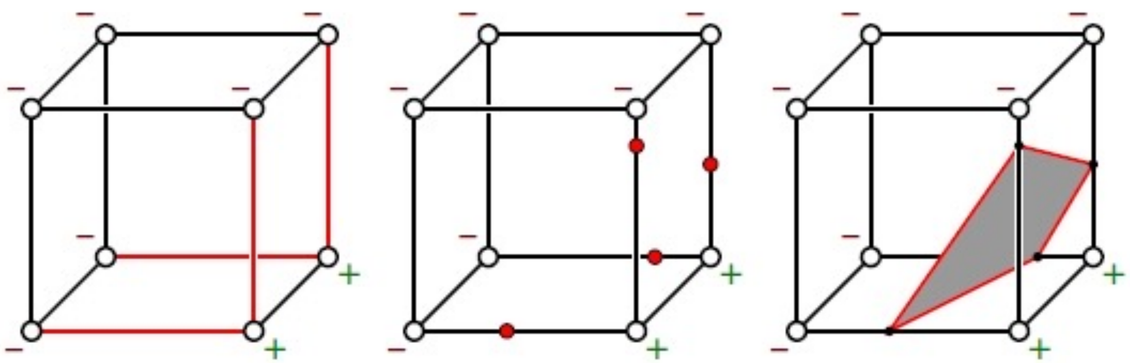


Marching cubes

The pioneer work of Hoppe *et al.* (1992)

- Contour tracing
  - Extract 0-set iso-surface from the scalar field
  - Marching cubes

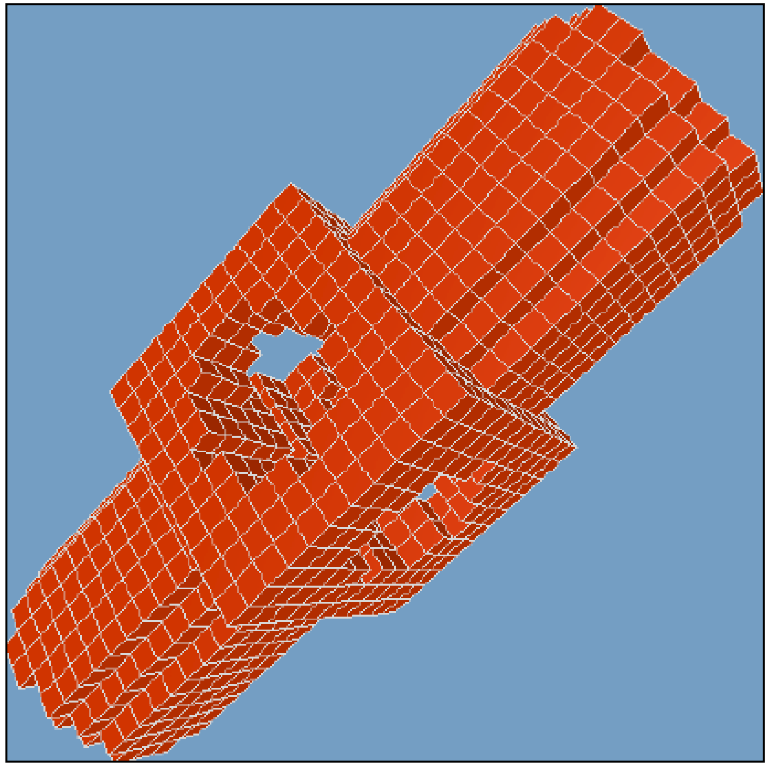


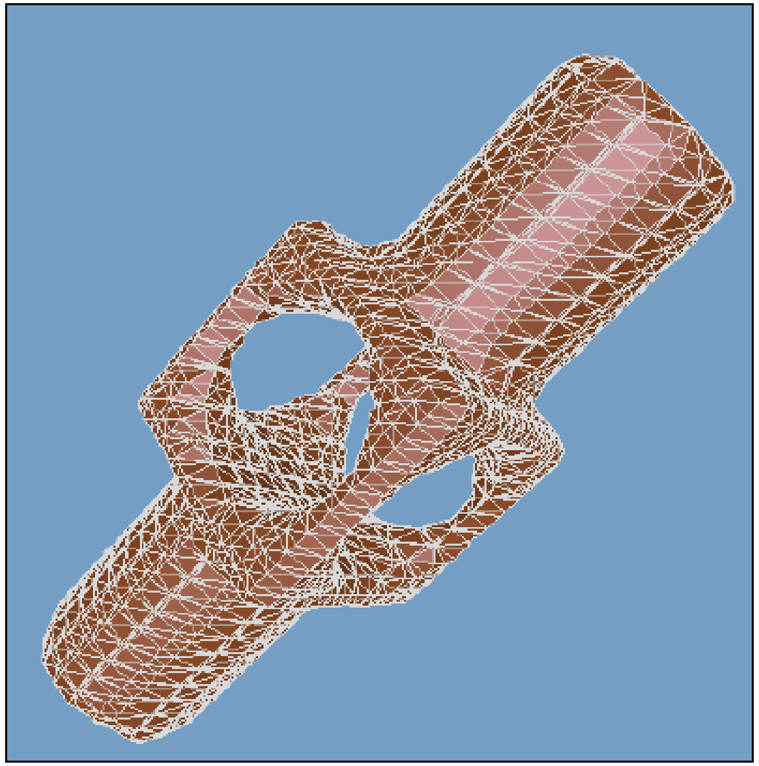


Marching cubes

The pioneer work of Hoppe *et al*. (1992)

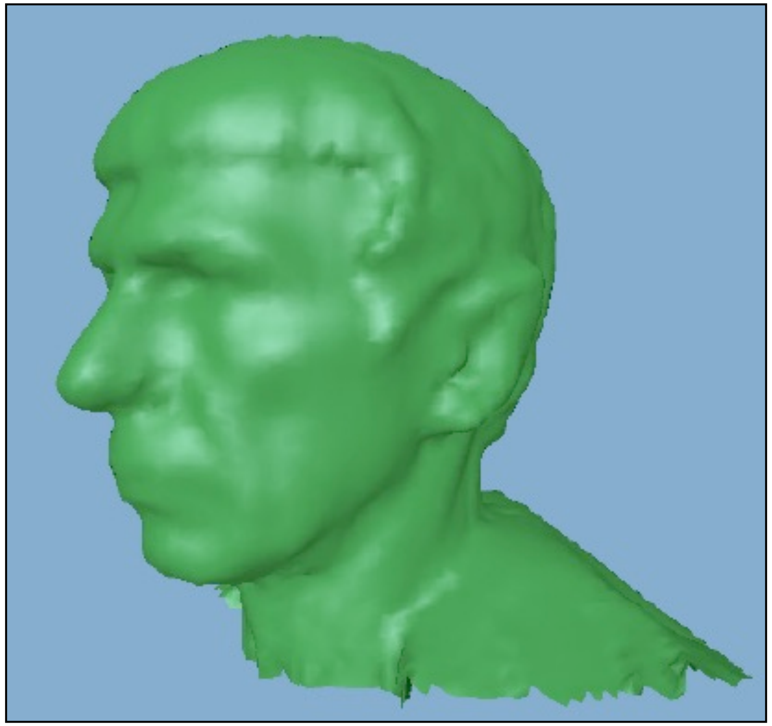
- Contour tracing

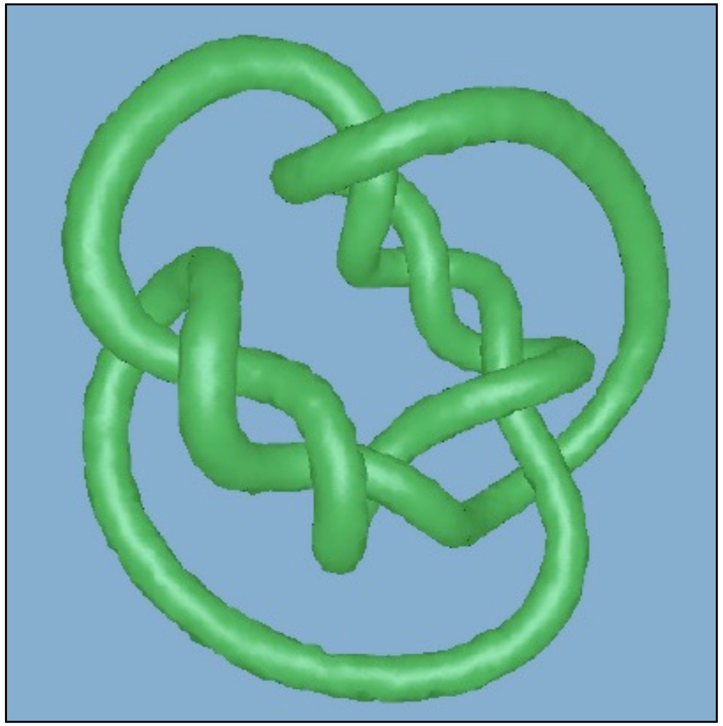




The pioneer work of Hoppe *et al*. (1992)

- Reconstruction results





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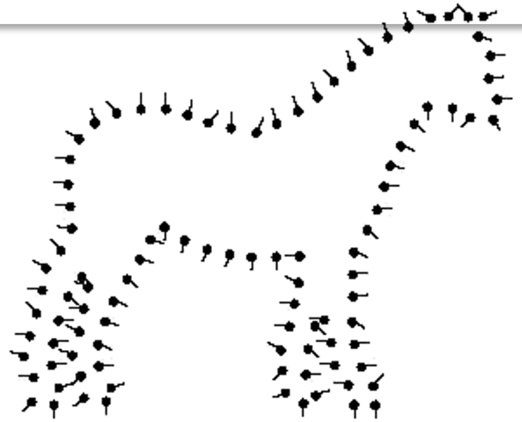
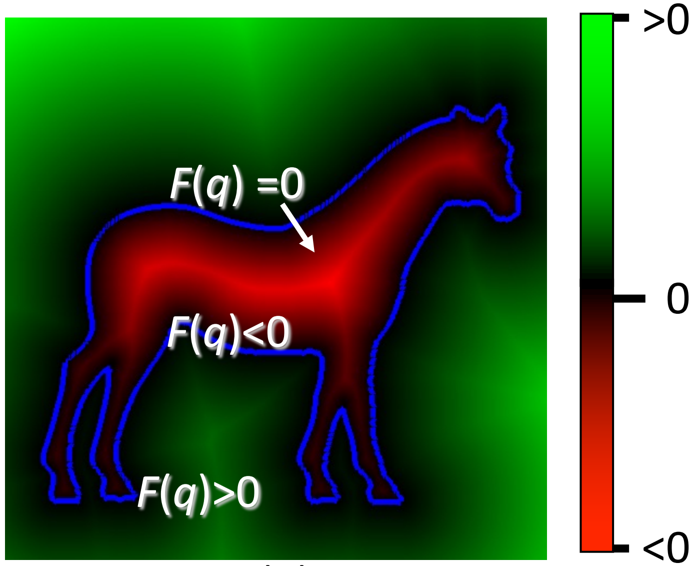
Poisson Reconstruction

- Implicit function fitting
  - Define a 3D scalar function
    - Zero values at the points
    - Positive values outside
    - Negative values inside
  - Extract the zero isosurface



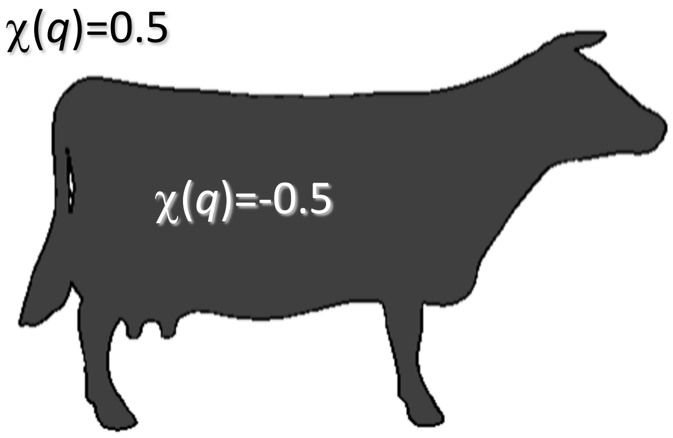
Sample points



F(*q*)

Poisson Reconstruction

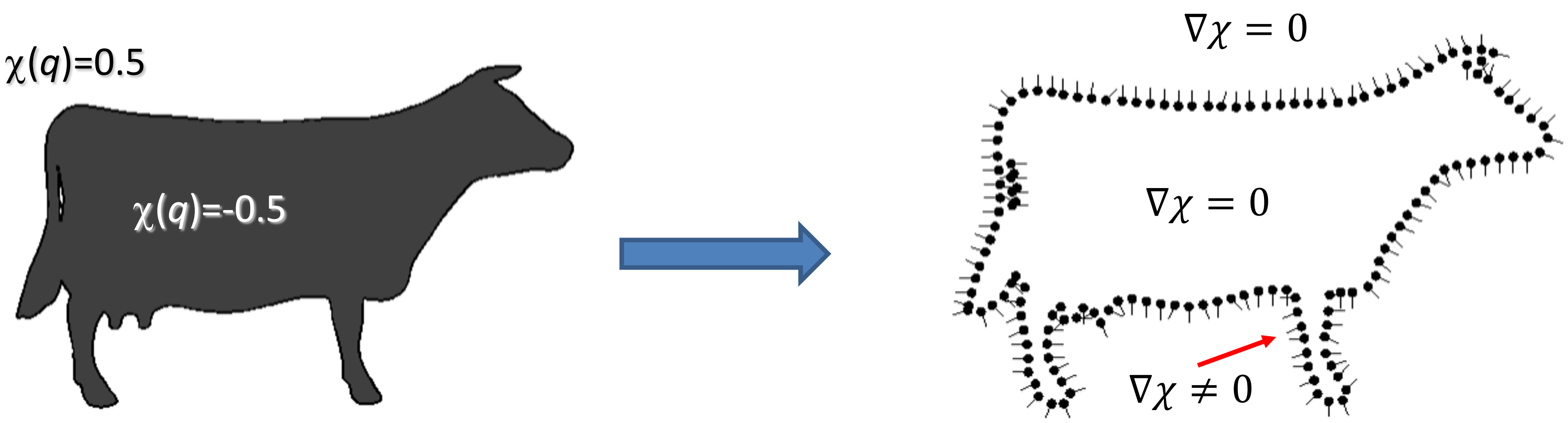
- The idea
  - The indicator function (χ)
    - Interior: constant negative value
    - Exterior: constant positive value



Poisson Reconstruction

- The idea
  - The indicator function (χ)
  - The gradient of the indicator function ($\nabla\chi$)
    - Zero everywhere except close to the boundary

Gradient?



- Gradient (∇f) is a vector at a point
  - Direction: the greatest rate of increase of ***f***
  - Magnitude: the rate of increase in that direction

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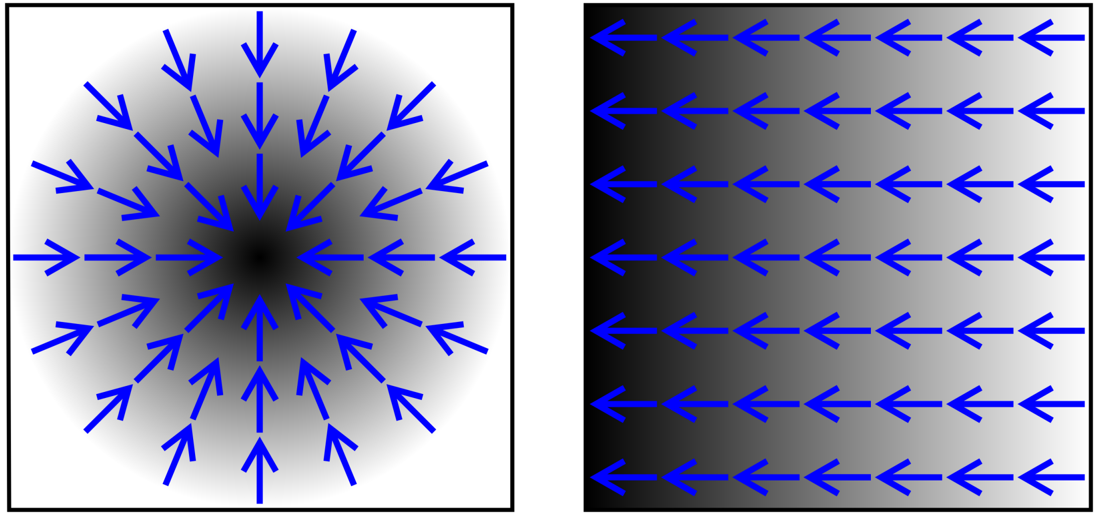
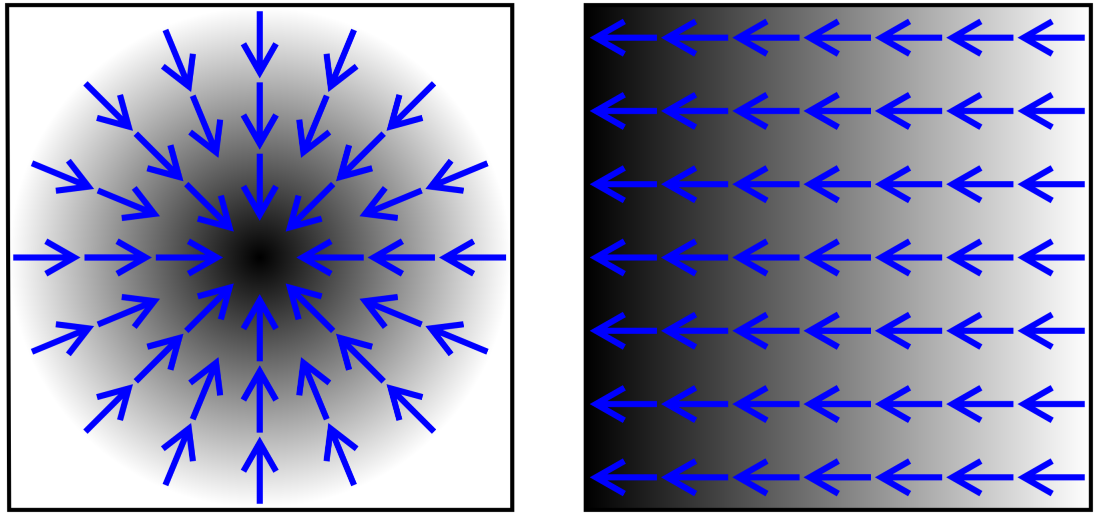
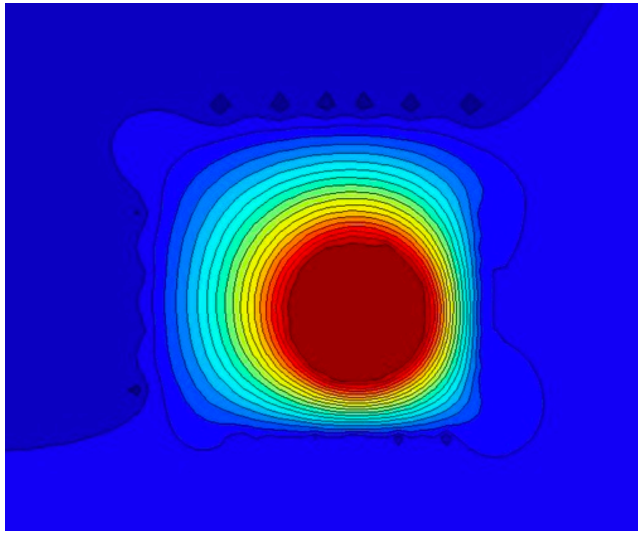


Gradient of colors

- Gradient (∇f) is a vector at a point
  - Direction: the greatest rate of increase of ***f***
  - Magnitude: the rate of increase in that direction



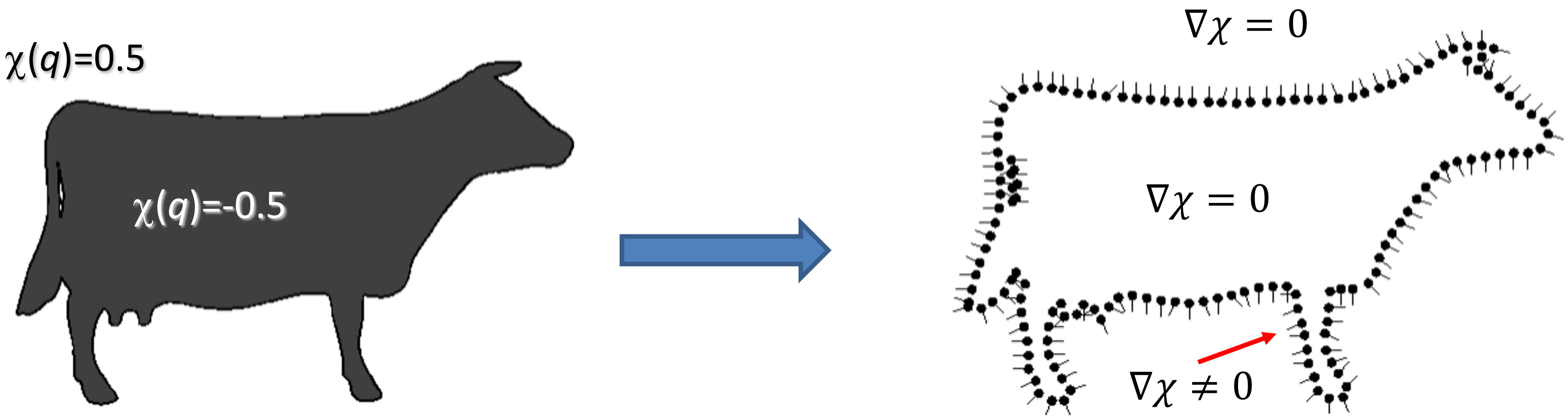
Gradient of colors



Heat source (2D)

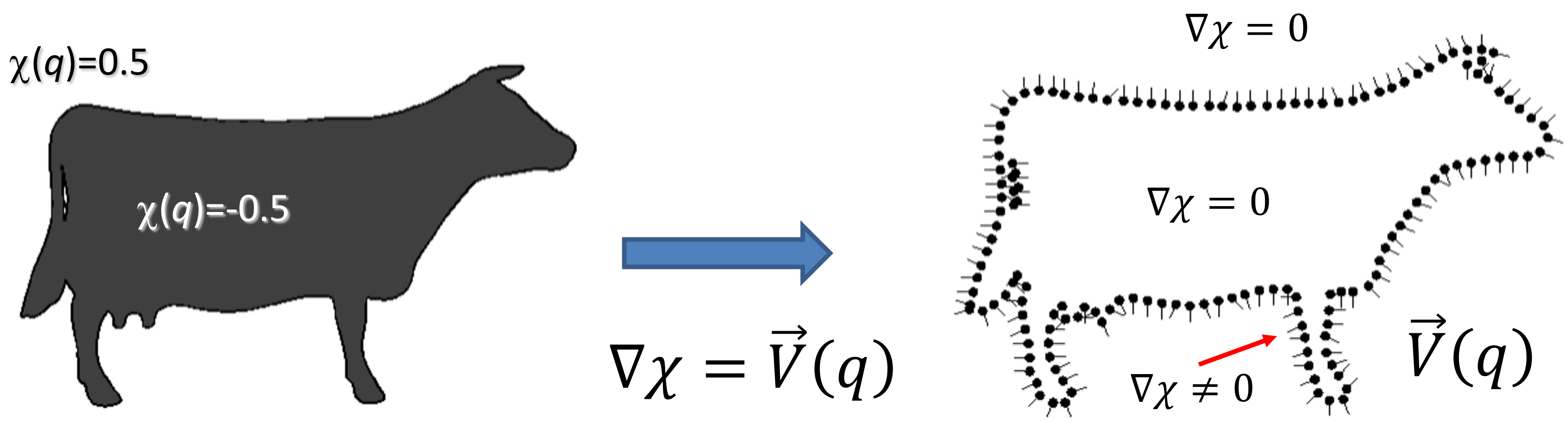
Poisson Reconstruction

- The idea
  - The indicator function (χ)
  - The gradient of the indicator function ($\nabla\chi$)
    - Zero everywhere except close to the boundary



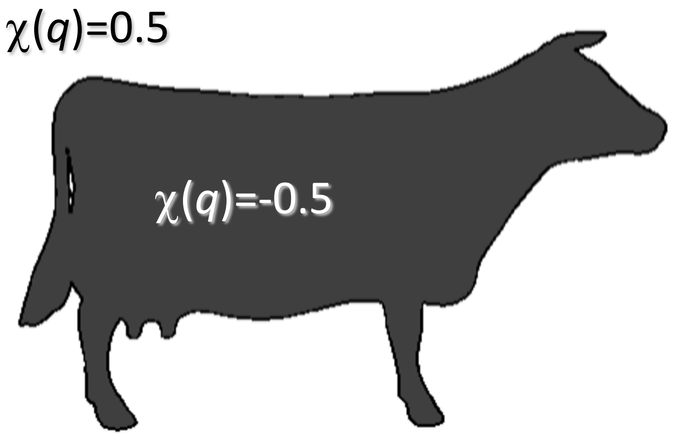
Poisson Reconstruction

- The idea
  - The indicator function (χ)
  - The gradient of the indicator function ($\nabla\chi$)
  - Oriented points $\vec{V}(q) \approx$ samples of gradient

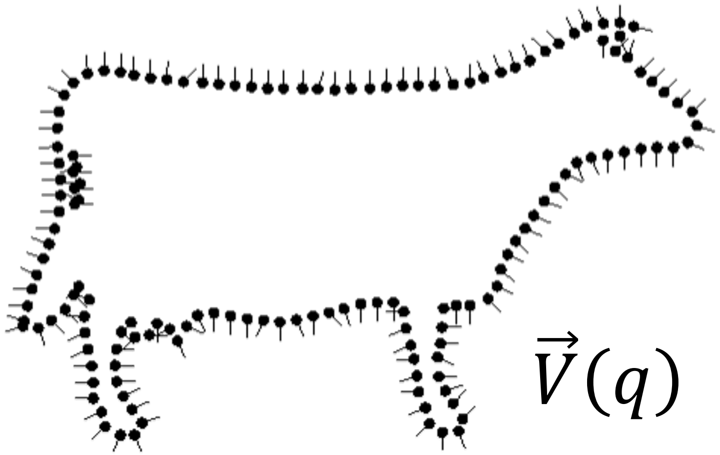


Poisson Reconstruction

- The idea
  - The indicator function (χ)
  - The gradient of the indicator function ($\nabla\chi$)
  - Oriented points $\vec{V}(q) \approx$ samples of gradient
  - Reconstruction

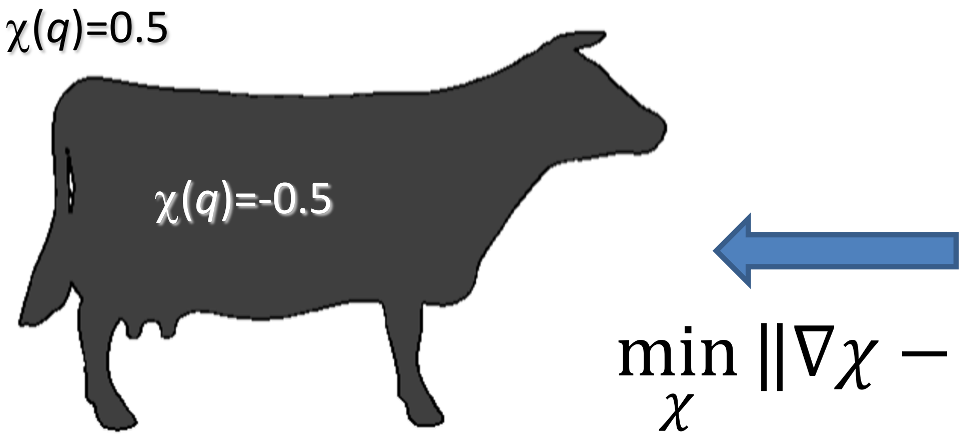


$$\nabla\chi = \vec{V}(q)$$

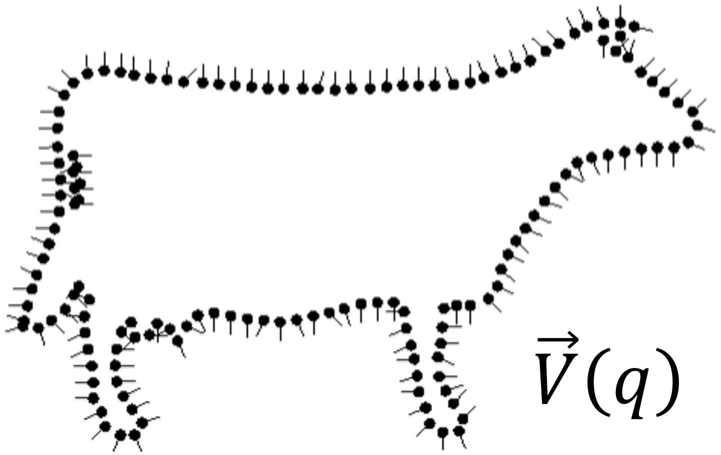


Poisson Reconstruction

- The idea
  - The indicator function (χ)
  - The gradient of the indicator function ($\nabla\chi$)
  - Oriented points $\vec{V}(q) \approx$ samples of gradient
  - Reconstruction: finding the indicator function



$$\min_{\chi} \|\nabla\chi - \vec{V}\|$$



$\vec{V}(q)$

Poisson Reconstruction

- Reconstruction: finding the indicator function
  - Continues vector field? $\nabla\chi = \vec{V}(q)$
  - The best least-squares approximate solution
  - Apply the divergence operator

$$\min_{\chi} \|\nabla\chi - \vec{V}\|$$

$$\Updownarrow \nabla\cdot$$

$$\nabla\cdot\nabla\chi = \nabla\cdot\vec{V} \iff \boxed{\Delta\chi = \nabla\cdot\vec{V}}$$

?

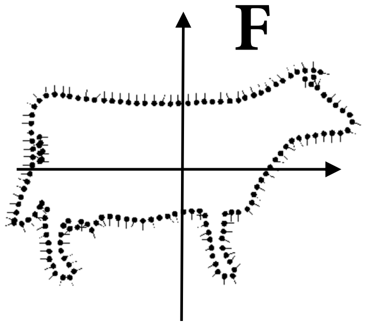
Poisson equation

Laplace operator (or Laplacian): ∇·∇, ∇^2, or Δ
- Divergence of the gradient of a function

Poisson Reconstruction

- Pipeline
  - Point samples -> Continuous vector field

$$\mathbf{F} = U\,\vec{i} + V\,\vec{j} + W\,\vec{k}$$



Poisson Reconstruction

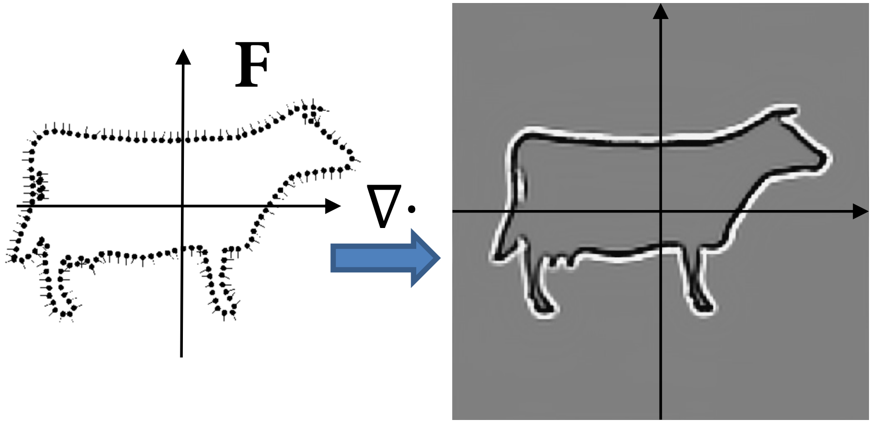
- Pipeline
  - Point samples -> Continuous vector field

$$\mathbf{F} = U\vec{i} + V\vec{j} + W\vec{k}$$

  - Compute the divergence ($\nabla \cdot$)

the sum of the first-order partial derivative

$$\operatorname{div}\mathbf{F} = \nabla \cdot \mathbf{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (U, V, W) = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}$$

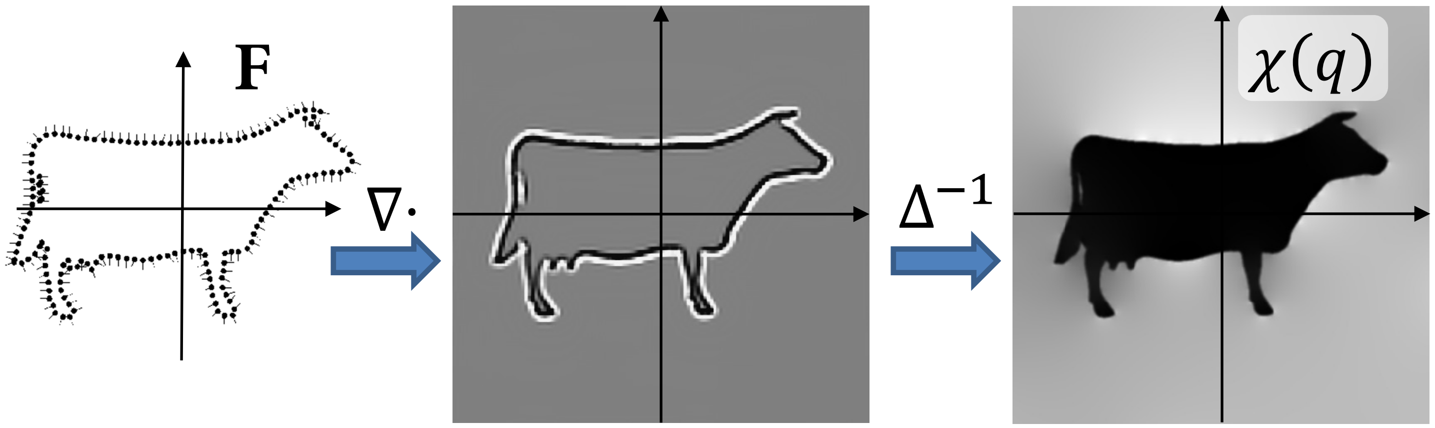


https://en.wikipedia.org/wiki/Divergence

Poisson Reconstruction

- Pipeline
  - Point samples -> Continuous vector field
  - Compute the divergence $(\nabla\cdot)$
  - Solve the Poisson equation

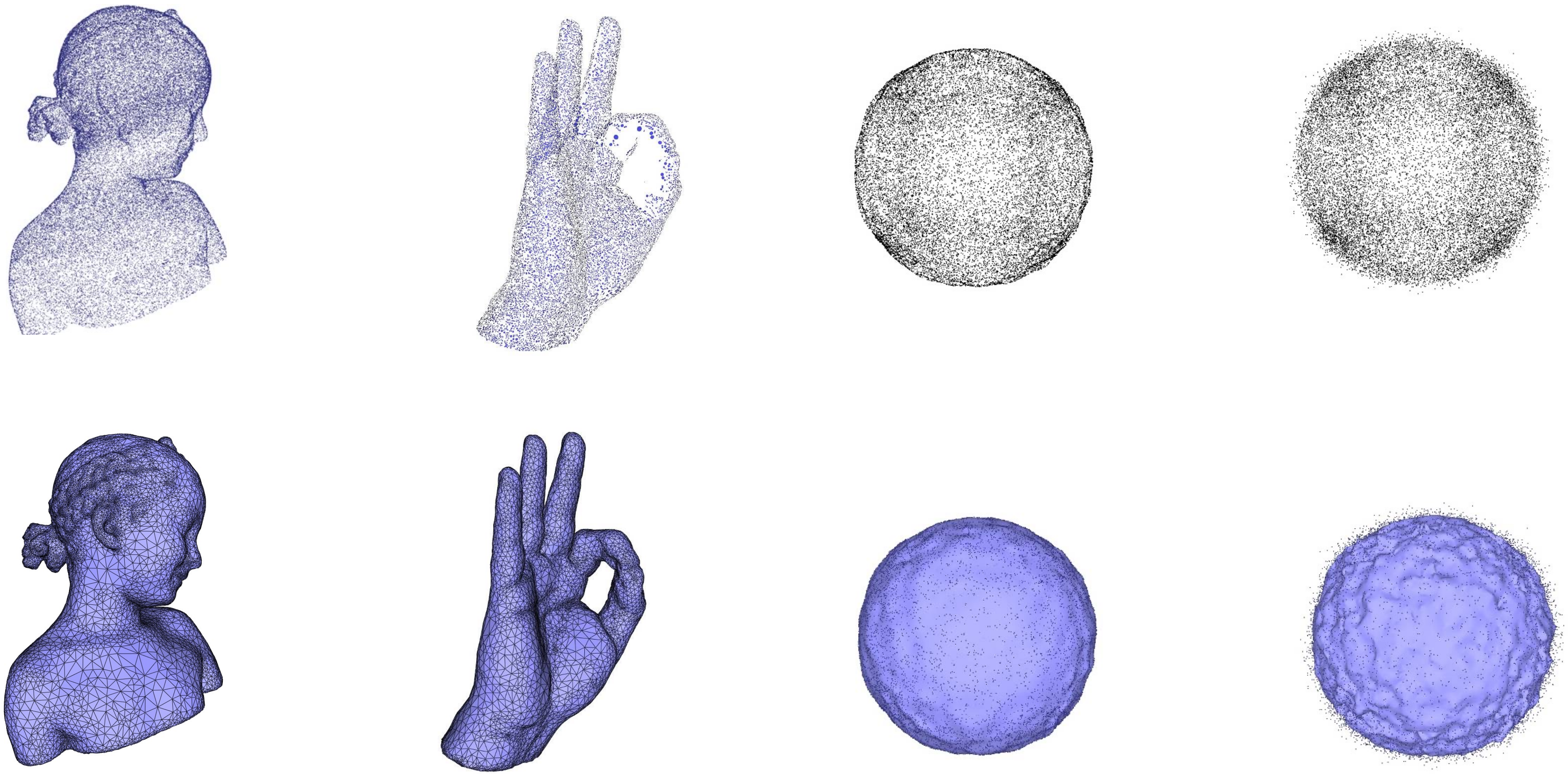
$$\Delta\chi = \nabla \cdot \mathbf{F}$$



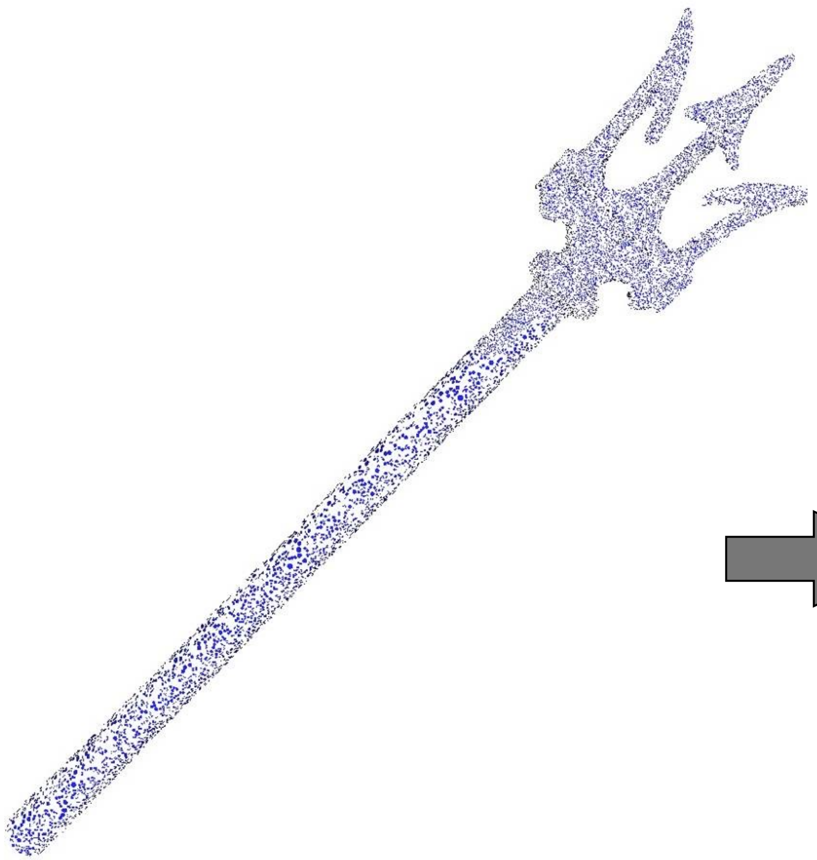
Poisson Reconstruction

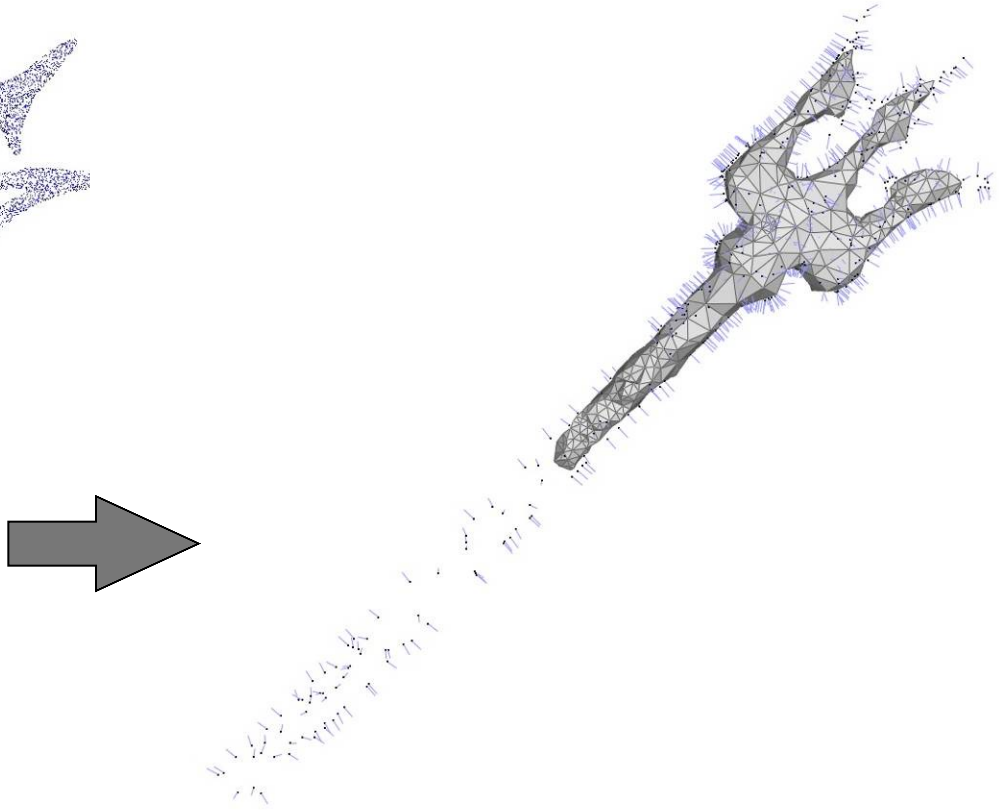
- Pipeline
  - Point samples -> Continuous vector field
  - Compute the divergence ($\nabla\cdot$)
  - Solve the Poisson equation
  - Iso-surface extraction
    - Marching cubes

Poisson Reconstruction



Poisson Reconstruction

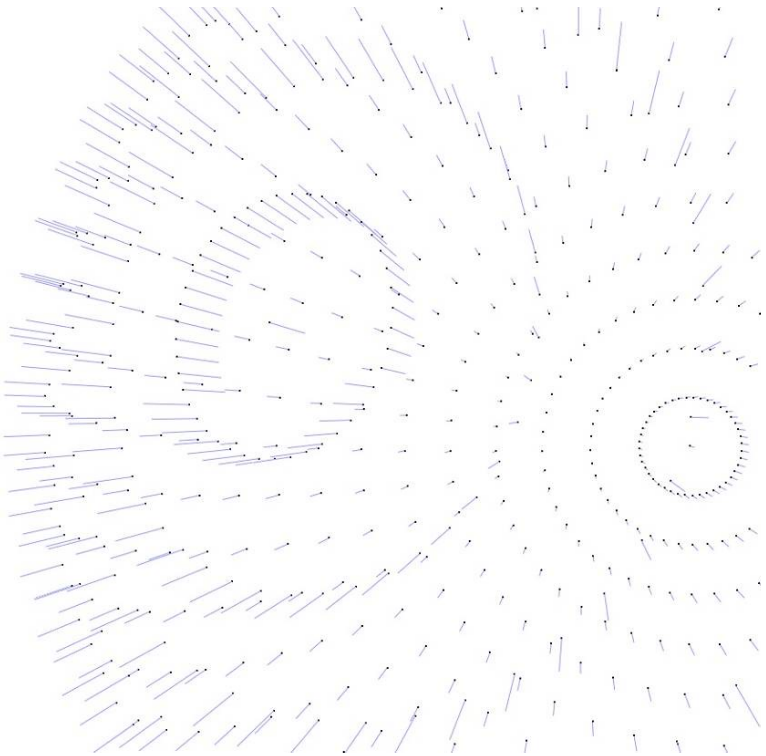




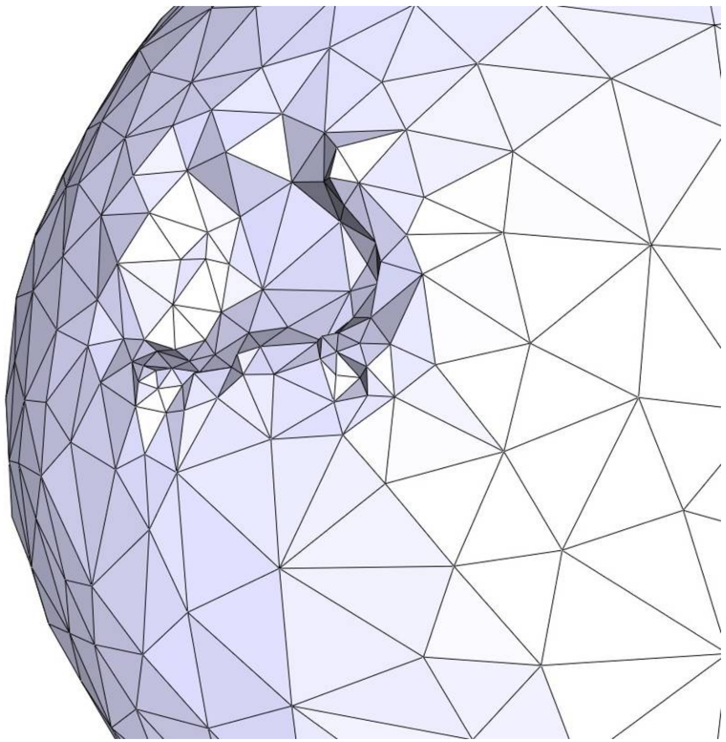
Left: 50K points sampled on Neptune trident

Right: point set simplified to 1K then reconstructed

Poisson Reconstruction

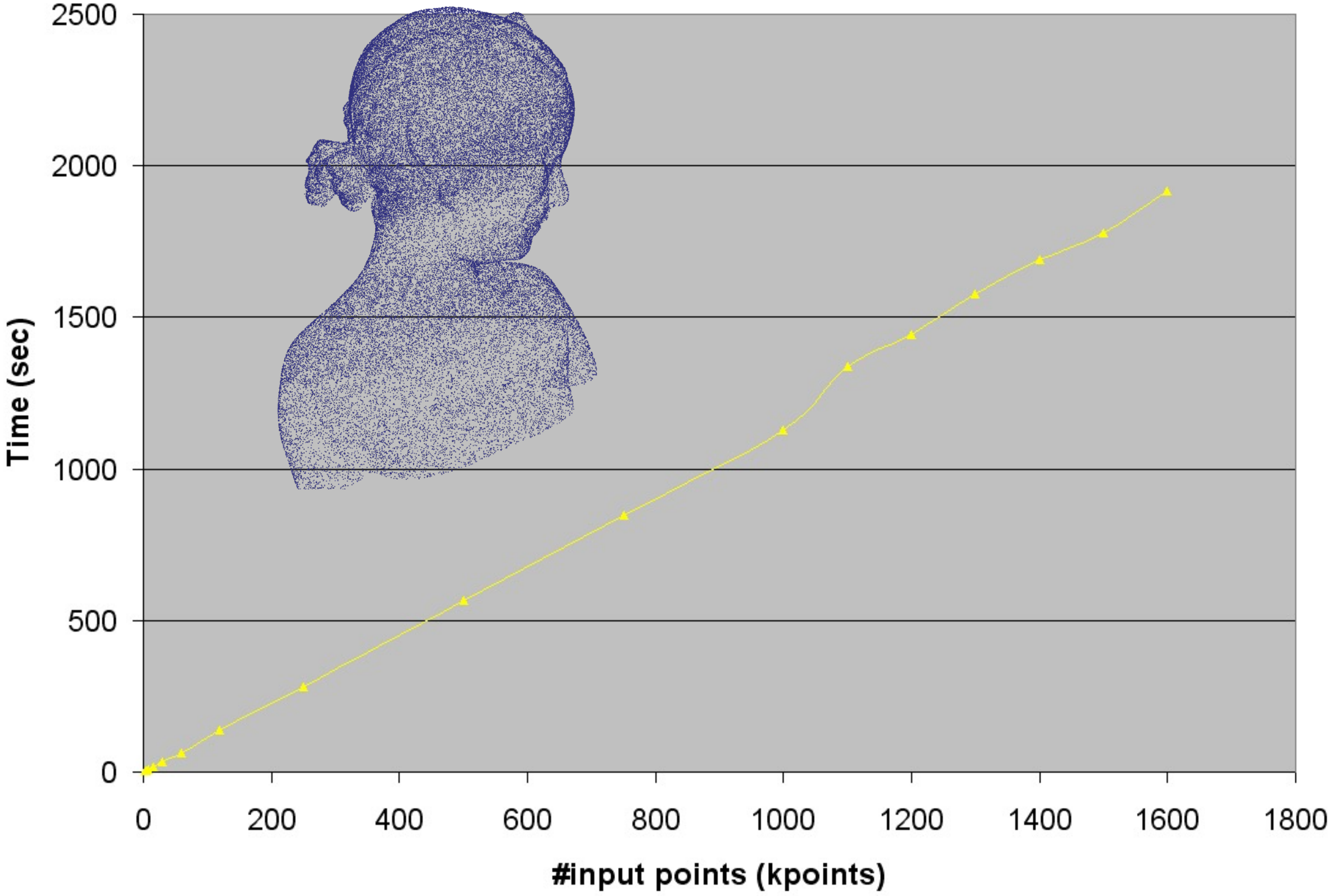


Left: points sampled on a sphere
with flipped normals

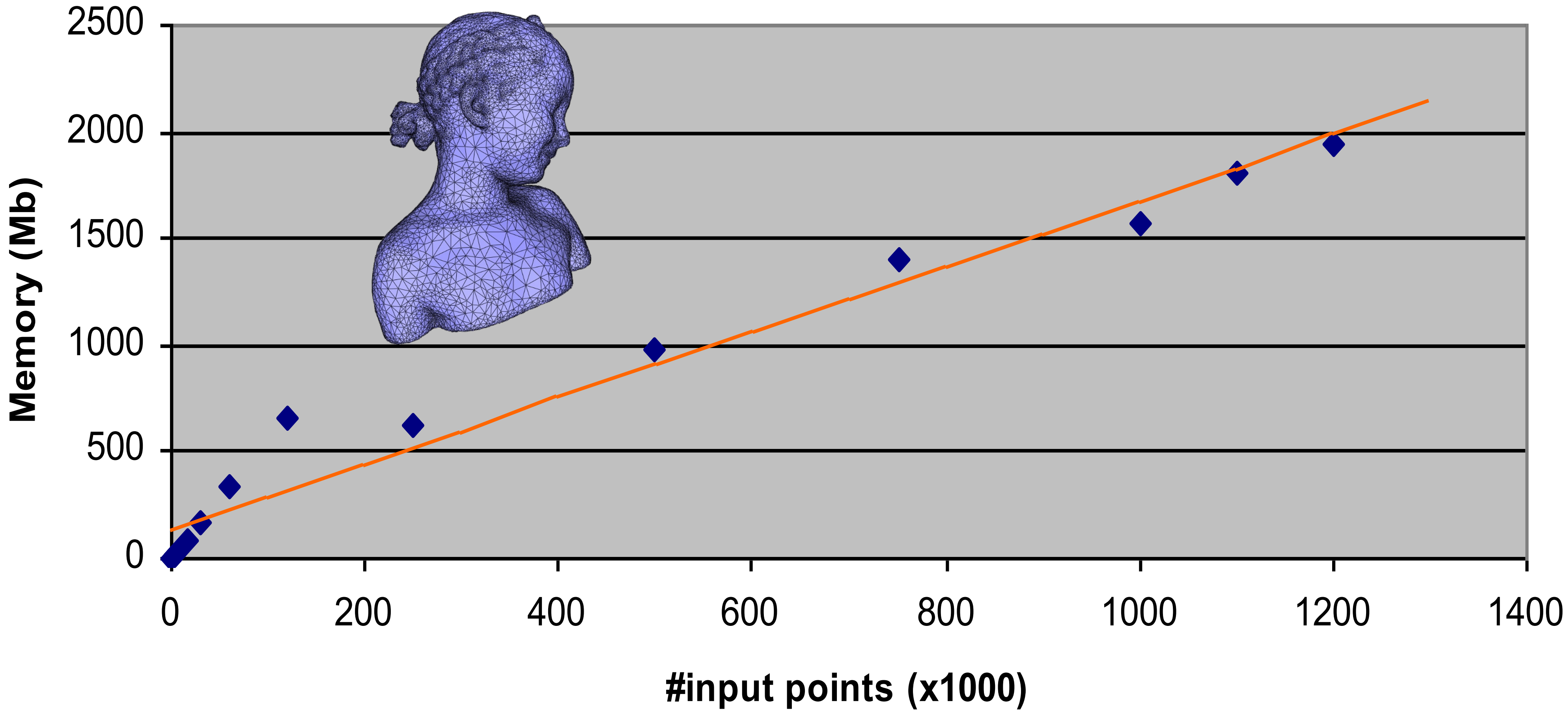


Right: reconstructed surface

Poisson duration wrt #input points

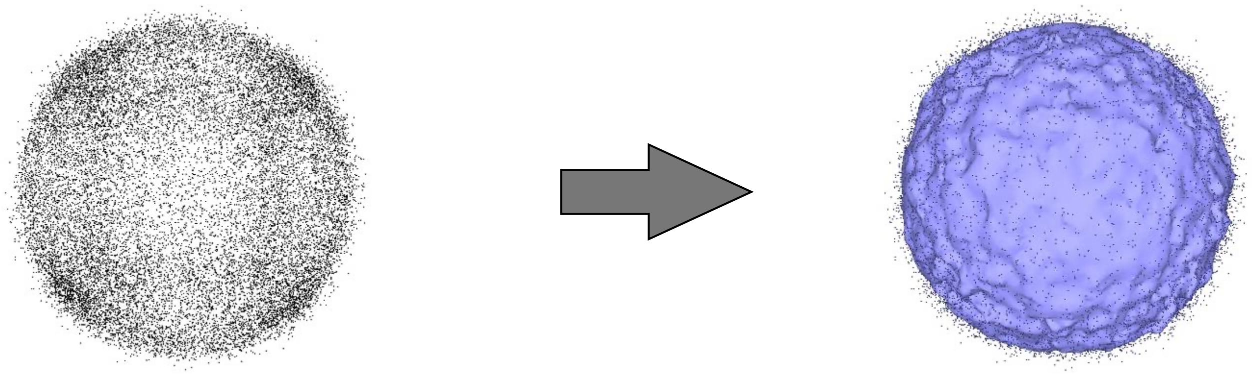


Memory wrt #input points



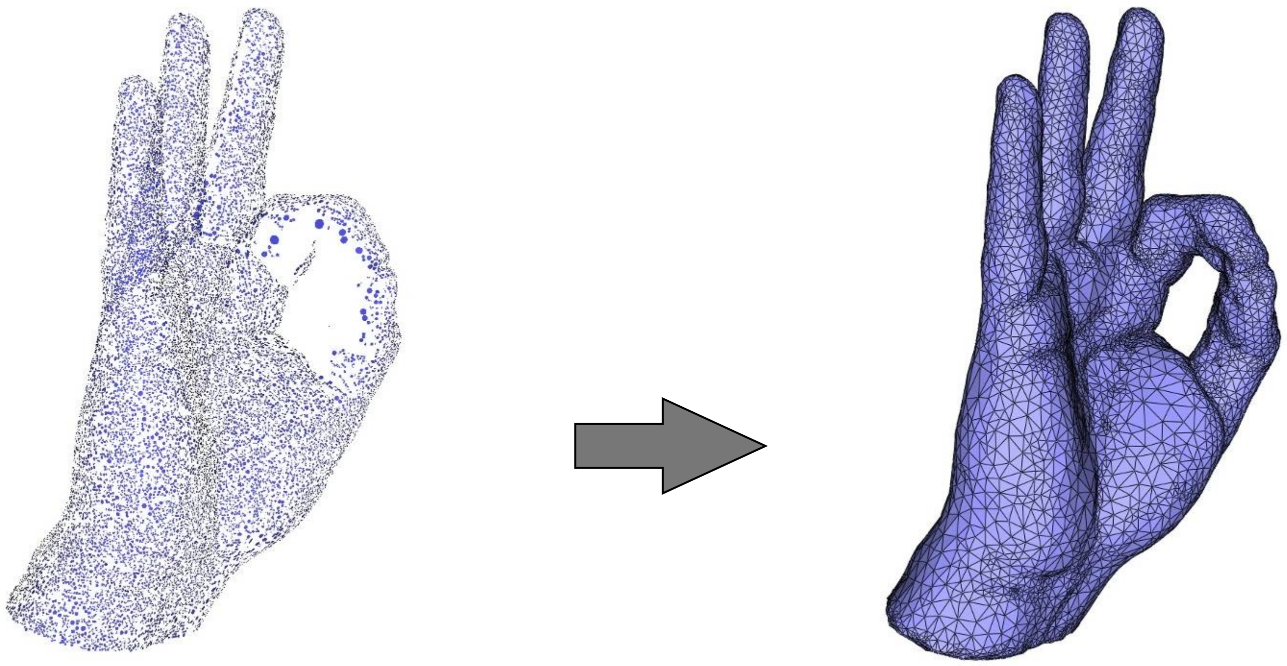
Poisson Reconstruction

- Properties
  - ✓ Supports noisy, non-uniform data



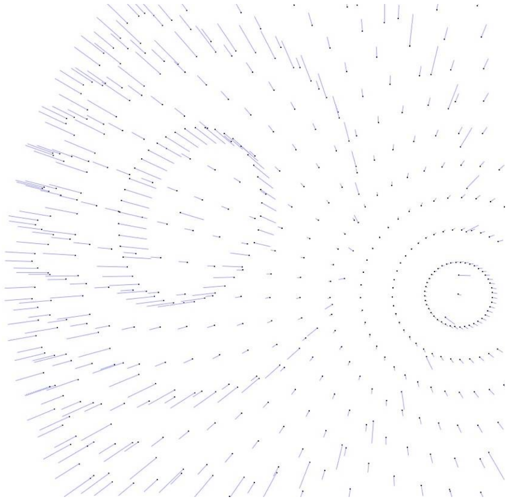
Poisson Reconstruction

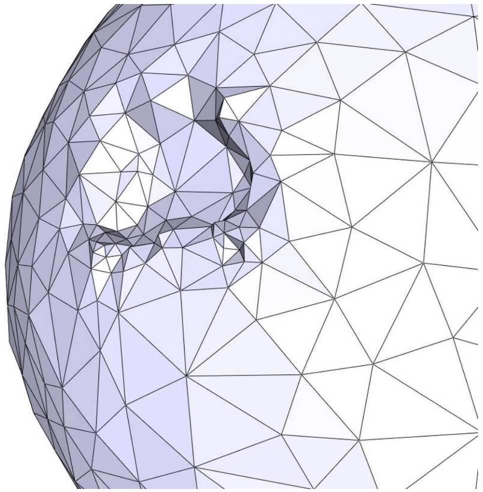
- Properties
  - ✓ Supports noisy, non-uniform data
  - ✓ Can fill reasonably large holes



Poisson Reconstruction

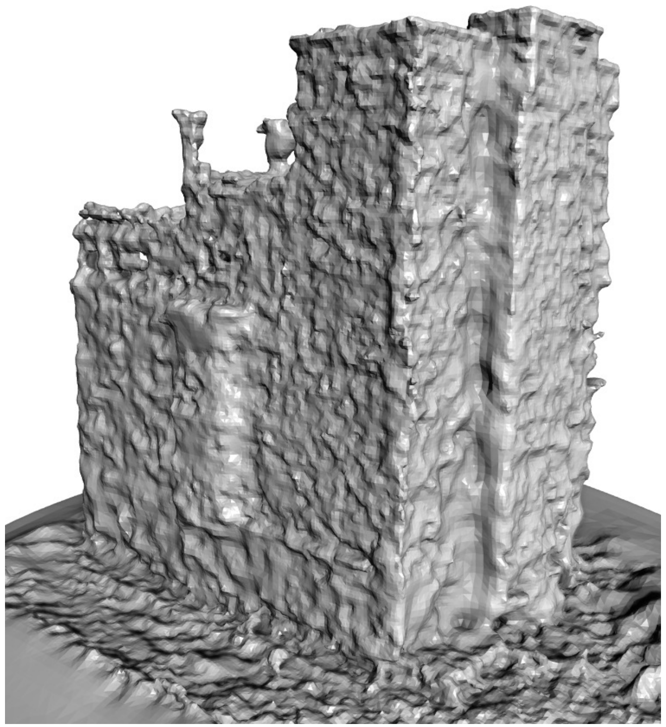
- Properties
  - ✓ Supports noisy, non-uniform data
  - ✓ Can fill reasonably large holes
- Limitations
  - It requires good normal information





Poisson Reconstruction

- Properties
  - ✓ Supports noisy, non-uniform data
  - ✓ Can fill reasonably large holes
- Limitations
  - It requires good normal information
  - Sharp features are oversmoothed
    - Not good for piecewise planar objects

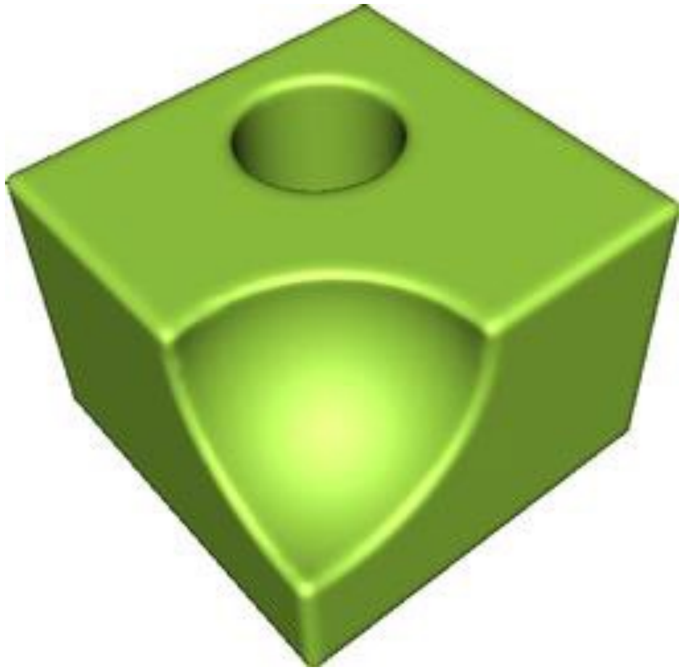


Today's Agenda

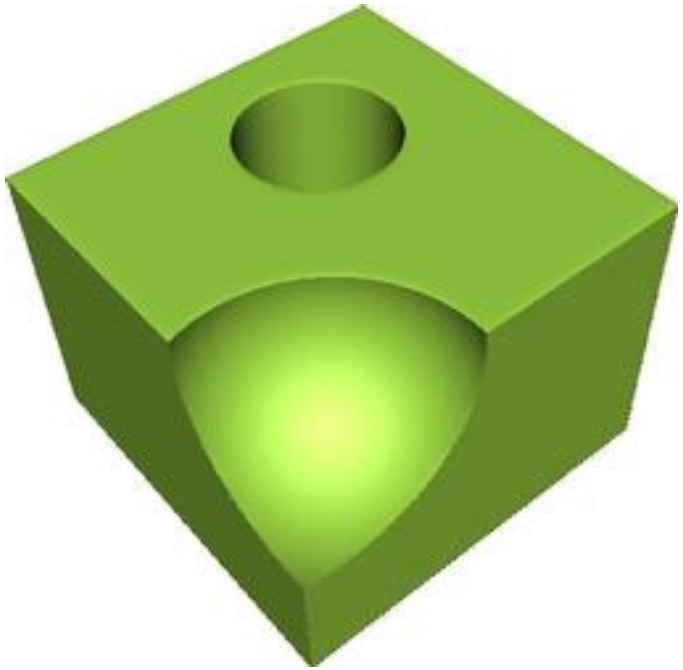
- Introduction
- Smooth object reconstruction
  - The pioneer work of Hoppe *et al*. (1992)
  - Poisson reconstruction [Kazhdan *et al*. 06]
  - Piecewise smooth reconstruction
- Piecewise planar object reconstruction [Nan and Wonka. 2017]

Piecewise Smooth Reconstruction

- Piecewise-smooth



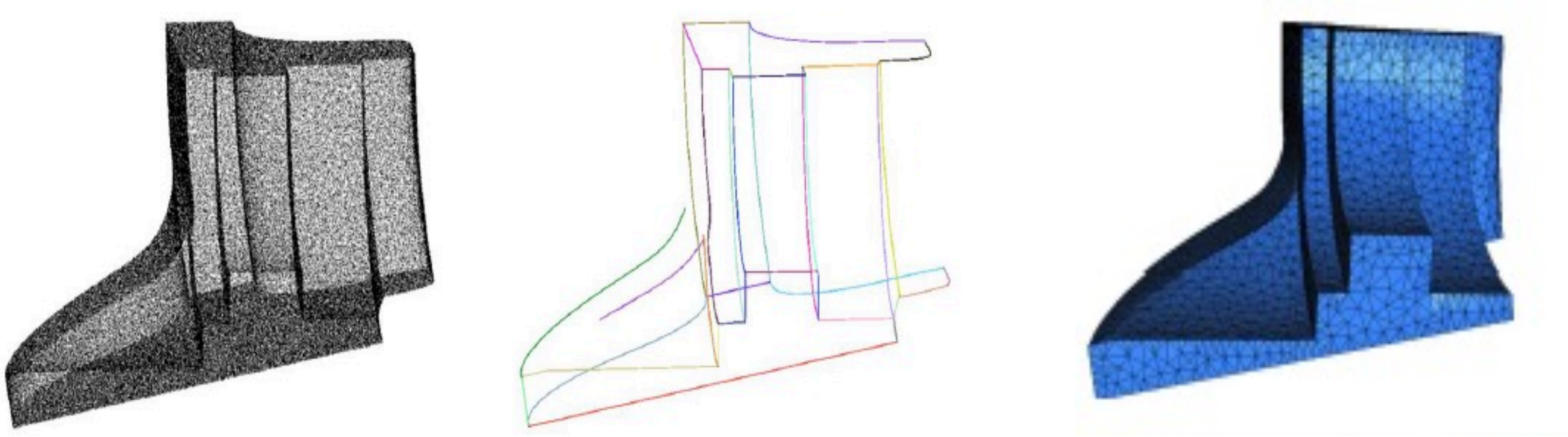
smooth



piecewise-smooth

Piecewise Smooth Reconstruction

- Feature detection
  - Extract a set of sharp features
  - Decompose the point cloud into smooth patches
- Smooth reconstruction patch by patch
- Stitch the patches

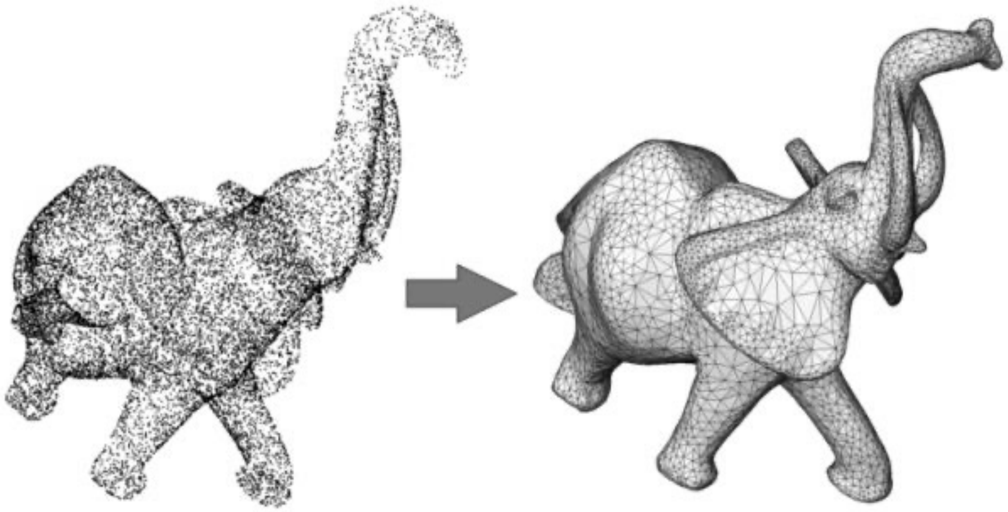


Today's Agenda

- Introduction
- Smooth object reconstruction
  - The pioneer work of Hoppe *et al*. (1992)
  - Poisson reconstruction [Kazhdan *et al*. 06]
  - Piecewise smooth reconstruction
- Piecewise planar object reconstruction

Polygonal Surface Reconstruction

- Surface Reconstruction Methods
  - Smooth surfaces
    - Fit noisy data; infer topology; fill (small) holes



Poisson Surface Reconstruction [Kazhdan *et al.* 06]

Polygonal Surface Reconstruction

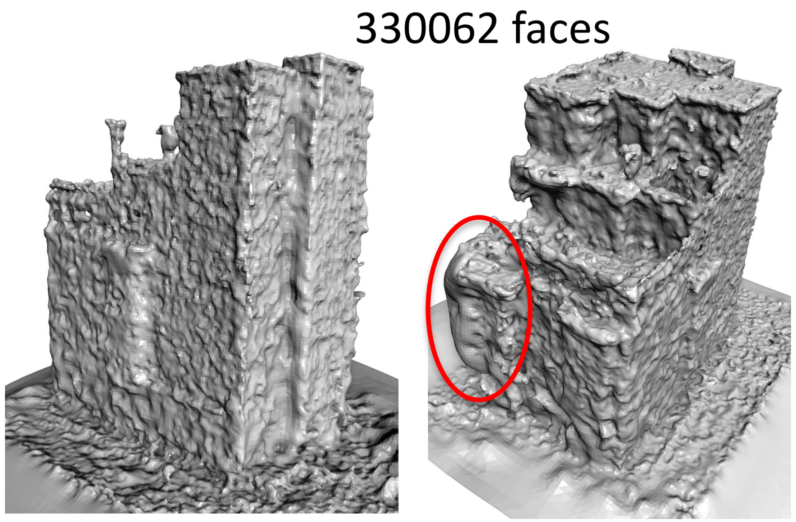
- Surface Reconstruction Methods
  - Smooth surfaces
  - Piecewise planar objects

Polygonal Surface Reconstruction

- Surface Reconstruction Methods
  - Smooth surfaces
  - Piecewise planar objects


Polygonal Surface Reconstruction

- Surface Reconstruction Methods
  - Smooth surfaces
  - Piecewise planar objects



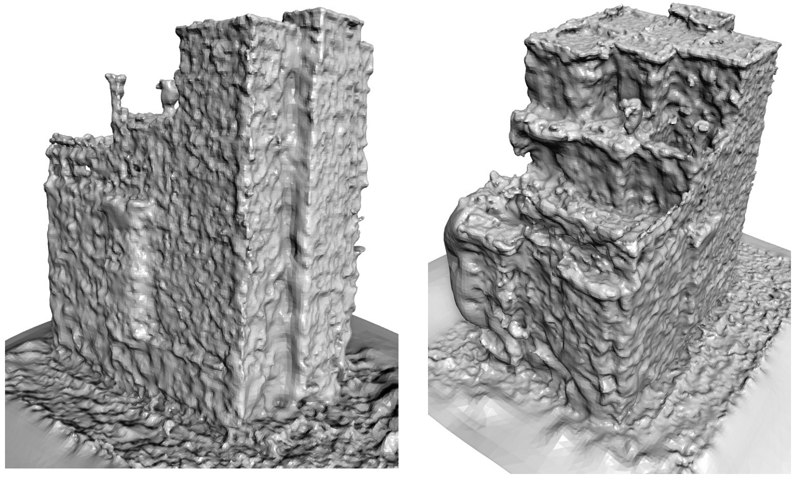
Result of [Kazhdan *et al.* 06]

- Unsatisfied results
  - Bumpy
  - Large number of faces
  - Unacceptable hole filling
- Rare direct applications
  - Post-processing required
    - Topologically correct
    - Simplified

Polygonal Surface Reconstruction

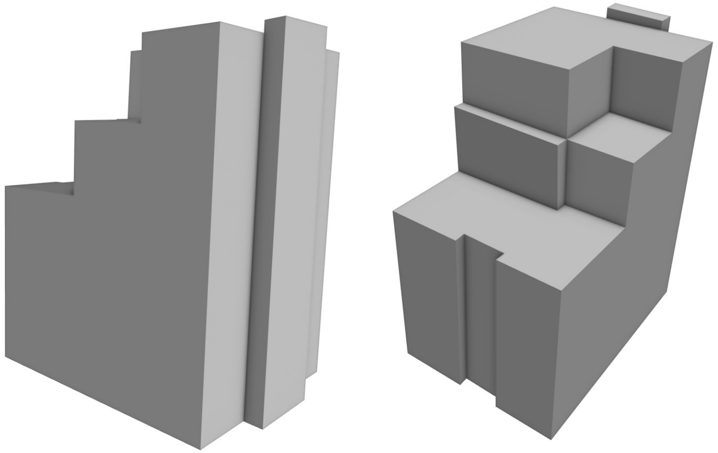
- Surface Reconstruction Methods
  - Smooth surfaces
  - Piecewise planar objects

330062 faces



Result of [Kazhdan *et al.* 06]

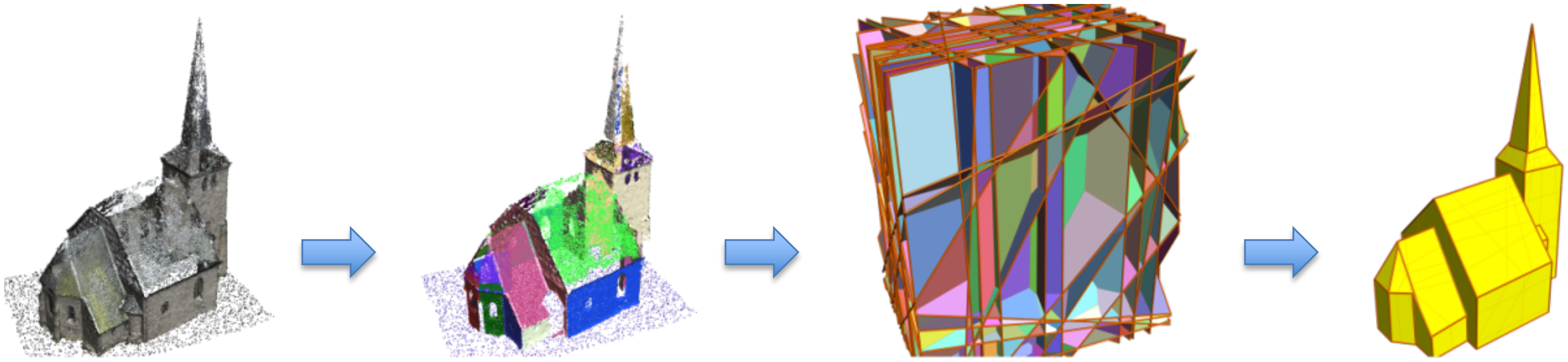
25 faces



[Nan and Wonka 17]

Polygonal Surface Reconstruction

- Overview

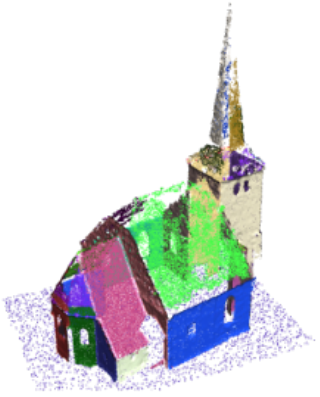


Input Planar segments Candidate faces Result

Polygonal Surface Reconstruction

- Overview

Input



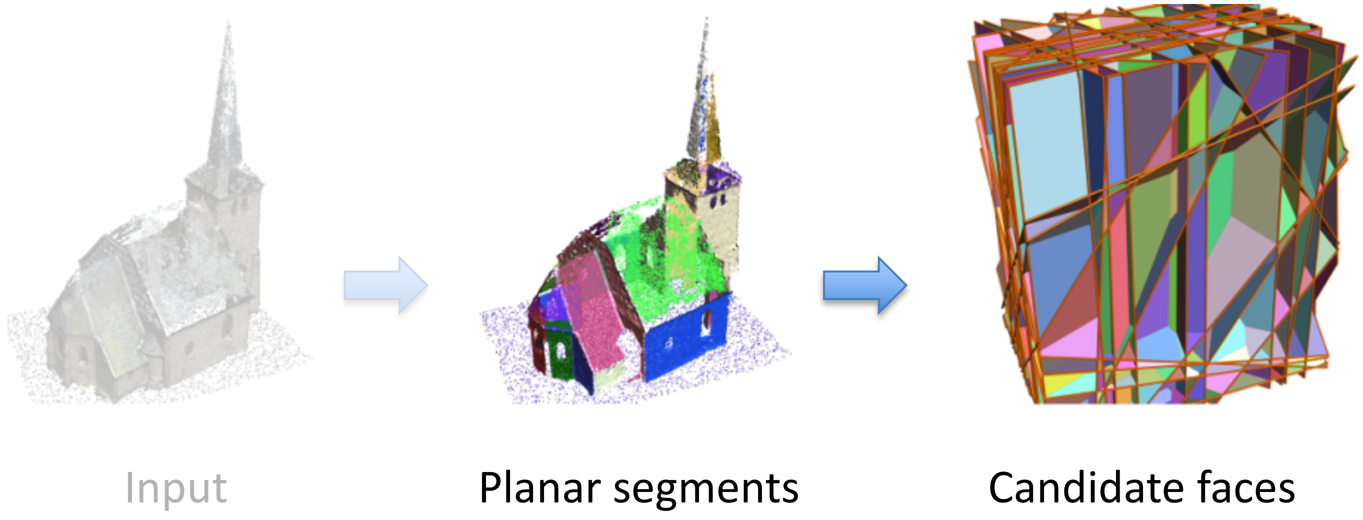
Planar segments

RANSAC algorithm

- Random 3 points -> plane
- Scoring, accept or reject
- Repeat
  - Plane from the remaining points
  - Stop if no plane can be extracted

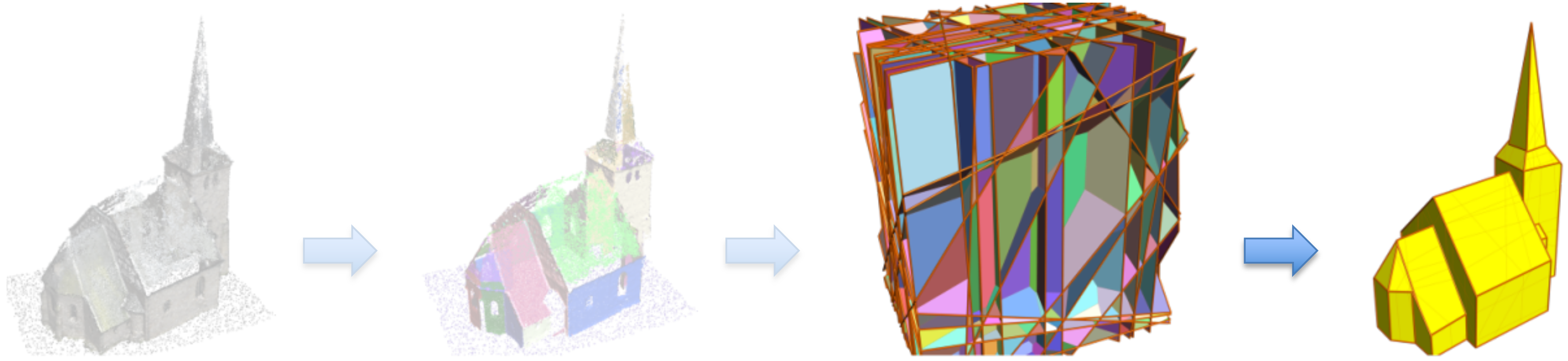
Polygonal Surface Reconstruction

- Candidate Generation
  - Supporting plane clipping
  - Pairwise intersection



Polygonal Surface Reconstruction

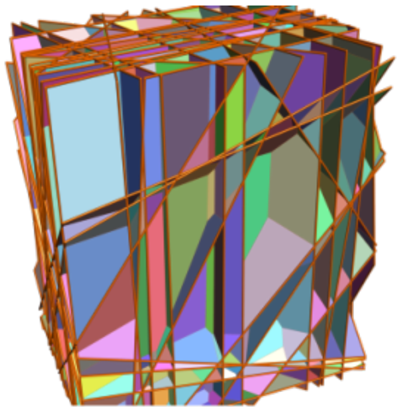
- Face Selection



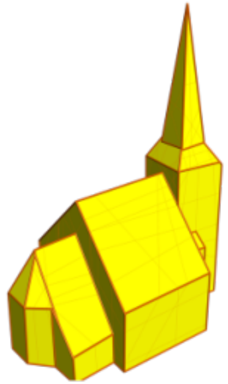
Input Planar segments Candidate faces Result

Polygonal Surface Reconstruction

- Face Selection
  - – Labeling problem
  - – Linear integer program



Candidate faces



Result

N candidate faces $F = \{f_i | 1 \le i \le N\}$

Variables: $x_i = \begin{cases} 1, & \text{face } f_i \text{ will be chosen} \\ 0, & \text{face } f_i \text{ will } \textbf{not} \text{ be chosen} \end{cases}$

Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
    - Favors selecting faces with more support
    - Percentage of unused points

$$E_f = 1 - \frac{1}{|P|} \sum_{i=1}^{N} x_i \cdot support(f_i)$$

Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
    - Favors selecting faces with more support
    - Percentage of unused points

$$E_f = 1 - \frac{1}{|P|} \sum_{i=1}^{N} x_i \cdot support(f_i)$$

Confidence weighted number of supporting point

$$support(f) = \sum_{p,f|dist(p,f)<\varepsilon} (1 - \frac{dist(p,f)}{\varepsilon}) \cdot conf(p)$$

Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
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Confidence weighted number of supporting point

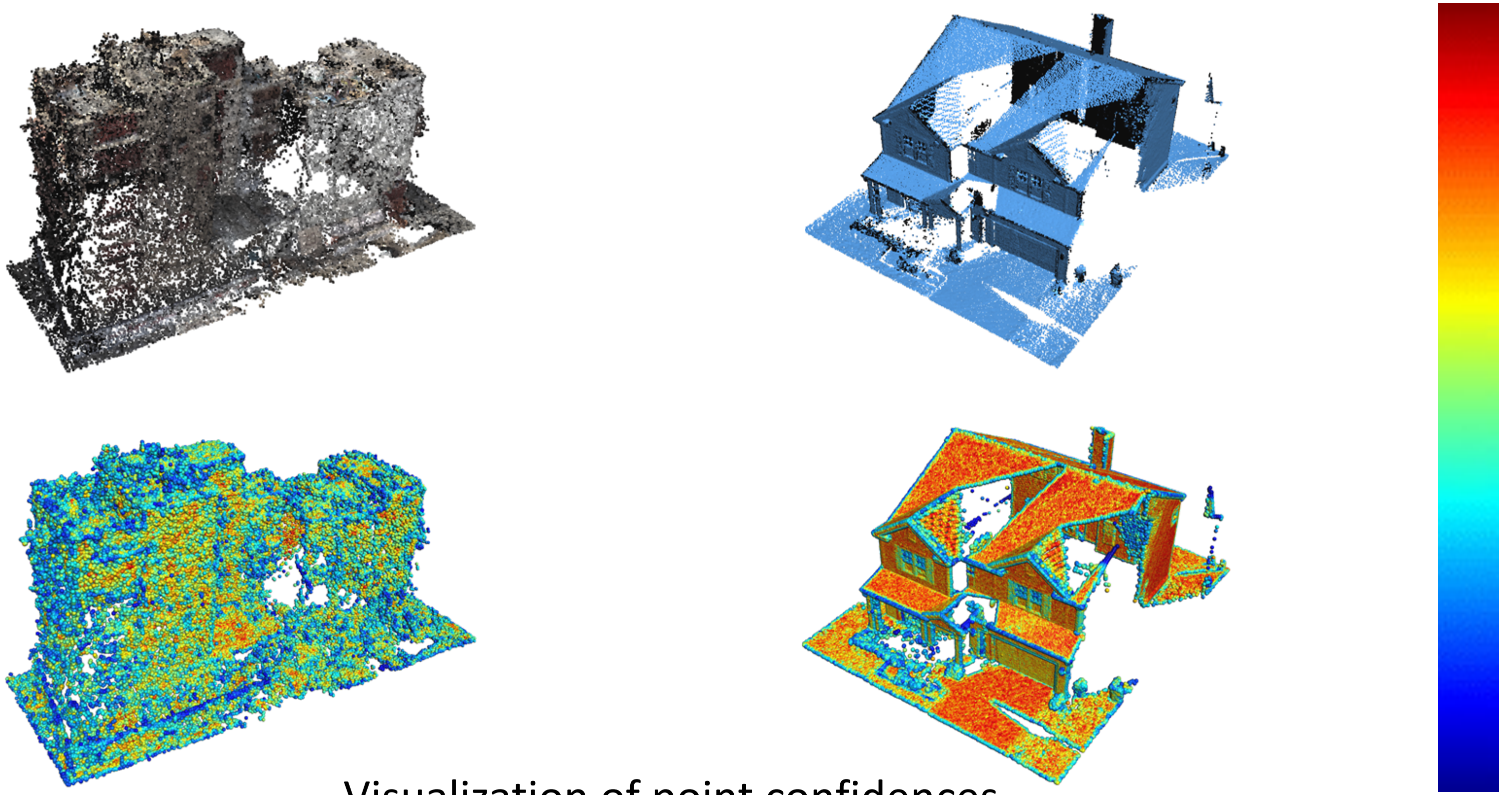
$$support(f) = \sum_{p,f|dist(p,f)<\varepsilon} (1 - \frac{dist(p,f)}{\varepsilon}) \cdot conf(p)$$

Point confidence

$$conf(p) = \frac{1}{3} \sum_{i=1}^{3} (1 - \frac{3\lambda_i^1}{\lambda_i^1 + \lambda_i^2 + \lambda_i^3}) \cdot \frac{\lambda_i^2}{\lambda_i^3} \qquad \lambda_i^1 \le \lambda_i^2 \le \lambda_i^3$$

Polygonal Surface Reconstruction

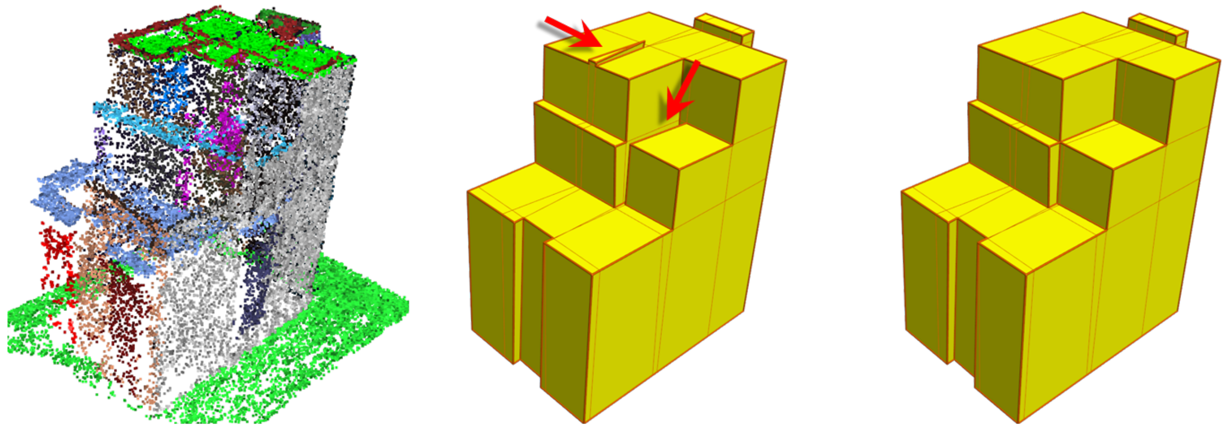
- Objective Function
  - Data fitting



Visualization of point confidences

Polygonal Surface Reconstruction

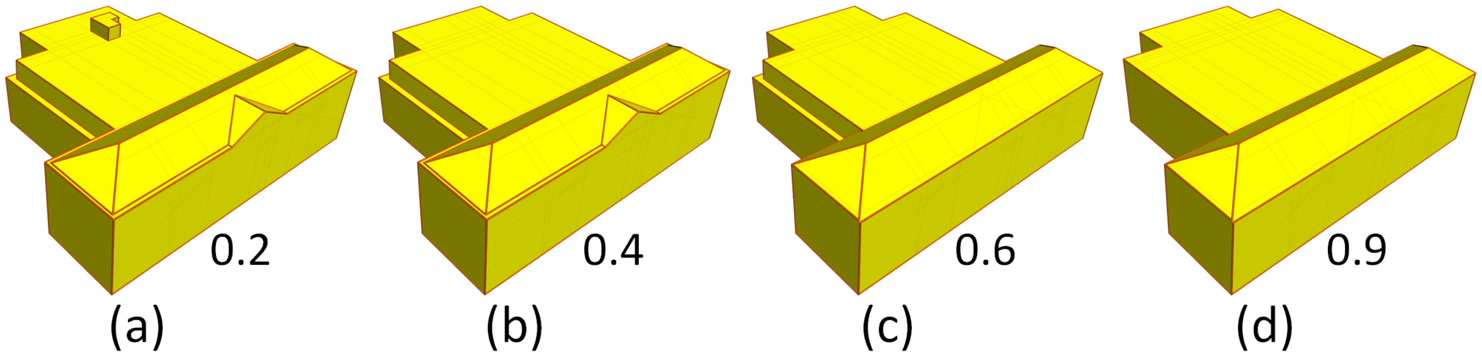
- Objective Function
  - Data fitting
  - Model complexity
    - Penalize sharp corners



(a) (b) (c)

Polygonal Surface Reconstruction

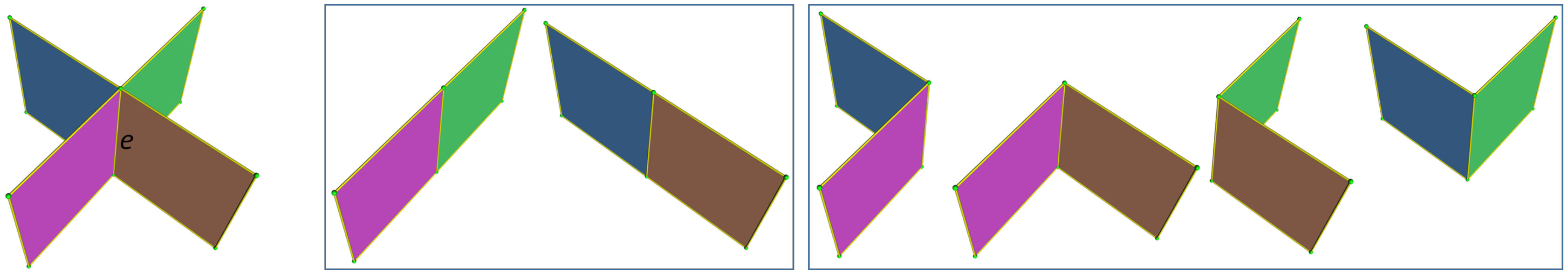
- Objective Function
  - Data fitting
  - Model complexity
    - Penalize sharp corners



Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
  - Model complexity
    - Penalize sharp corners

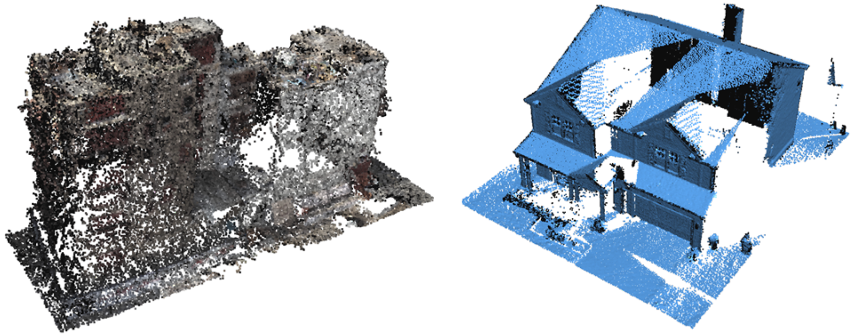
$$E_m = \frac{1}{|E|} \sum_{i=1}^{|E|} corner(e_i)$$

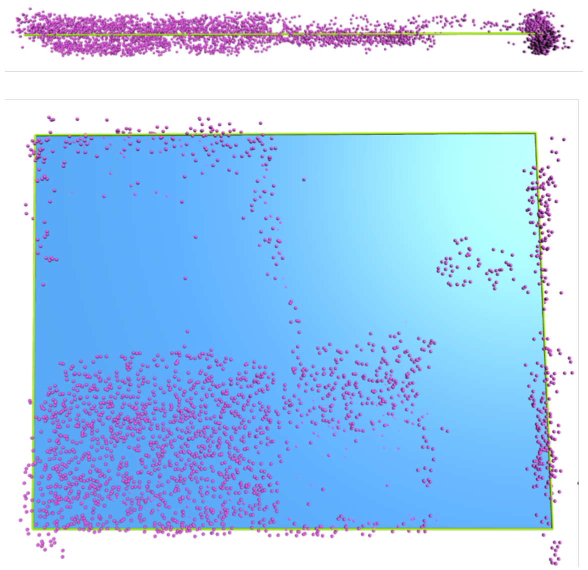


Intersecting two faces

Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
  - Model complexity
  - Point coverage

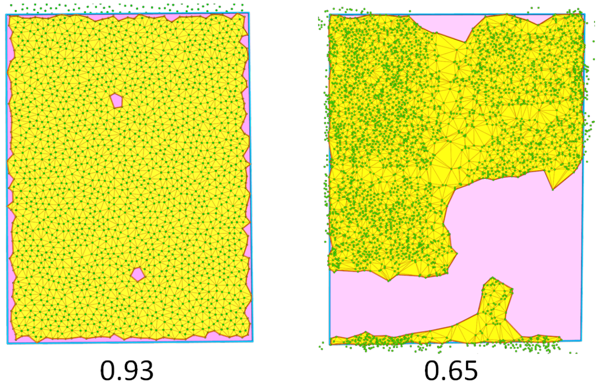




Polygonal Surface Reconstruction

- Objective Function
  - Data fitting
  - Model complexity
  - Point coverage

$$E_c = \frac{1}{area(M)} \sum_{i=1}^{N} x_i \cdot (area(f_i) - area(M_i^{\alpha})),$$



Polygonal Surface Reconstruction

- Face Selection
  - Linear integer program

$$\min_{\mathbf{x}} \quad \lambda_f \cdot \underline{E_f} + \lambda_m \cdot \underline{E_m} + \lambda_c \cdot \underline{E_c}$$

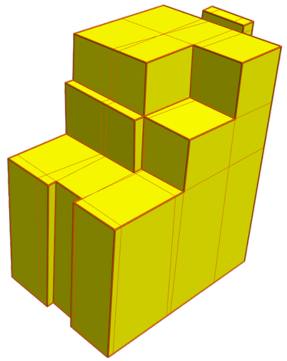
Data fitting

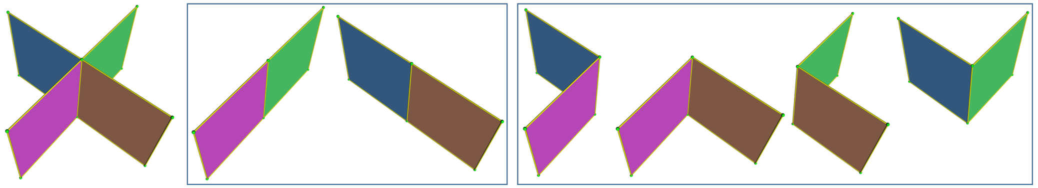
Model complexity

Point coverage

Polygonal Surface Reconstruction

- Face Selection
  - Linear integer program
    - Constraints
      - Watertight
      - Manifold





$$\min_{\mathbf{x}} \quad \lambda_f \cdot E_f + \lambda_m \cdot E_m + \lambda_c \cdot E_c$$

$$\text{s.t.} \quad \begin{cases} \sum_{j \in \mathcal{N}(e_i)} x_j = 2 \quad \text{or} \quad 0, & 1 \leq i \leq |E| \\ x_i \in \{0, 1\}, & 1 \leq i \leq N \end{cases}$$

Polygonal Surface Reconstruction

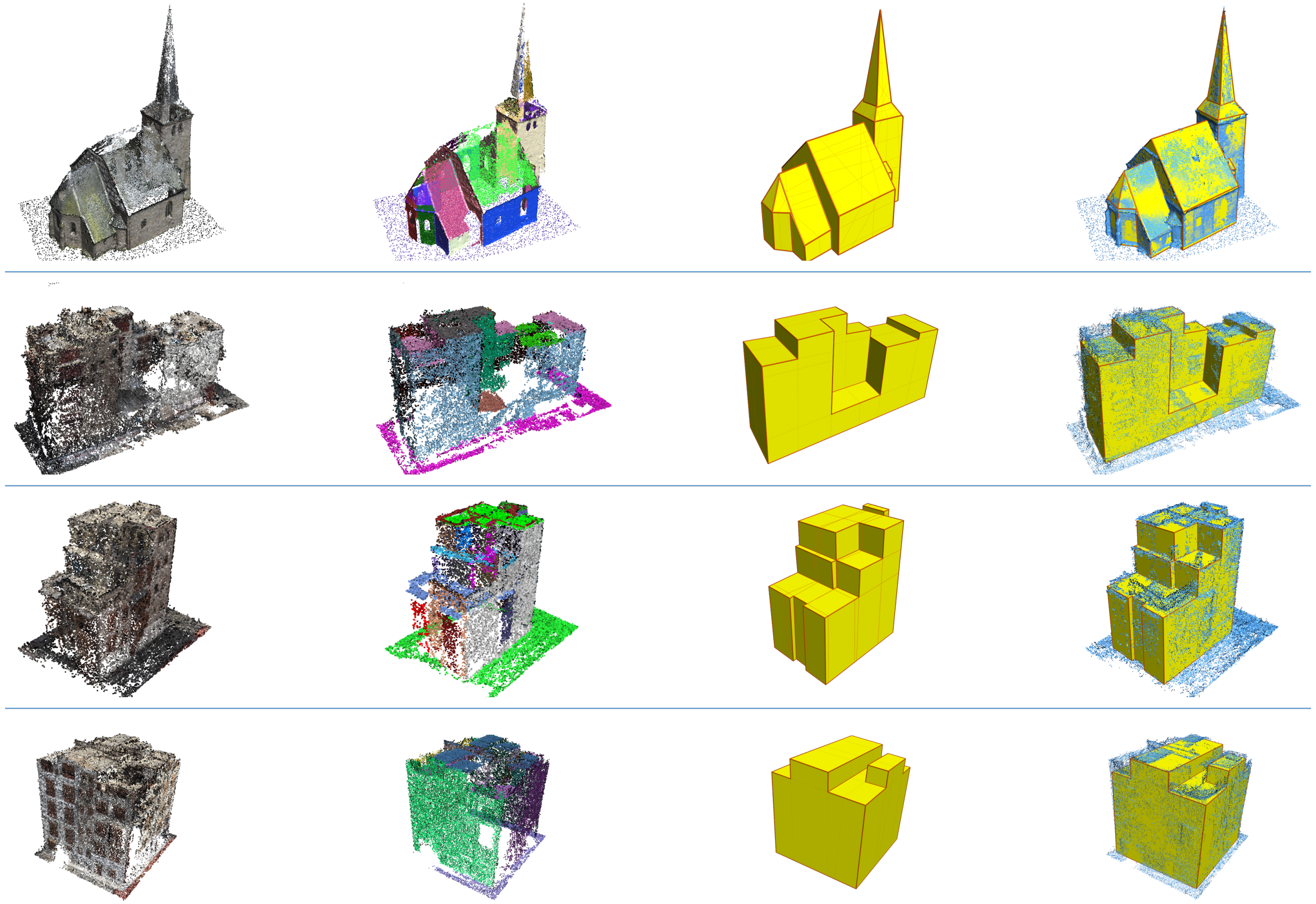
- Face Selection
  - Linear integer program
    - Constraints
    - Solvers (SCIP, CBC, GLPK, Gurobi…)

$$\min_{\mathbf{x}} \quad \lambda_f \cdot E_f + \lambda_m \cdot E_m + \lambda_c \cdot E_c$$

$$\text{s.t.} \quad \begin{cases} \sum_{j \in \mathcal{N}(e_i)} x_j = 2 \quad \text{or} \quad 0, & 1 \leq i \leq |E| \\ x_i \in \{0, 1\}, & 1 \leq i \leq N \end{cases}$$

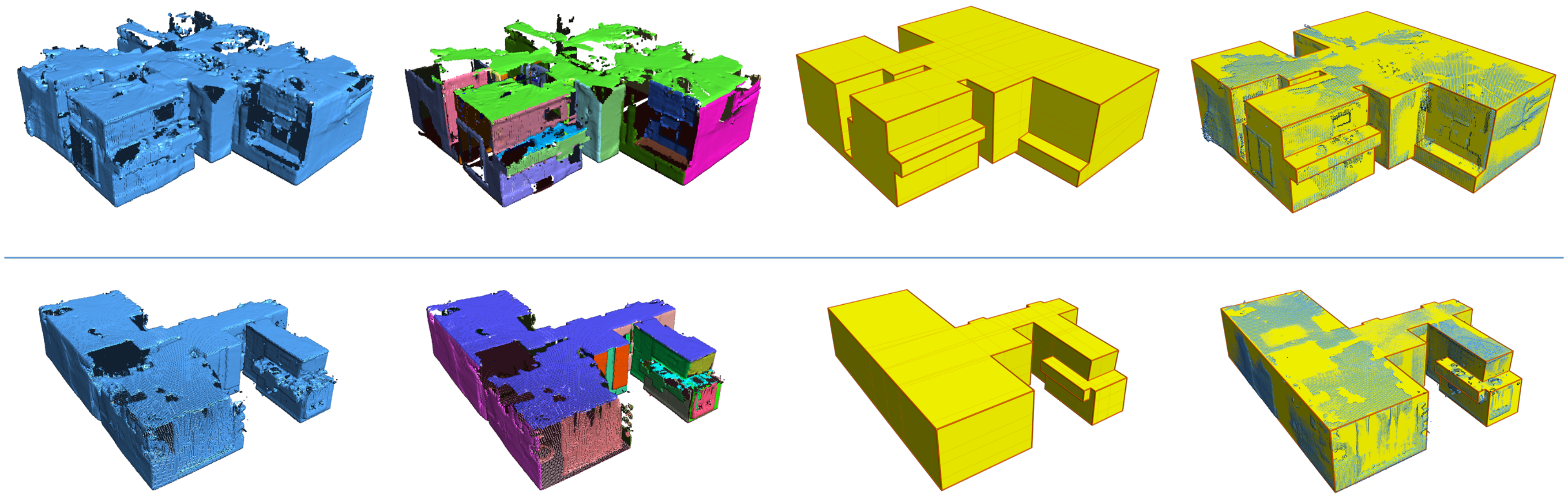
Polygonal Surface Reconstruction

- Reconstruction Results



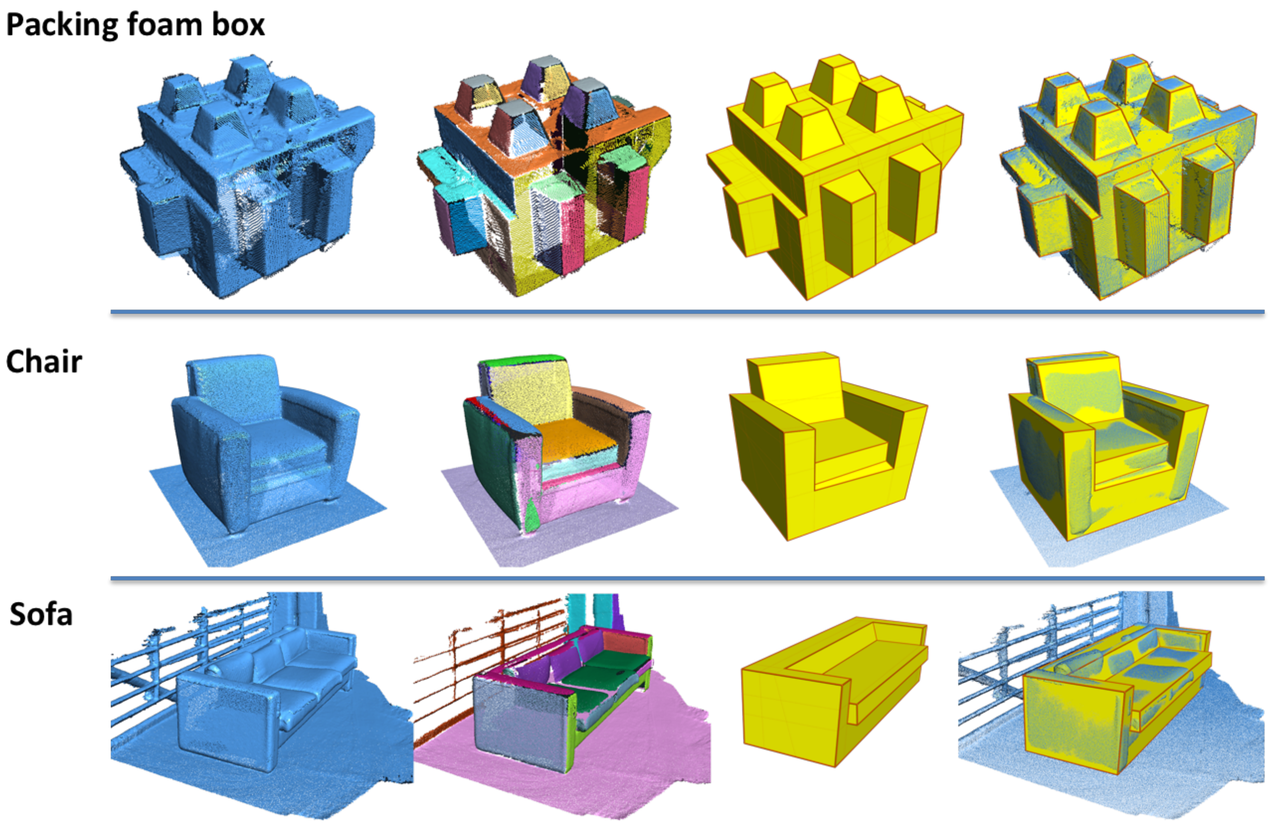
Polygonal Surface Reconstruction

- Reconstruction Results



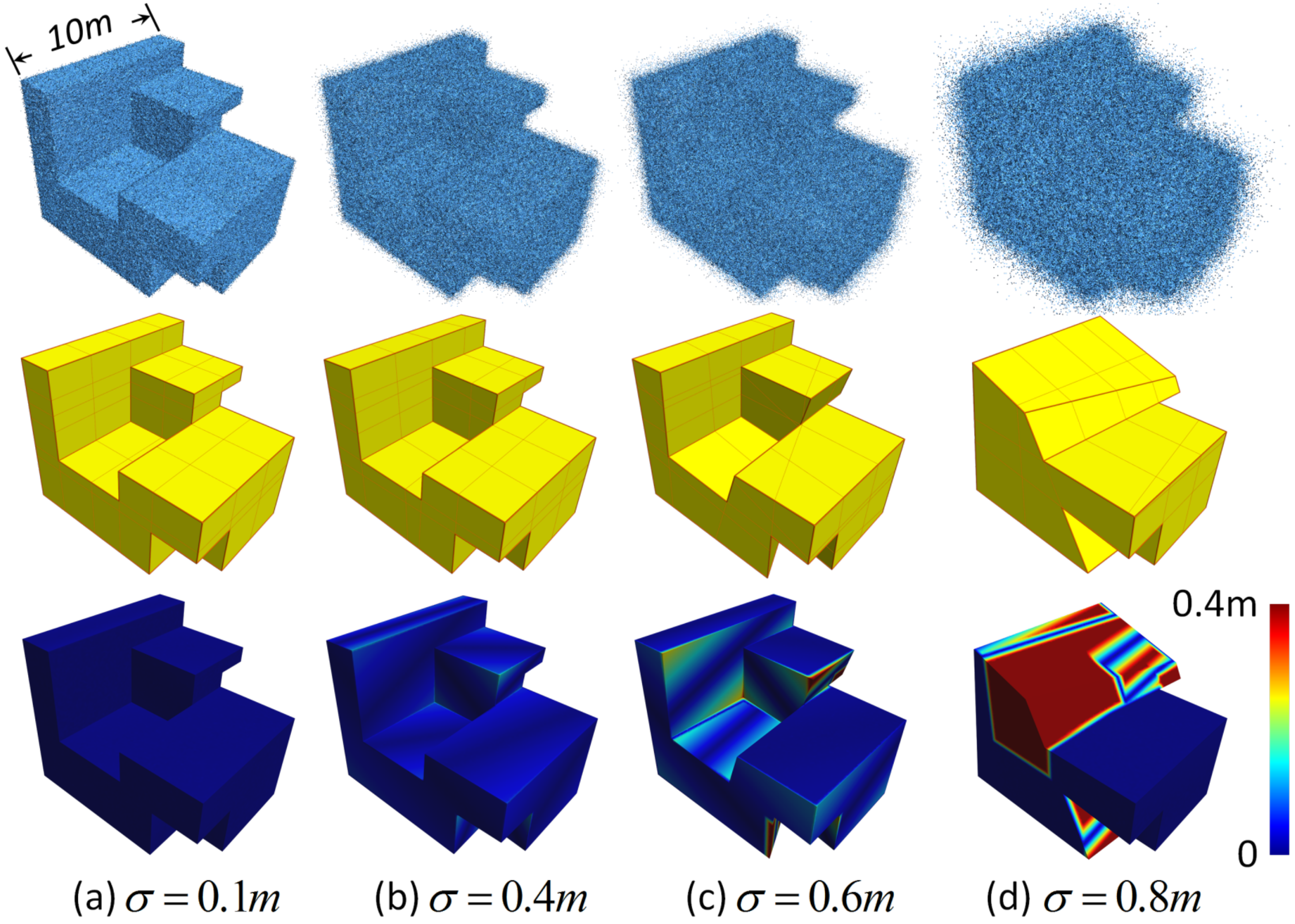
Piecewise Planar Reconstruction

- Reconstruction Results



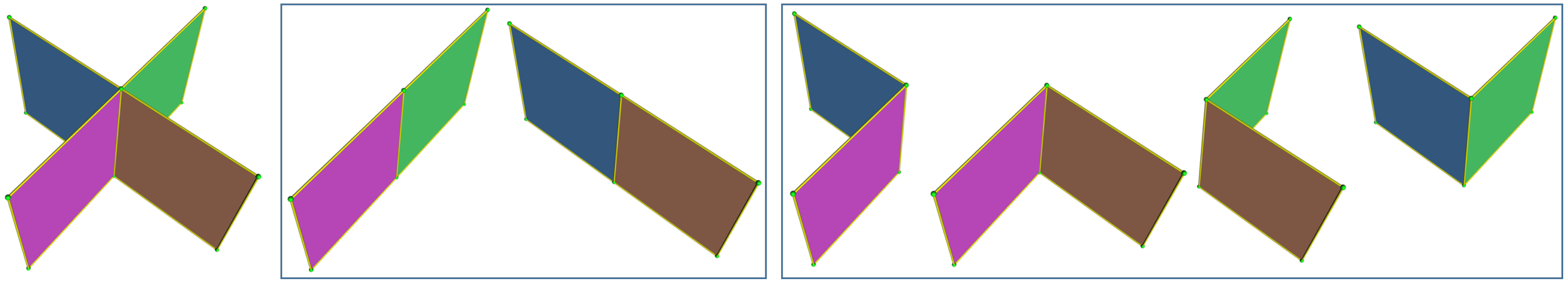
Piecewise Planar Reconstruction

- Robustness to noise



Limitations

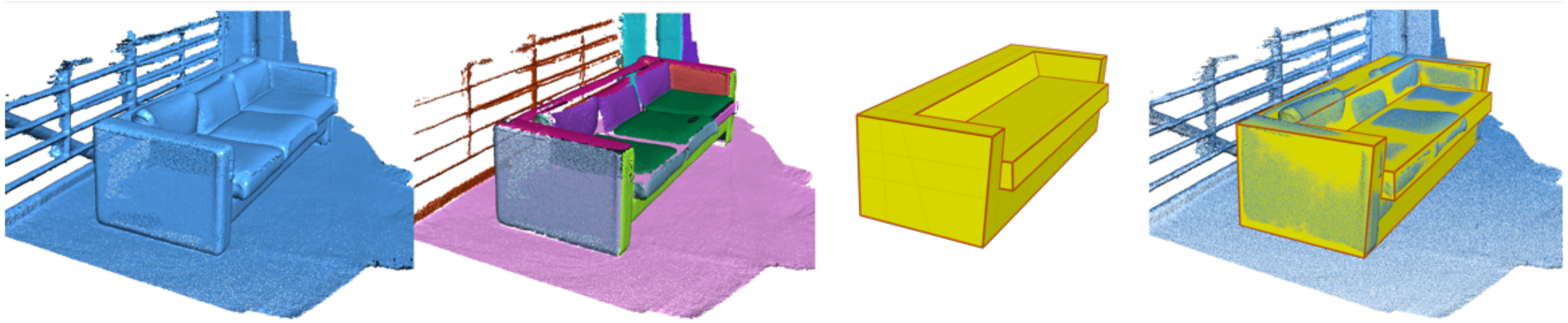
- Open surfaces



$$\text{s.t.} \quad \begin{cases} \sum_{j \in \mathcal{N}(e_i)} x_j = 2 \quad \text{or} \quad 0, & 1 \leq i \leq |E| \\ x_i \in \{0, 1\}, & 1 \leq i \leq N \end{cases}$$

Limitations

- Open surfaces



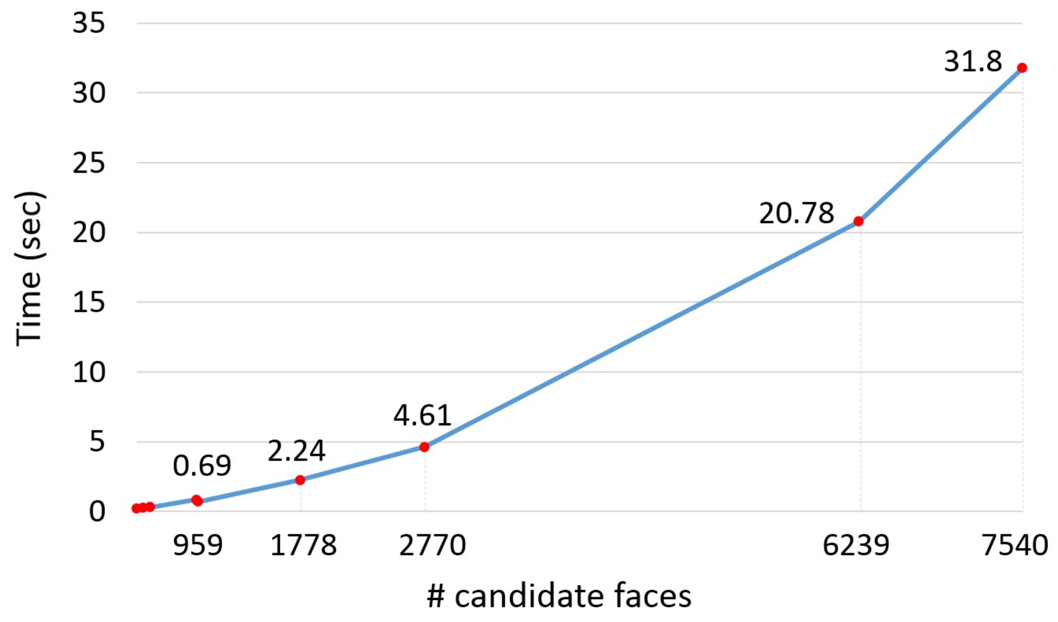
$$
\text{s.t.} \quad \begin{cases} \sum_{j \in \mathcal{N}(e_i)} x_j = 2 \quad \text{or} \quad 0, & 1 \leq i \leq |E| \\ x_i \in \{0, 1\}, & 1 \leq i \leq N \end{cases}
$$

Limitations

- Open surfaces
- Finer surface details
  - Fence
  - Façade decorations
  - Door handle
  - …

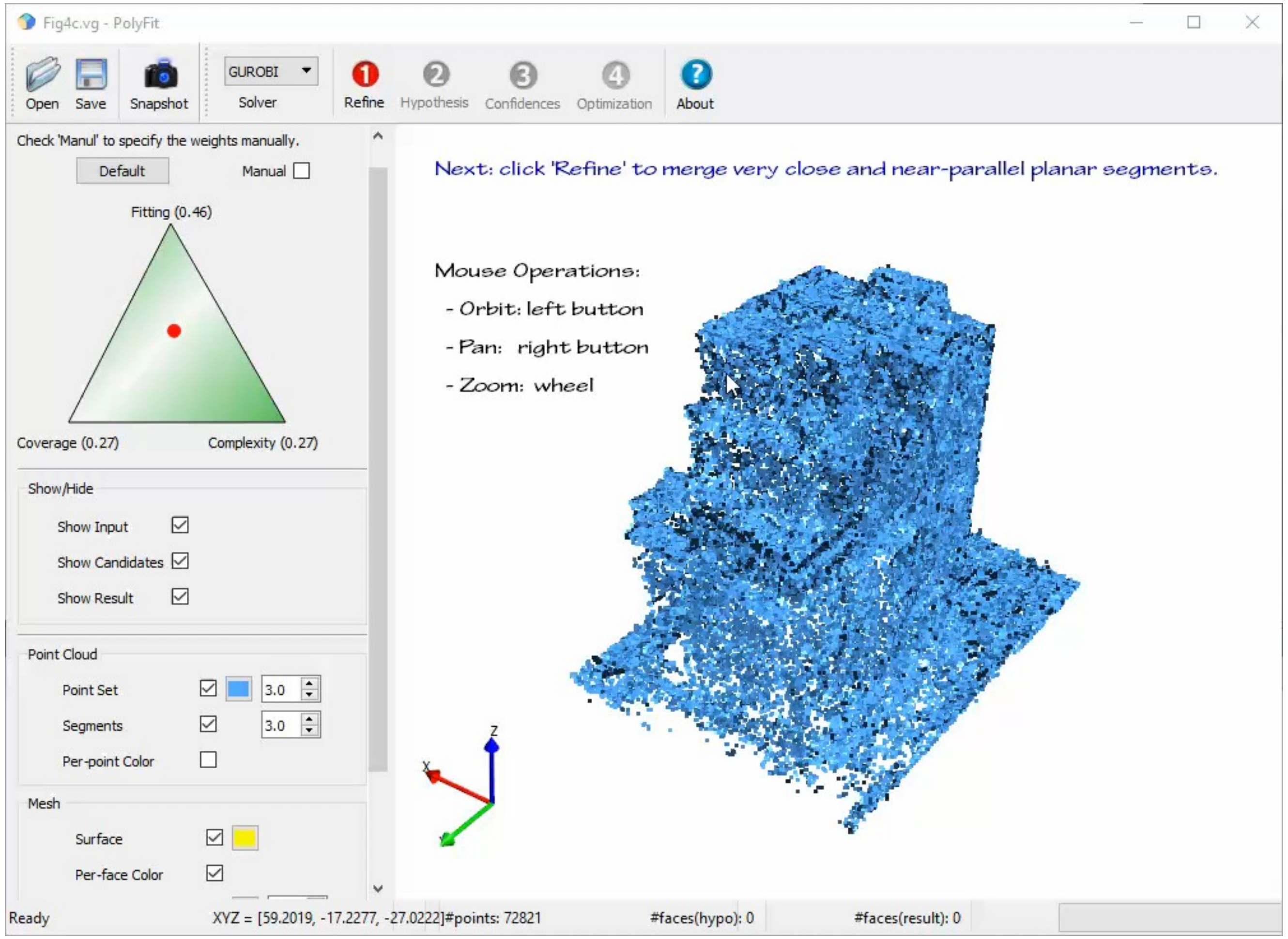
Limitations

- Open surfaces
- Finer surface details
- Complexity of the algorithm



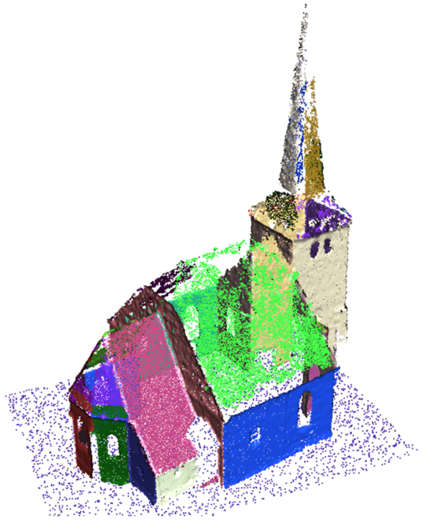
Polygonal Surface Reconstruction

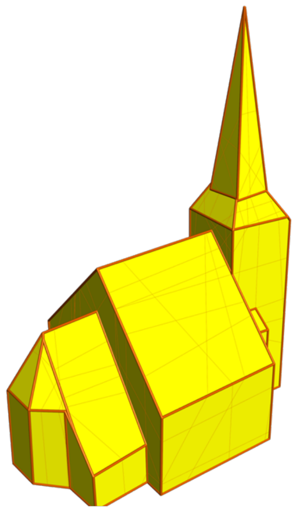
- Demo



Source Code (in C++) https://github.com/LiangliangNan/PolyFit

Questions?





Next Lecture

- Machine learning for point cloud segmentation/classification