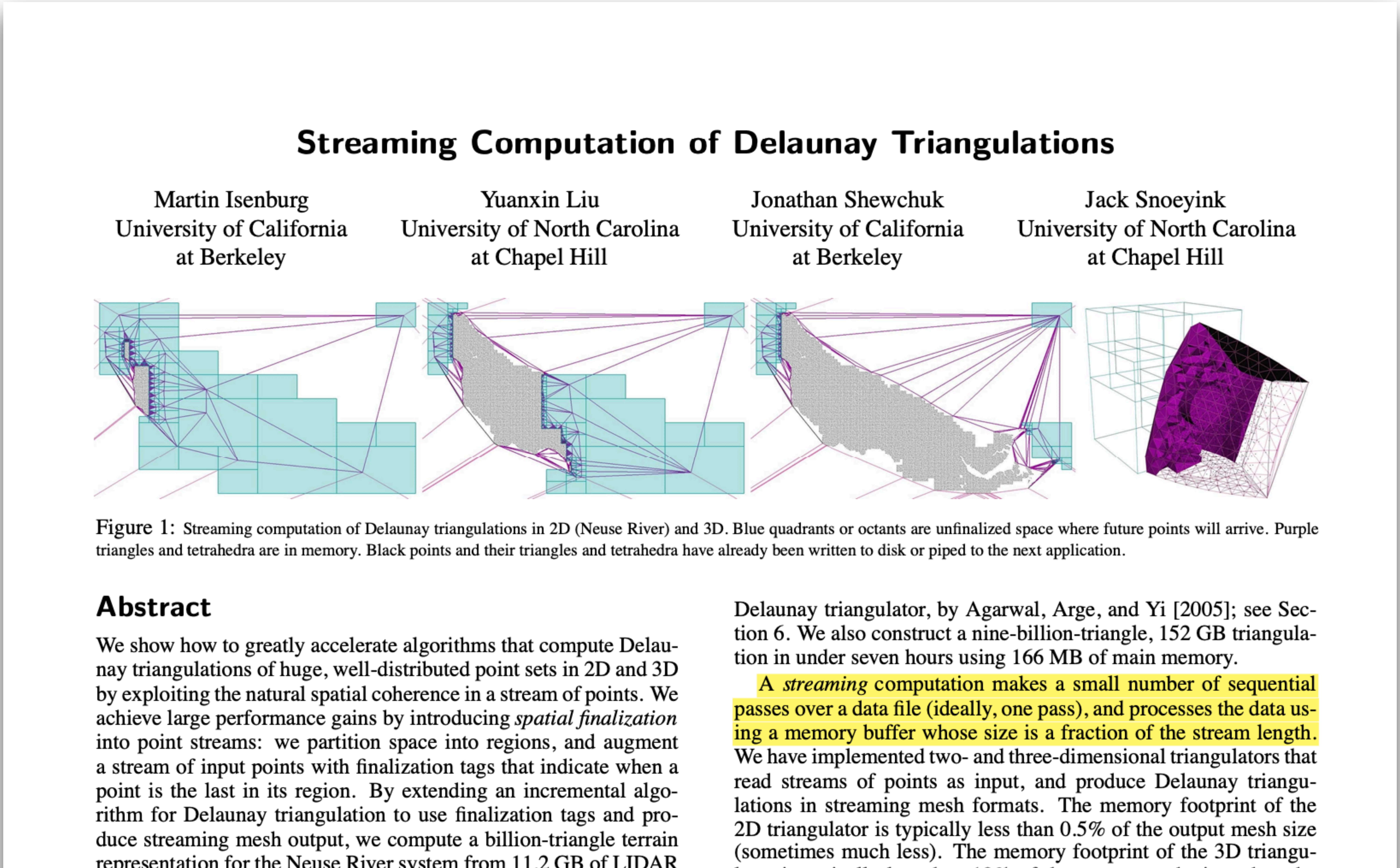
What I describe today is basically this paper



Streaming Computation of Delaunay Triangulations

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University of North Carolina
at Chapel Hill

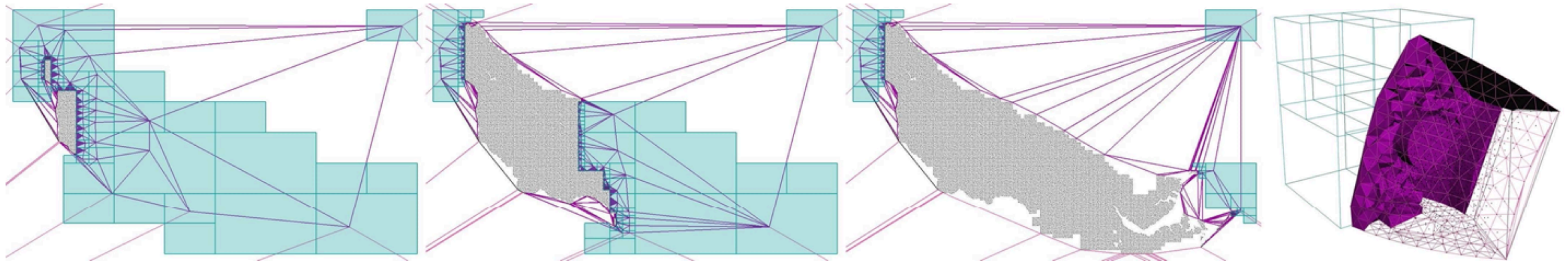


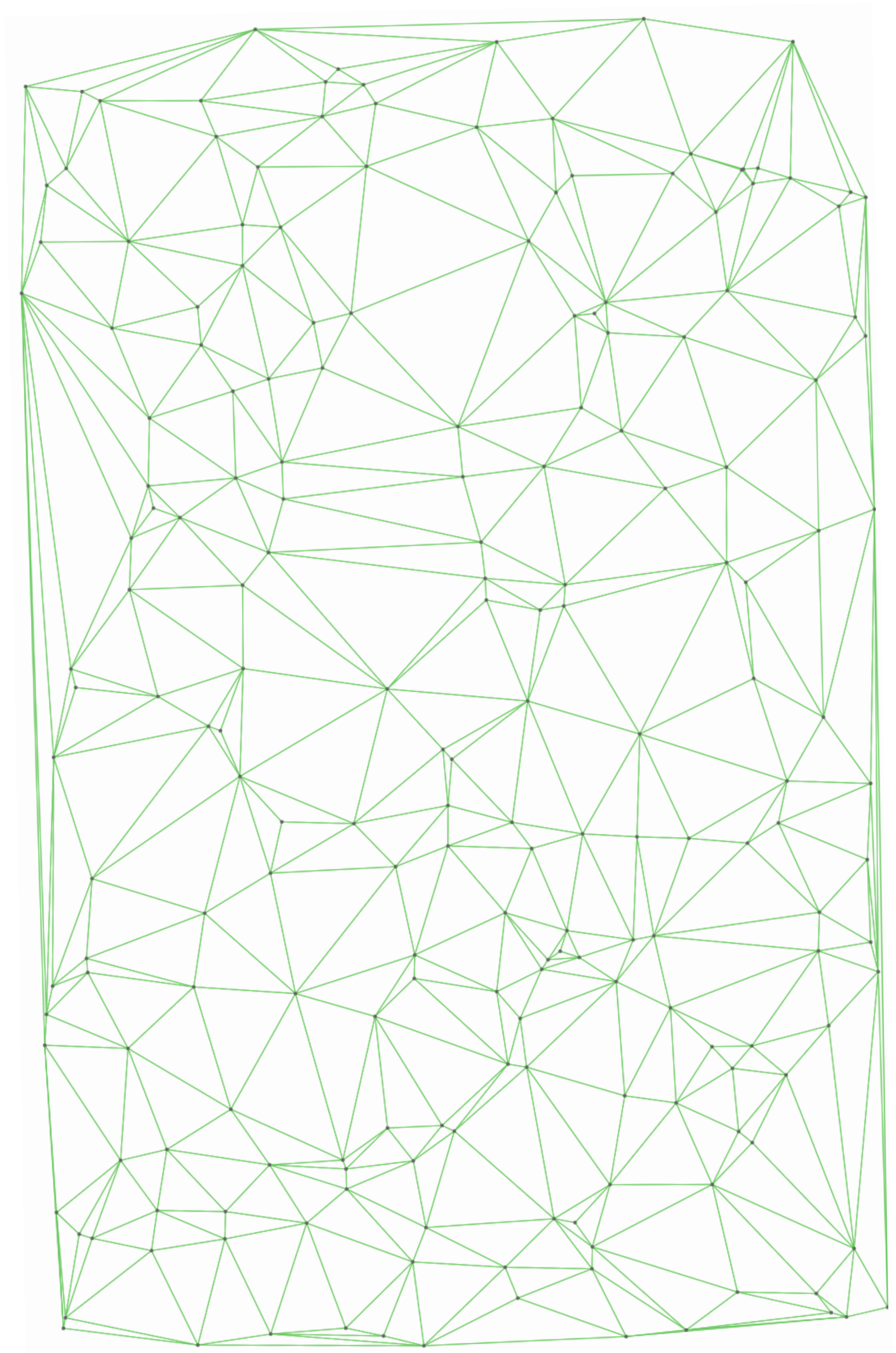
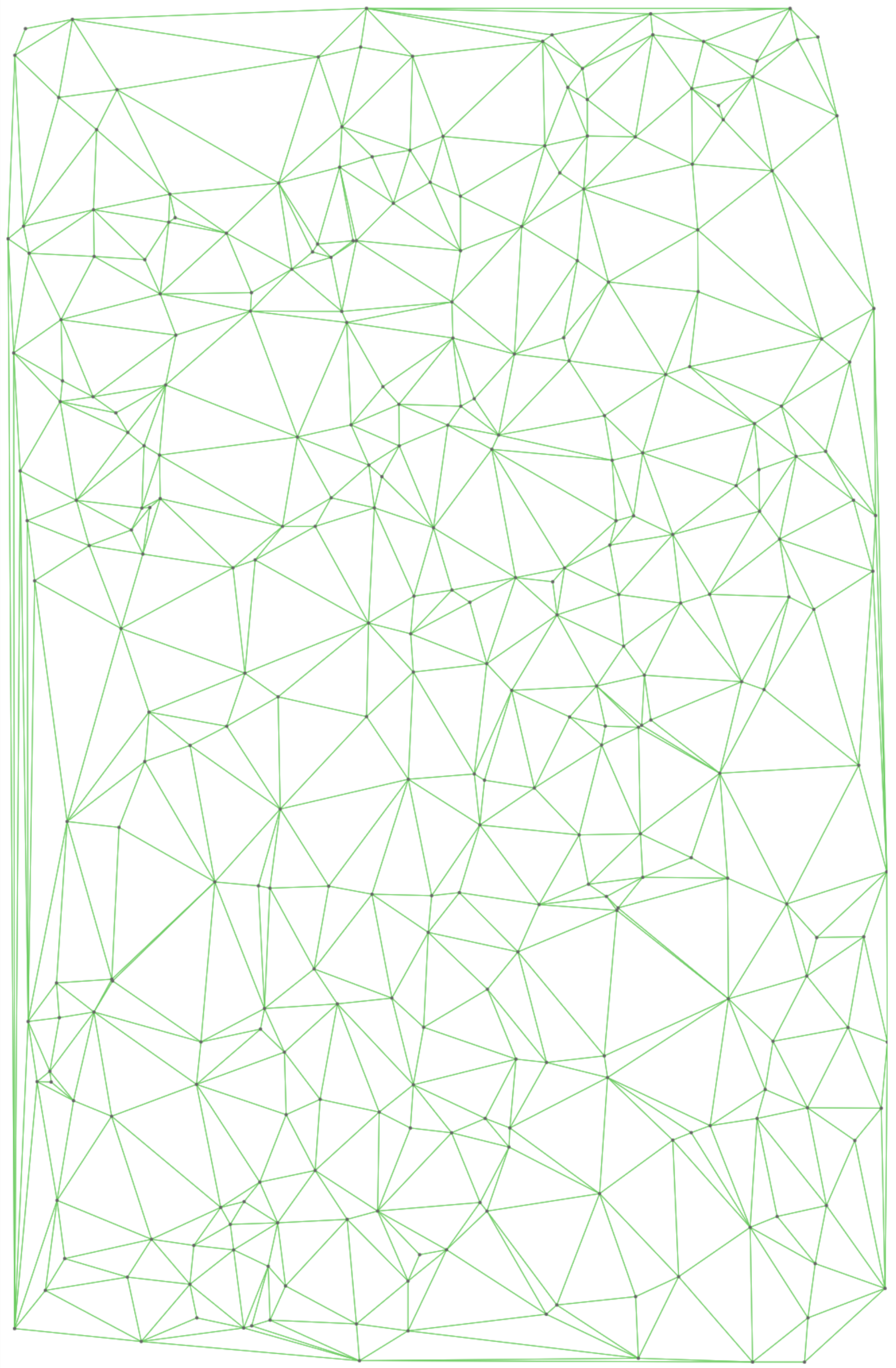
Figure 1: Streaming computation of Delaunay triangulations in 2D (Neuse River) and 3D. Blue quadrants or octants are unfinalized space where future points will arrive. Purple triangles and tetrahedra are in memory. Black points and their triangles and tetrahedra have already been written to disk or piped to the next application.

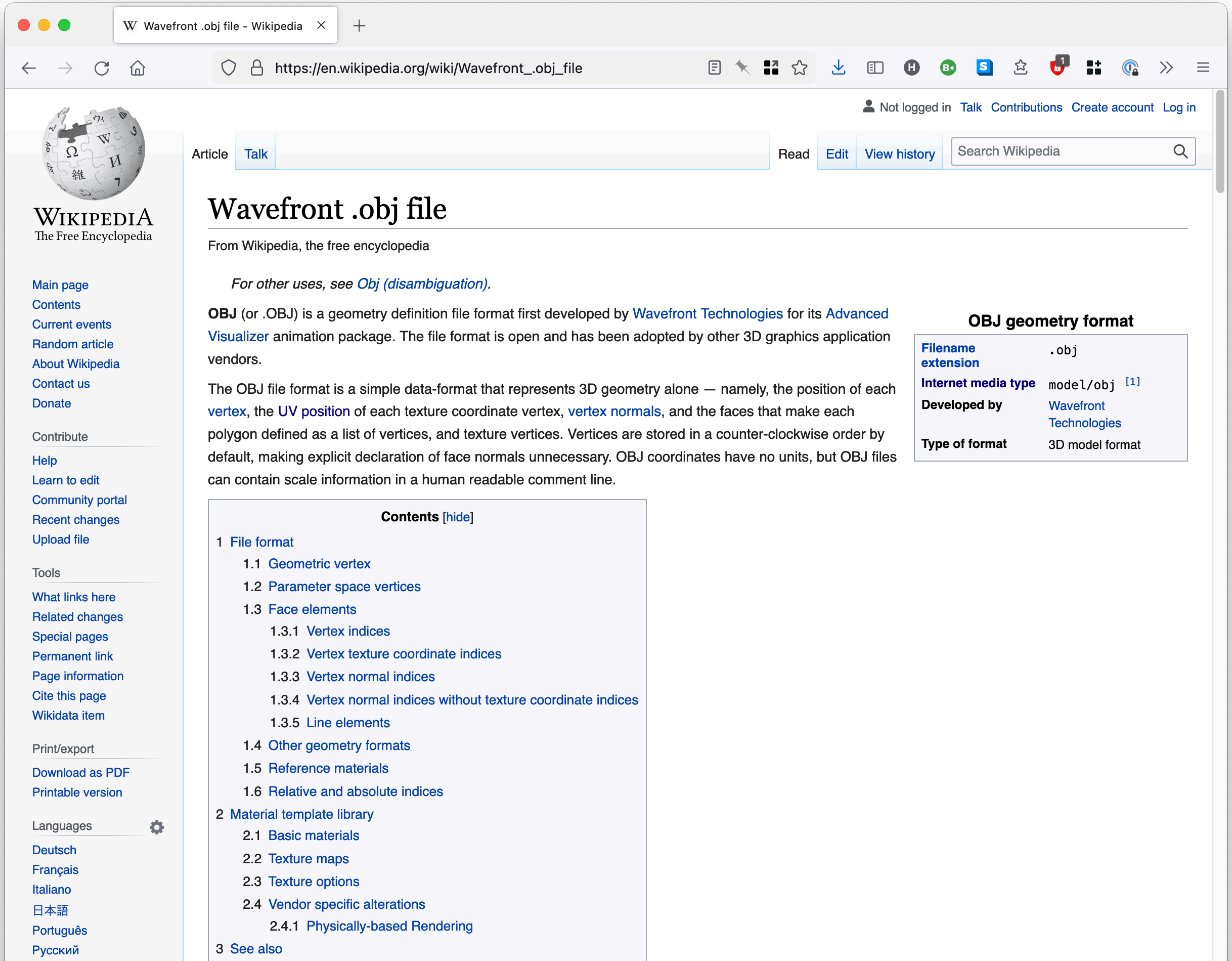
Abstract

We show how to greatly accelerate algorithms that compute Delaunay triangulations of huge, well-distributed point sets in 2D and 3D by exploiting the natural spatial coherence in a stream of points. We achieve large performance gains by introducing *spatial finalization* into point streams: we partition space into regions, and augment a stream of input points with finalization tags that indicate when a point is the last in its region. By extending an incremental algorithm for Delaunay triangulation to use finalization tags and produce streaming mesh output, we compute a billion-triangle terrain representation for the Neuse River system from 11.2 GB of LIDAR

Delaunay triangulator, by Agarwal, Arge, and Yi [2005]; see Section 6. We also construct a nine-billion-triangle, 152 GB triangulation in under seven hours using 166 MB of main memory.

A *streaming* computation makes a small number of sequential passes over a data file (ideally, one pass), and processes the data using a memory buffer whose size is a fraction of the stream length. We have implemented two- and three-dimensional triangulators that read streams of points as input, and produce Delaunay triangulations in streaming mesh formats. The memory footprint of the 2D triangulator is typically less than 0.5% of the output mesh size (sometimes much less). The memory footprint of the 3D triangu-


Wavefront .obj file - Wikipedia

https://en.wikipedia.org/wiki/Wavefront_.obj_file

Wavefront .obj file

From Wikipedia, the free encyclopedia

For other uses, see Obj (disambiguation).

OBJ (or .OBJ) is a geometry definition file format first developed by Wavefront Technologies for its Advanced Visualizer animation package. The file format is open and has been adopted by other 3D graphics application vendors.

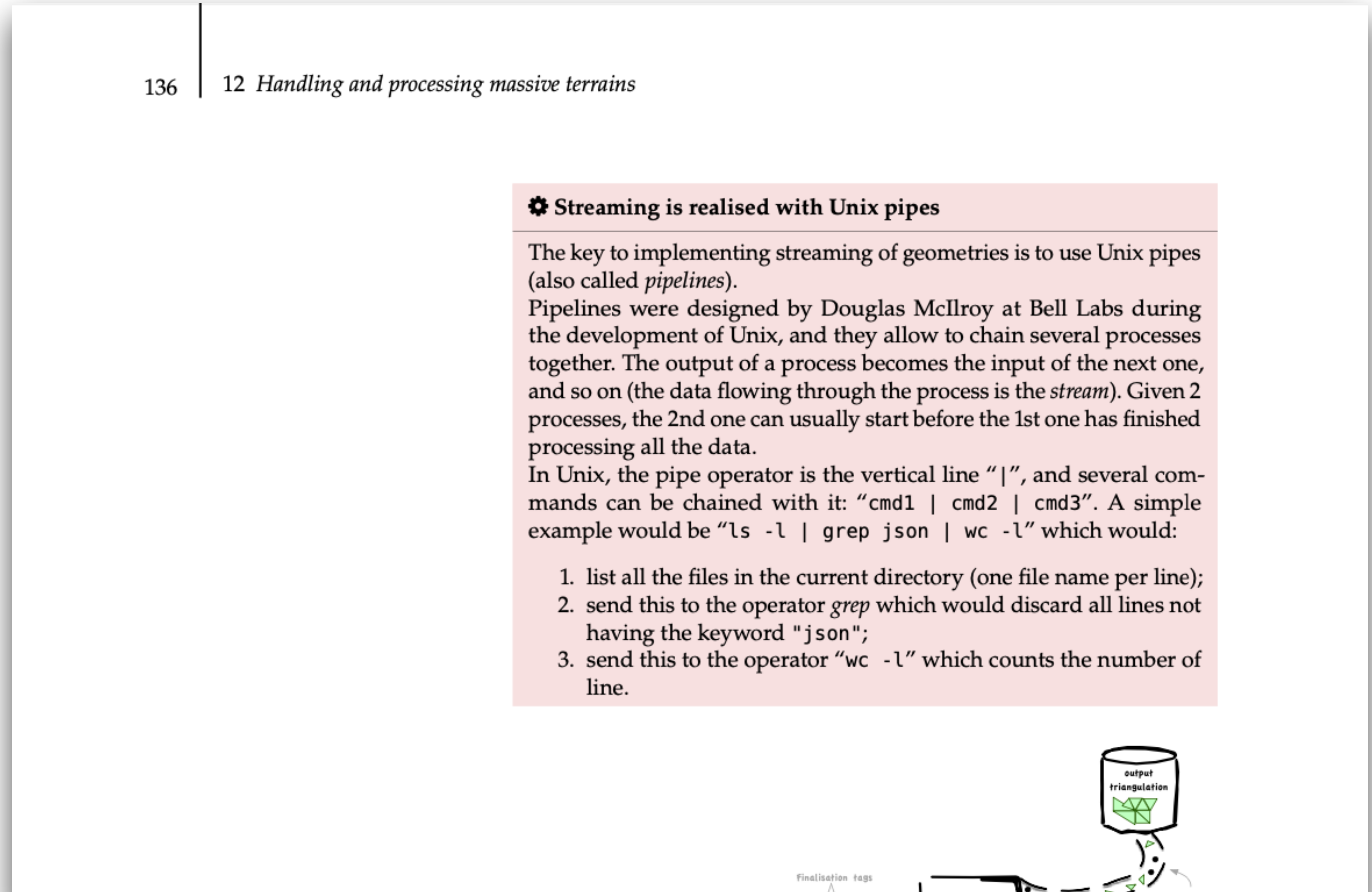
The OBJ file format is a simple data-format that represents 3D geometry alone — namely, the position of each vertex, the UV position of each texture coordinate vertex, vertex normals, and the faces that make each polygon defined as a list of vertices, and texture vertices. Vertices are stored in a counter-clockwise order by default, making explicit declaration of face normals unnecessary. OBJ coordinates have no units, but OBJ files can contain scale information in a human readable comment line.

OBJ geometry format

Filename extension	`.obj`
Internet media type	`model/obj` [1]
Developed by	Wavefront Technologies
Type of format	3D model format

Contents [hide]

Unix pipes for streaming



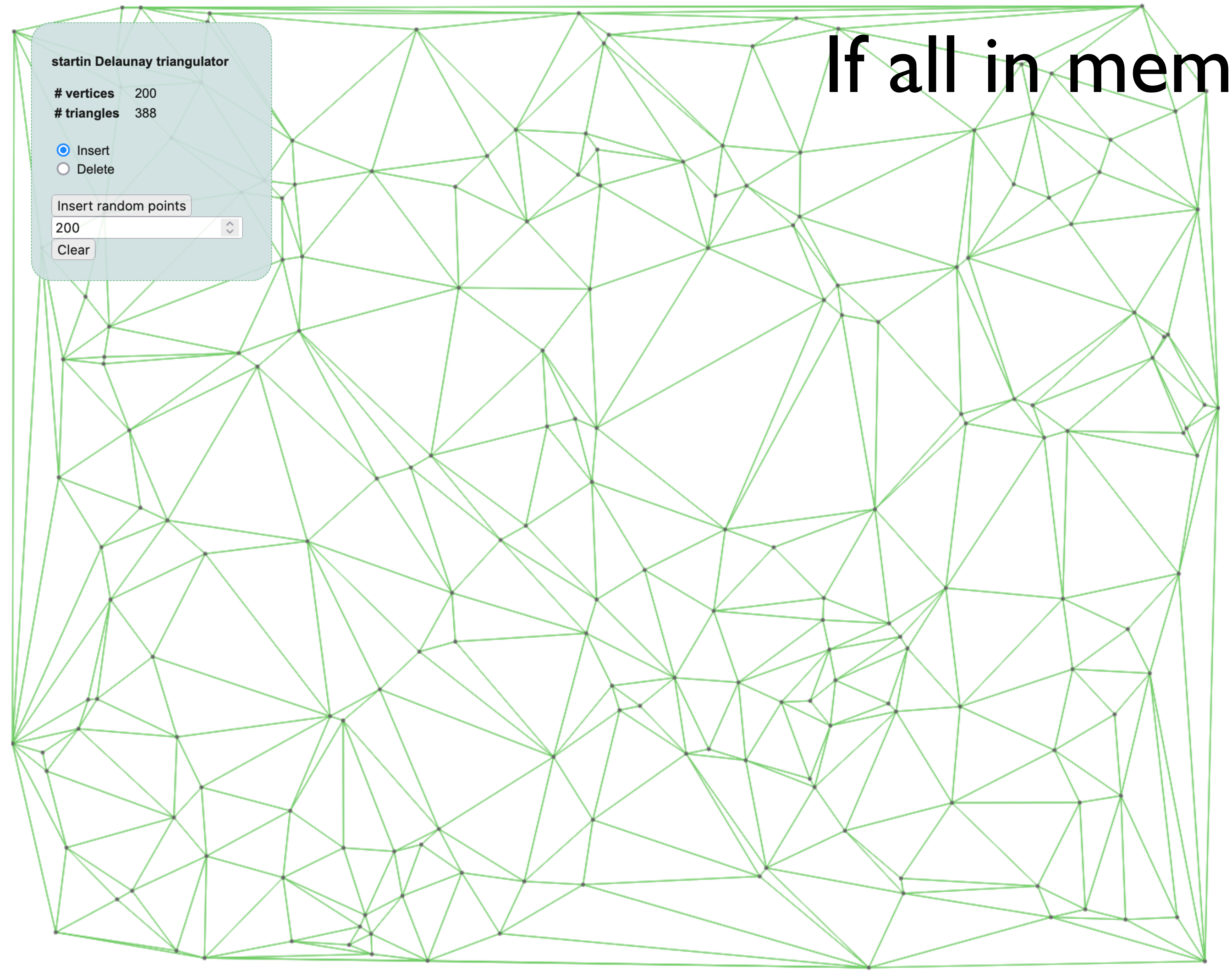
136 | 12 *Handling and processing massive terrains*

Streaming is realised with Unix pipes

The key to implementing streaming of geometries is to use Unix pipes (also called *pipelines*).
Pipelines were designed by Douglas McIlroy at Bell Labs during the development of Unix, and they allow to chain several processes together. The output of a process becomes the input of the next one, and so on (the data flowing through the process is the *stream*). Given 2 processes, the 2nd one can usually start before the 1st one has finished processing all the data.
In Unix, the pipe operator is the vertical line "`|`", and several commands can be chained with it: "`cmd1 | cmd2 | cmd3`". A simple example would be "`ls -l | grep json | wc -l`" which would:

1. list all the files in the current directory (one file name per line);
2. send this to the operator *grep* which would discard all lines not having the keyword "`json`";
3. send this to the operator "`wc -l`" which counts the number of line.

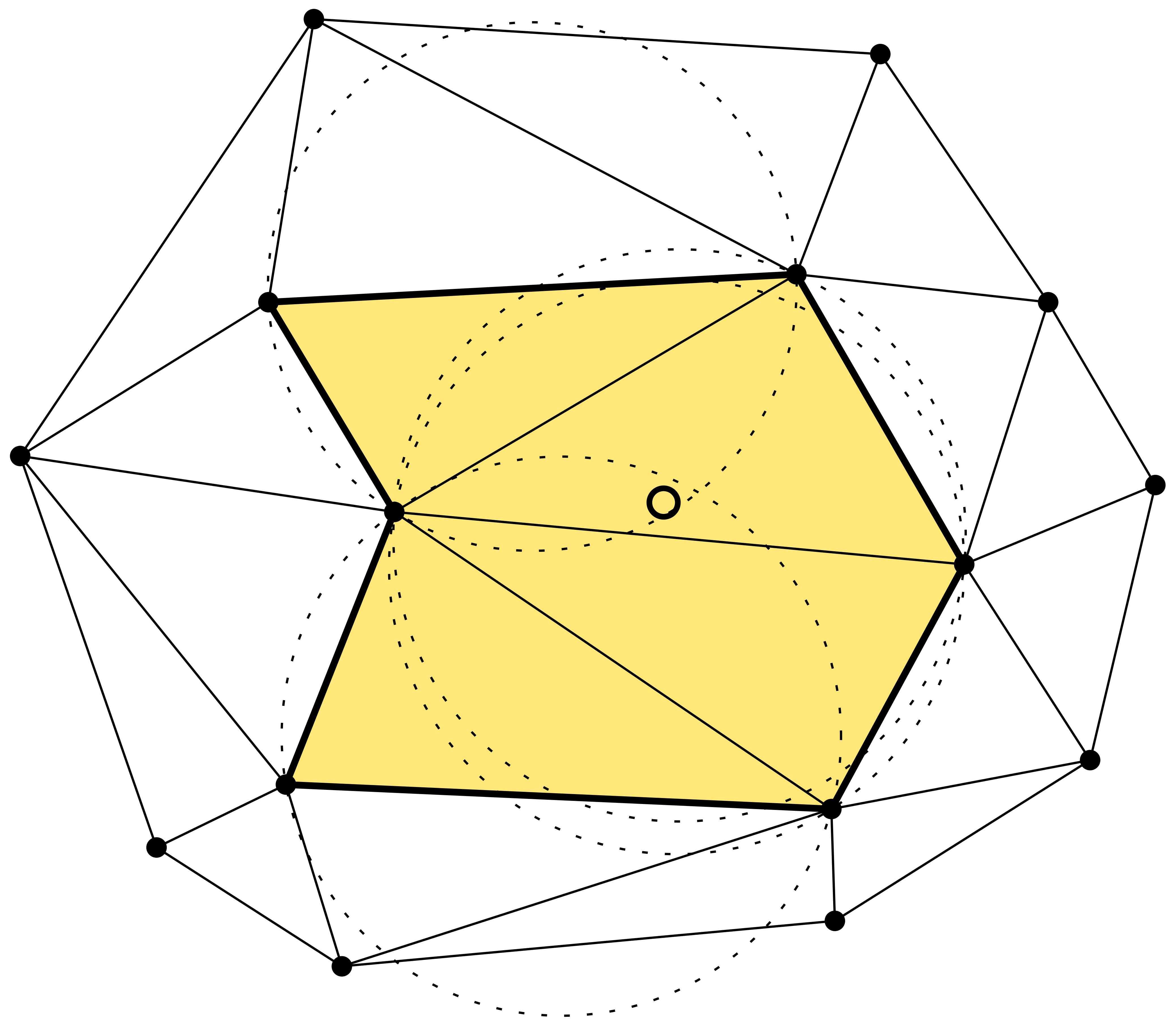
If all in memory it's "easy"

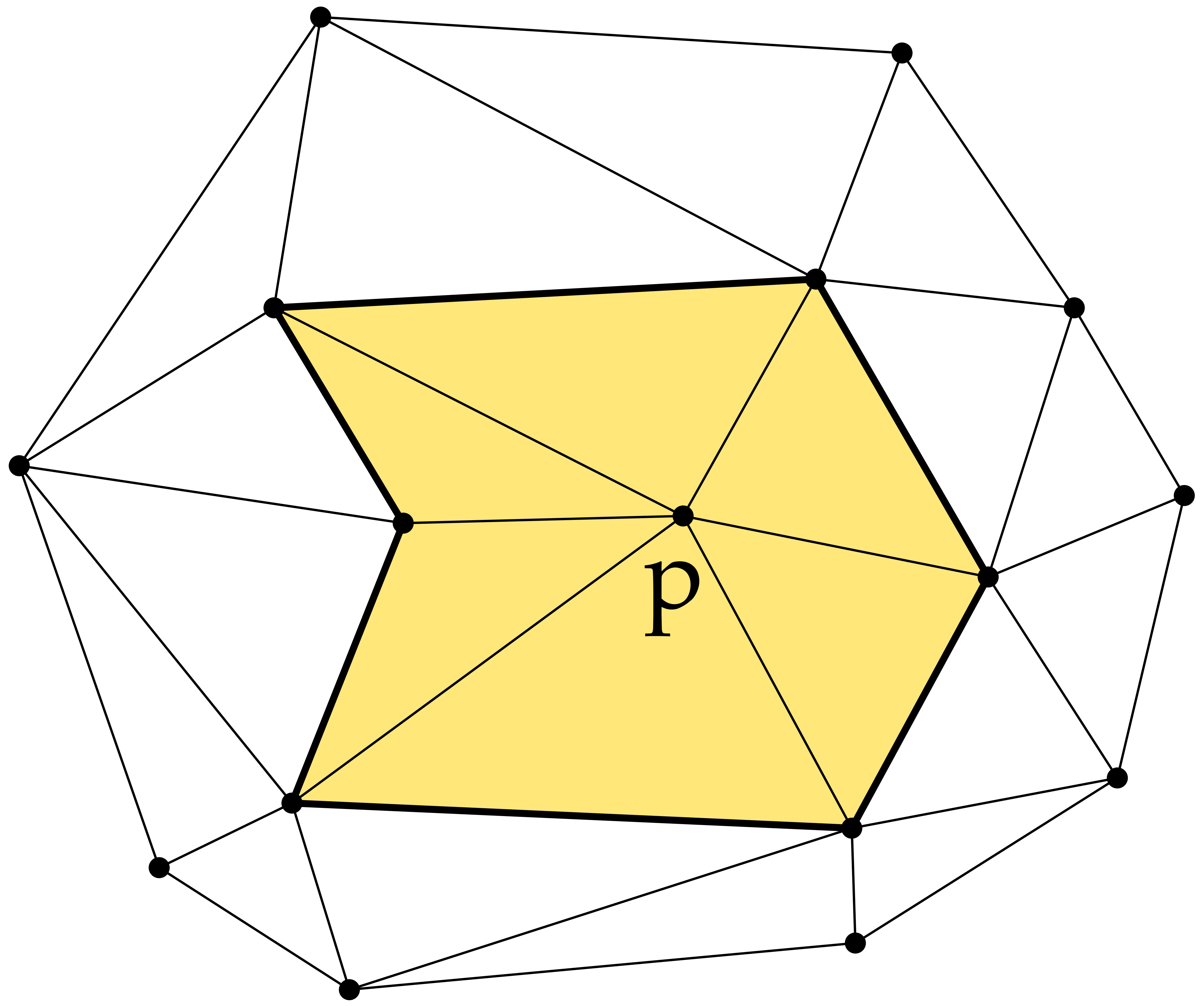

startin Delaunay triangulator

# vertices	200
# triangles	388

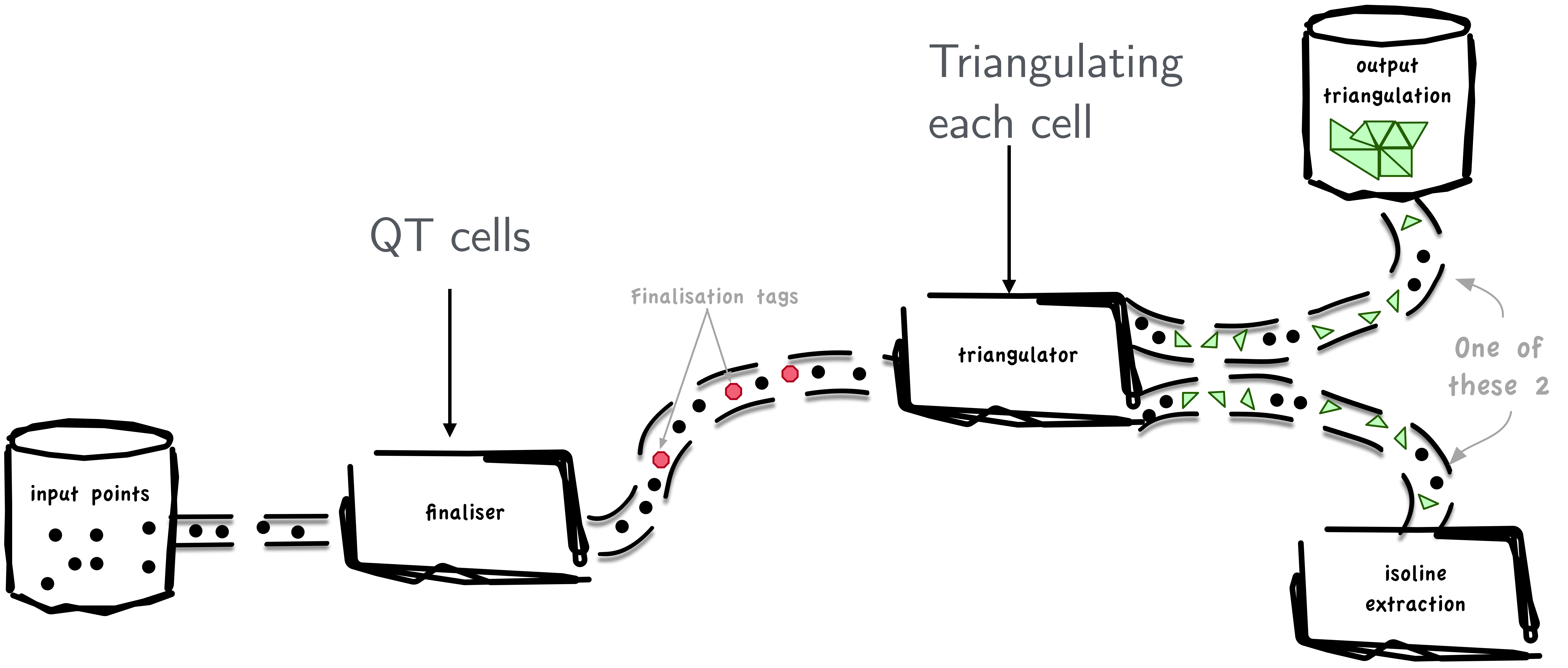
[✓ Insert] Delete

Insertion deletes all the triangles whose circumcircle contains p

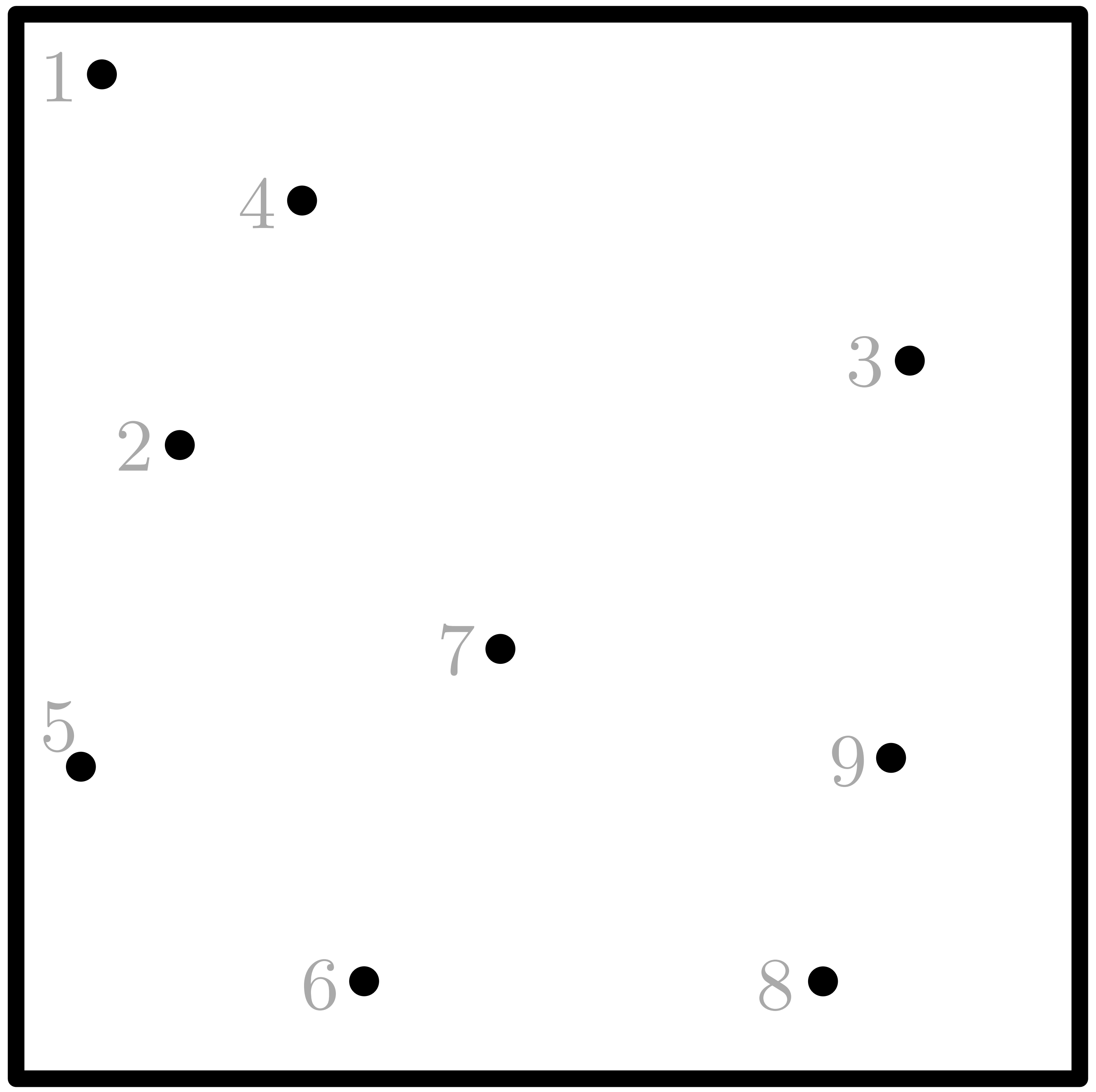




Streaming DT architecture

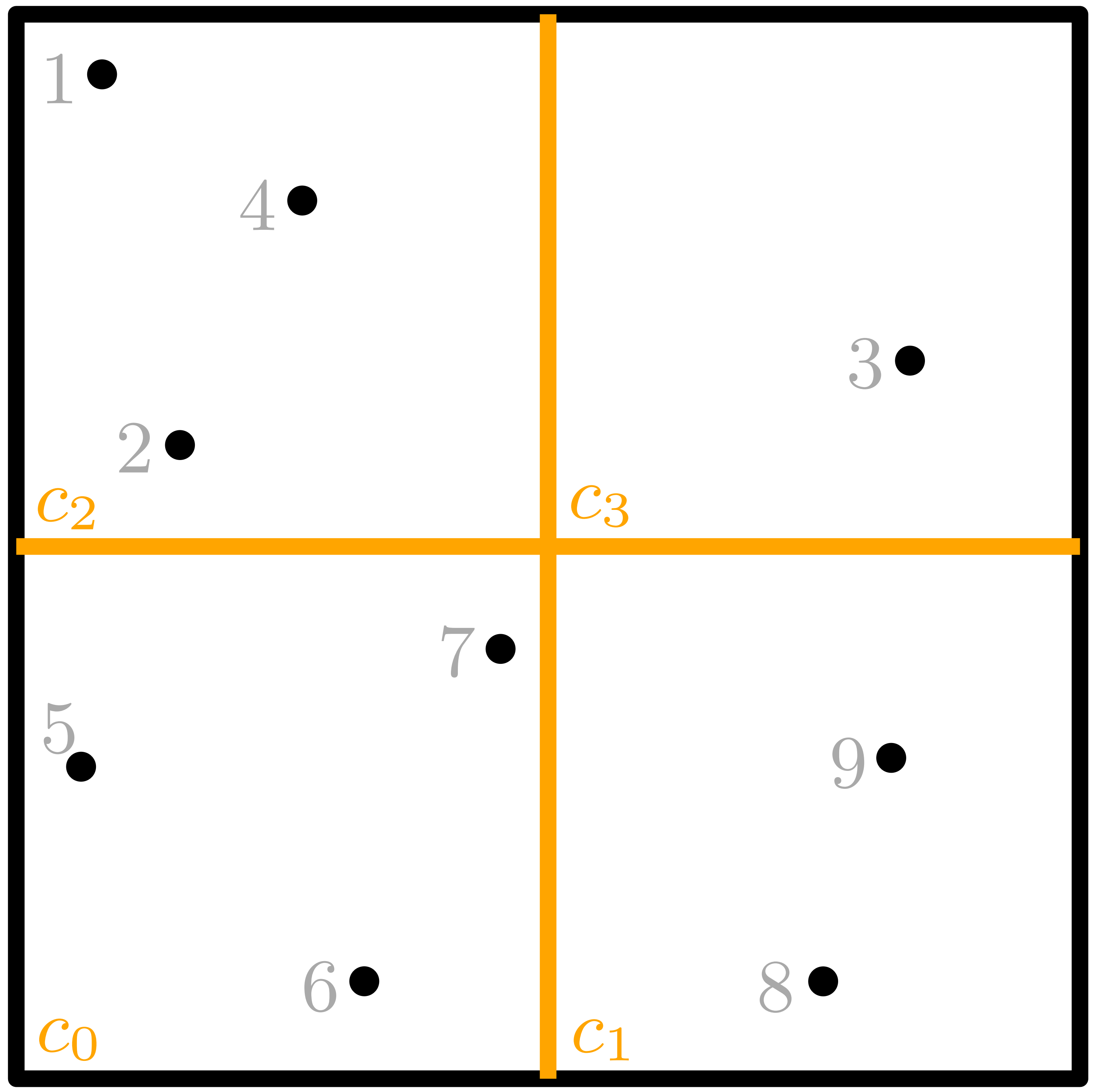


Finaliser based on quadtree



1. x y z
2. x y z
3. x y z
4. x y z
5. x y z
6. x y z
7. x y z
8. x y z
9. x y z

9 points and a ”normal” stream



1. x y z
2. x y z
3. x y z

finalise c_3

4. x y z

finalise c_2

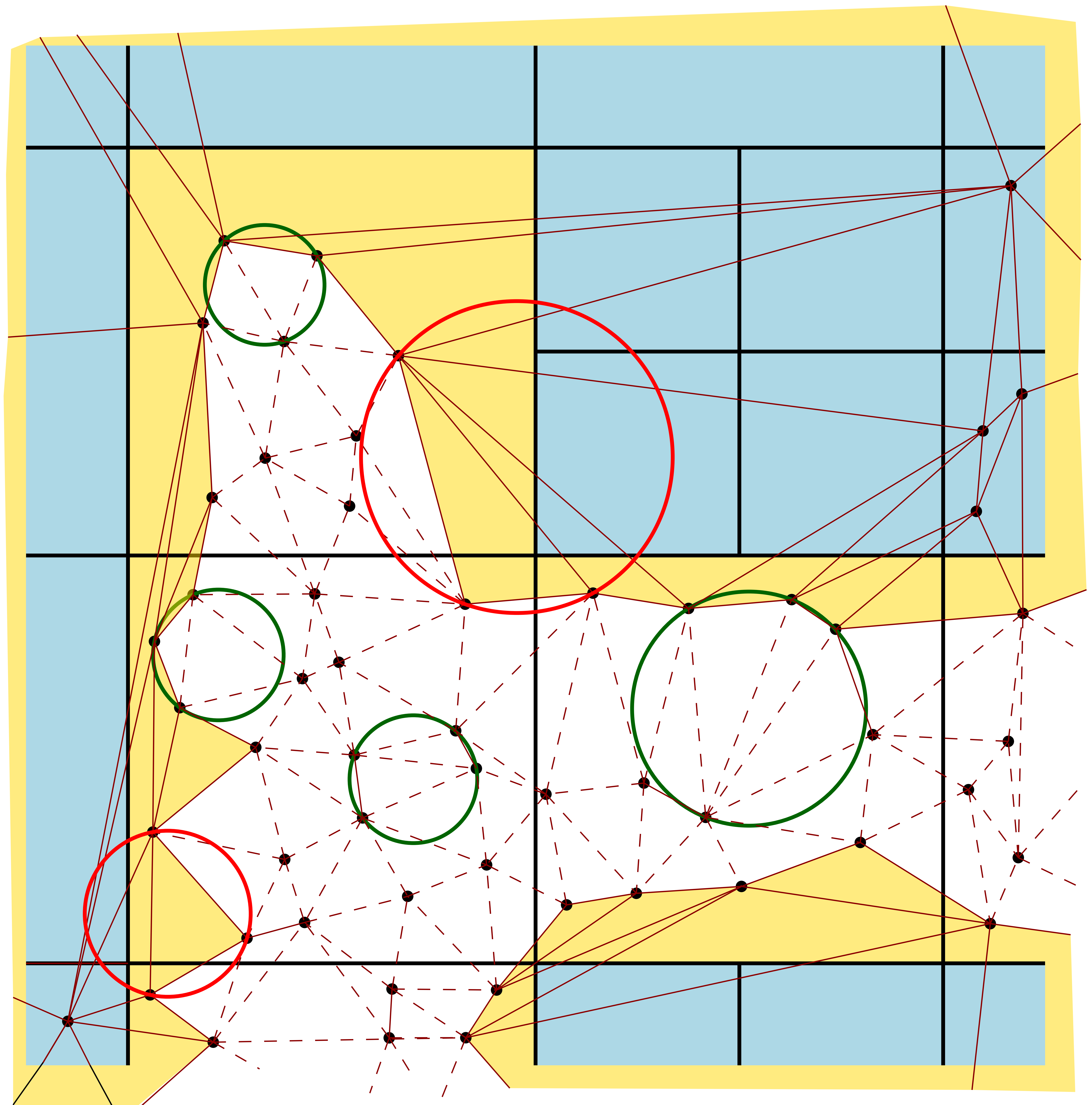
5. x y z
6. x y z
7. x y z

finalise c_0

8. x y z
9. x y z

finalise c_1

9 points and a stream with finalisation tags

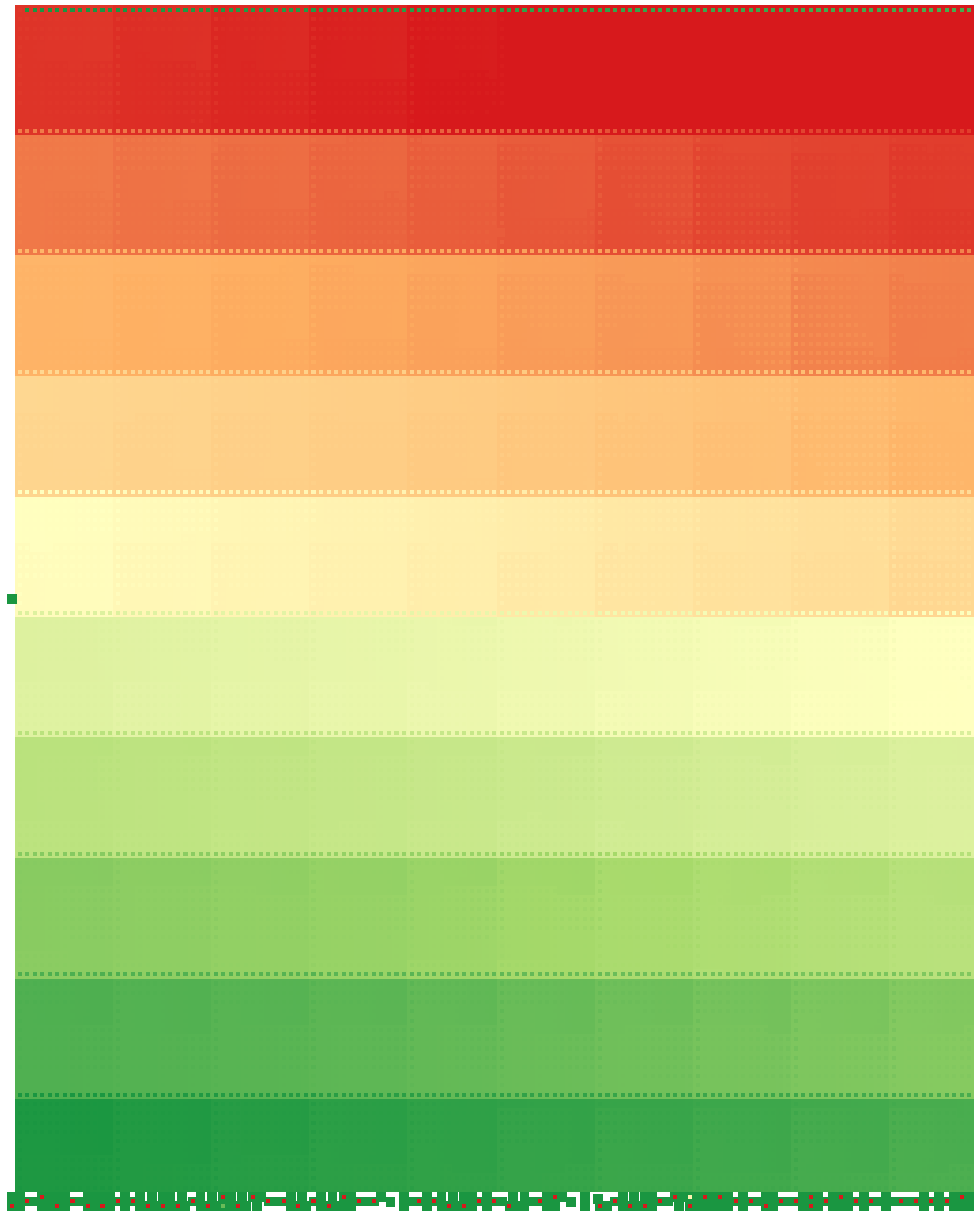


blue cell = not finalised cell

Yellow tr = in memory

White tr = written to stream

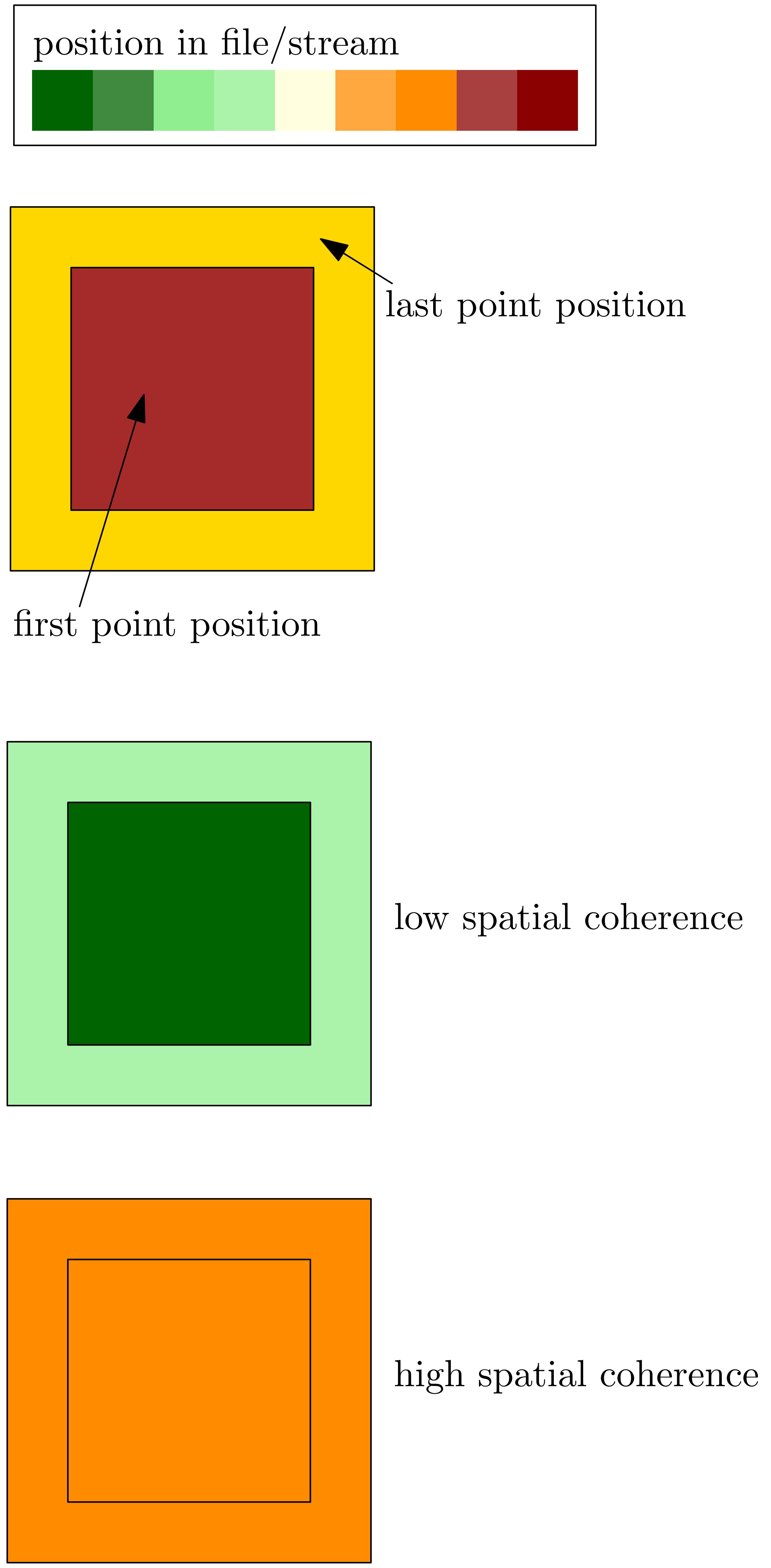
Spatial coherence



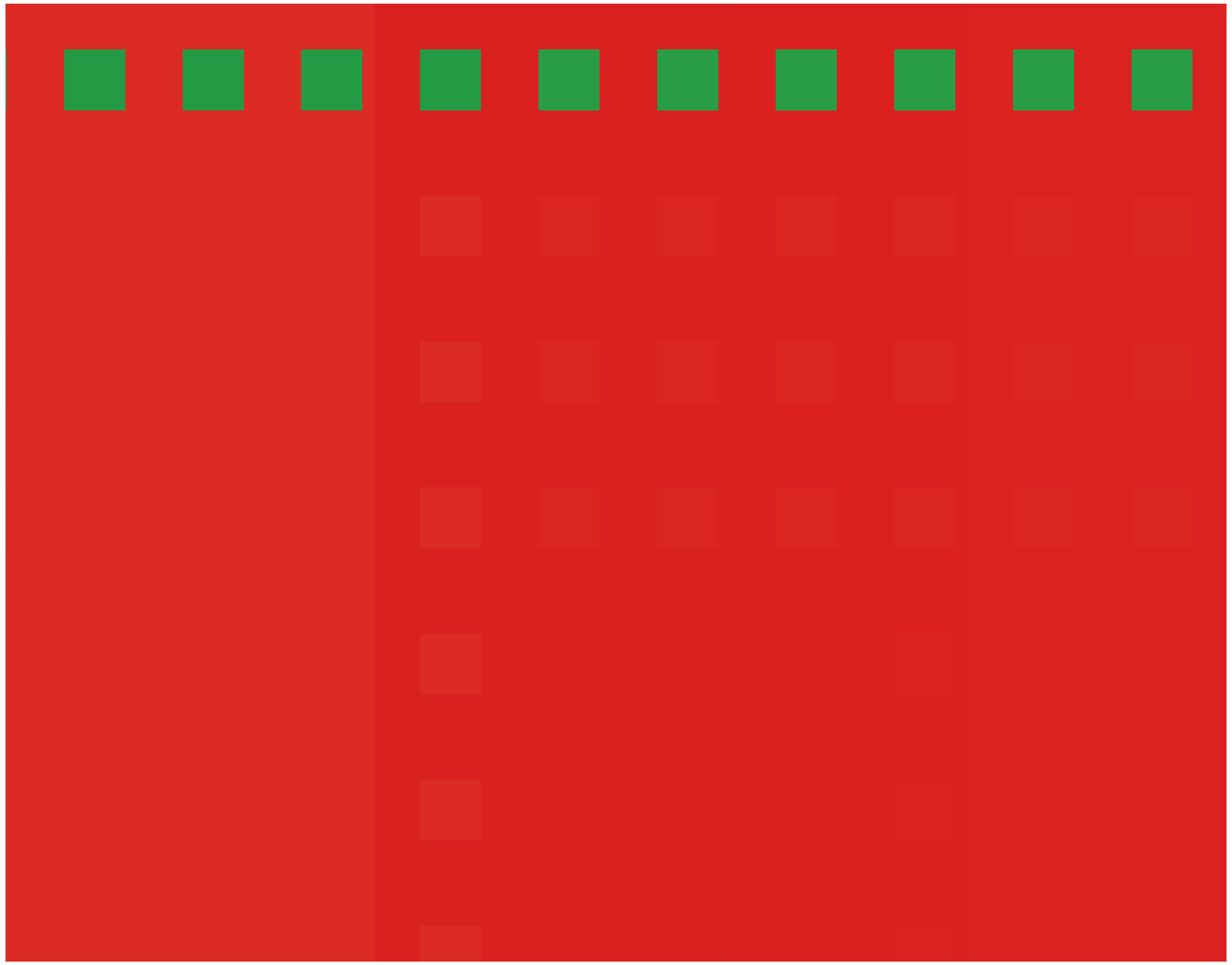
AHN3 – 37EN1

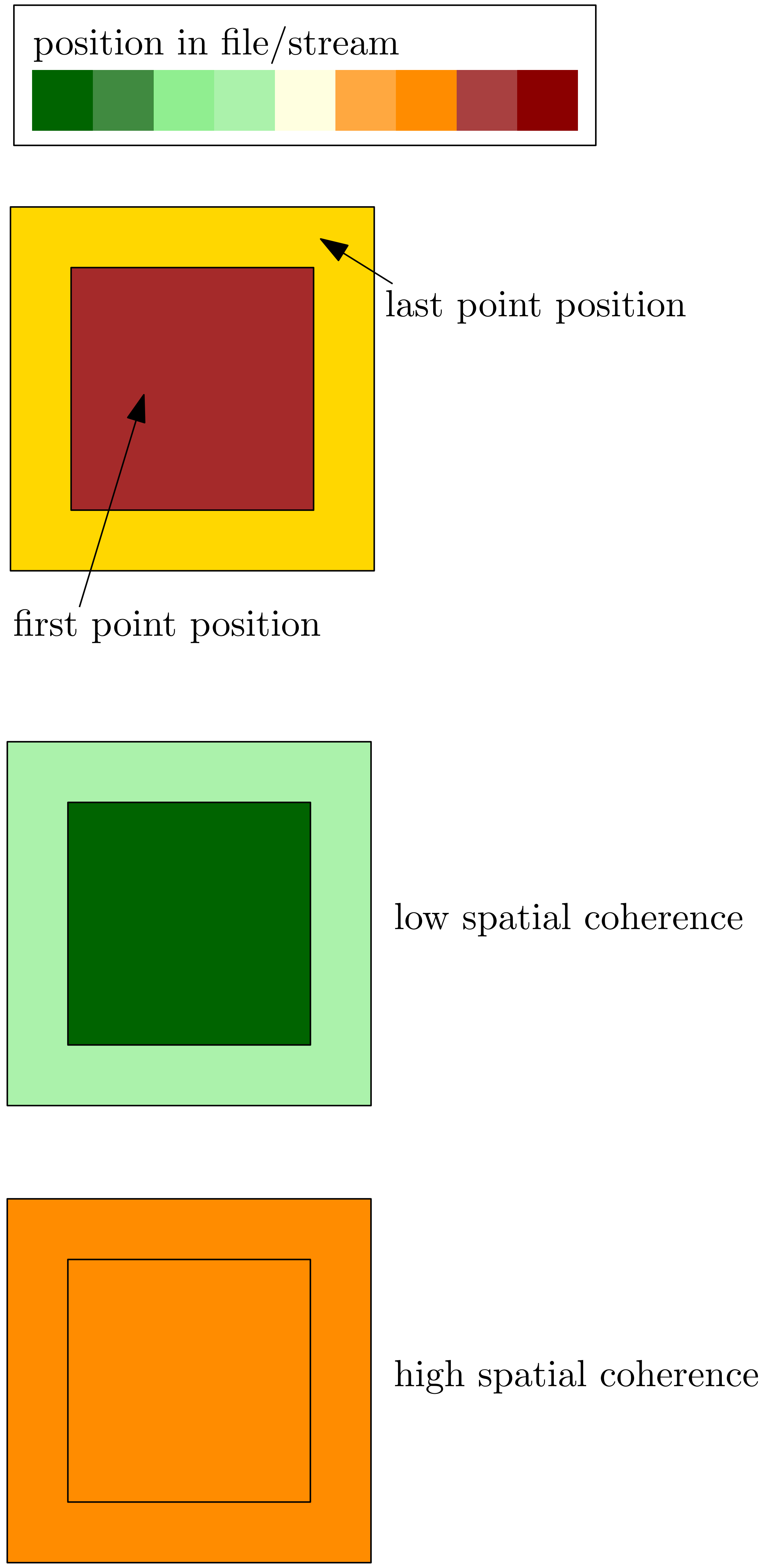
order in file

🟩	0
🟩	120M
🟨	240M
🟧	360M
🟥	480M

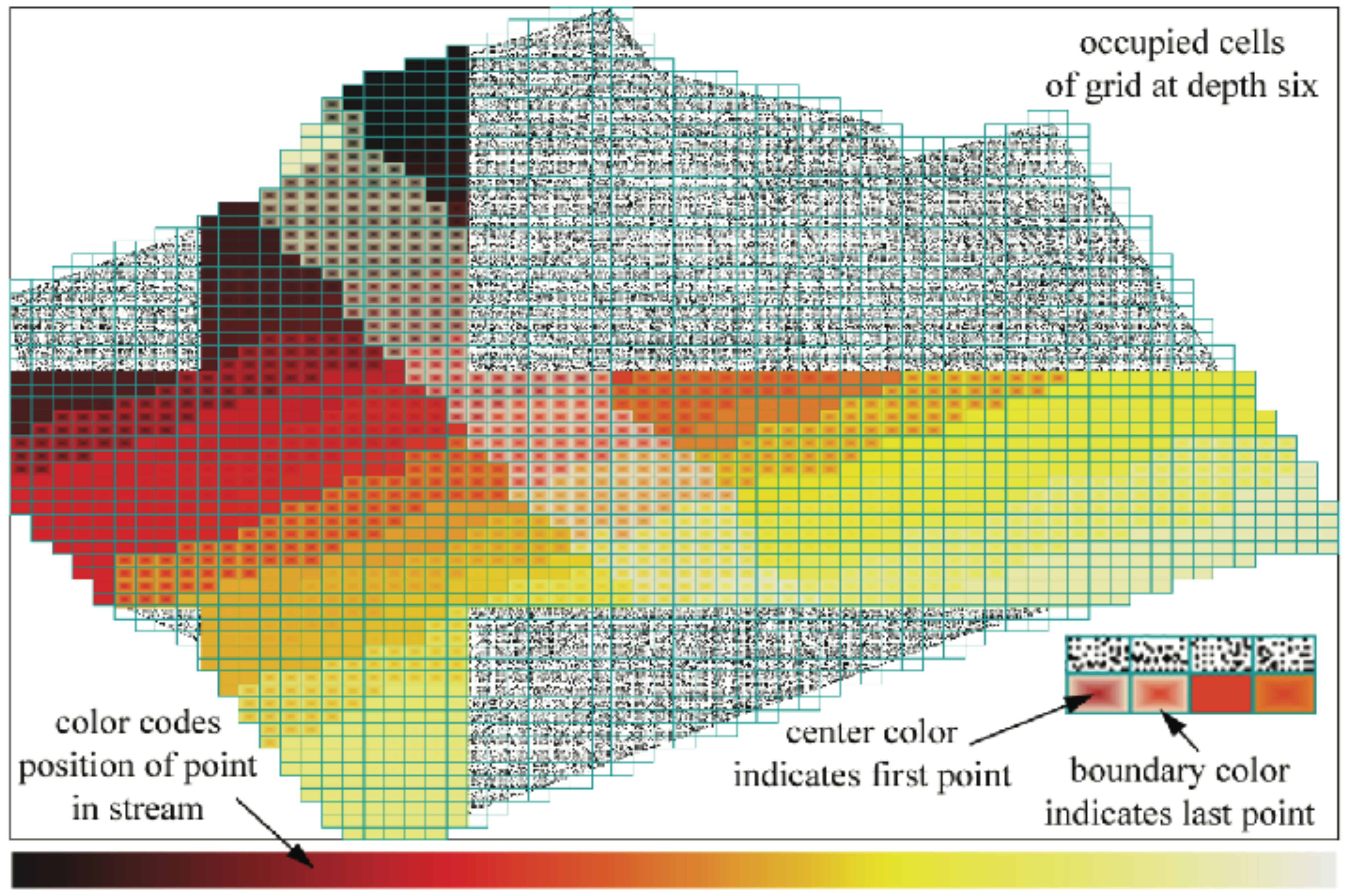


Spatial coherence

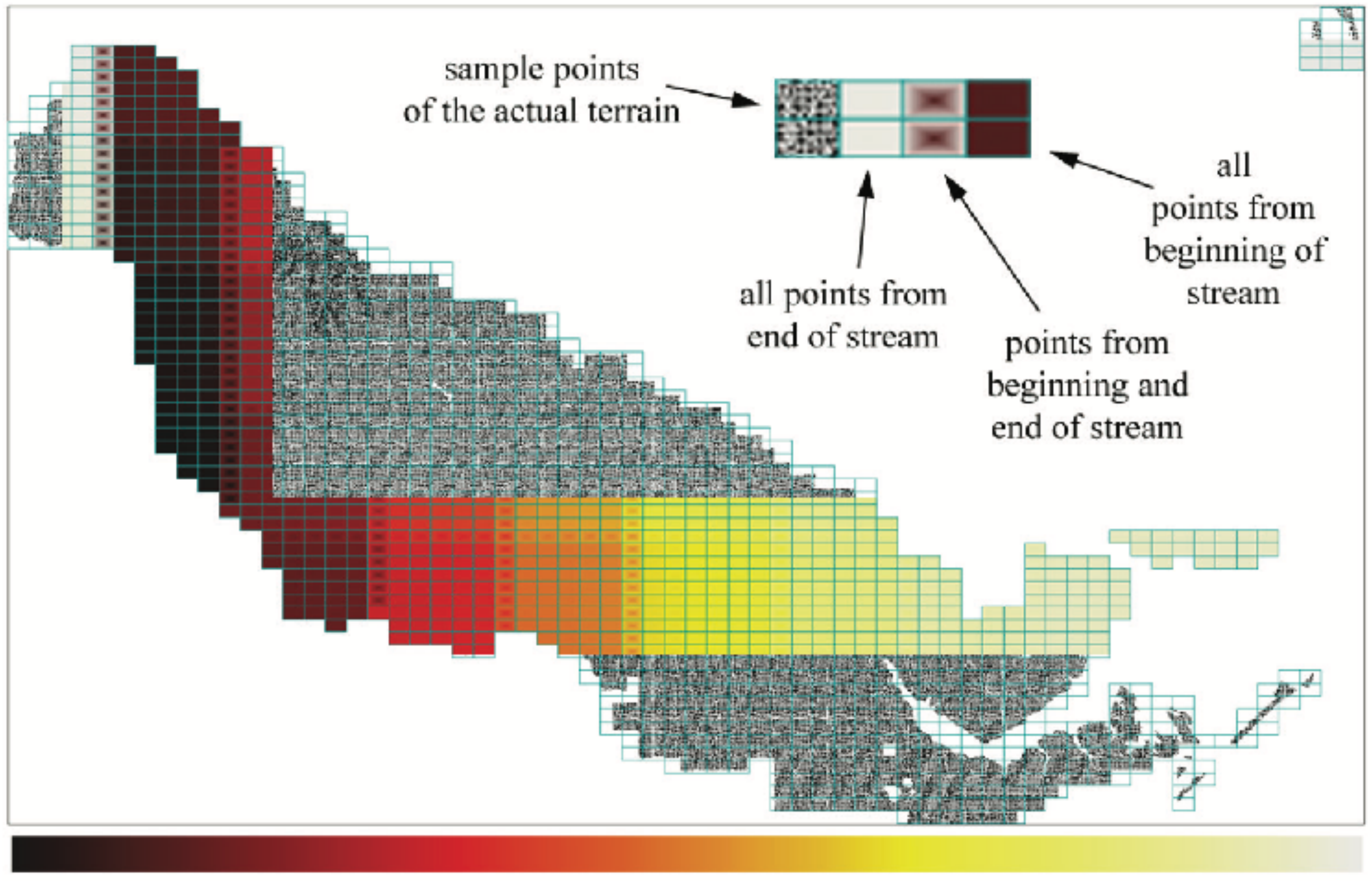




Spatial coherence

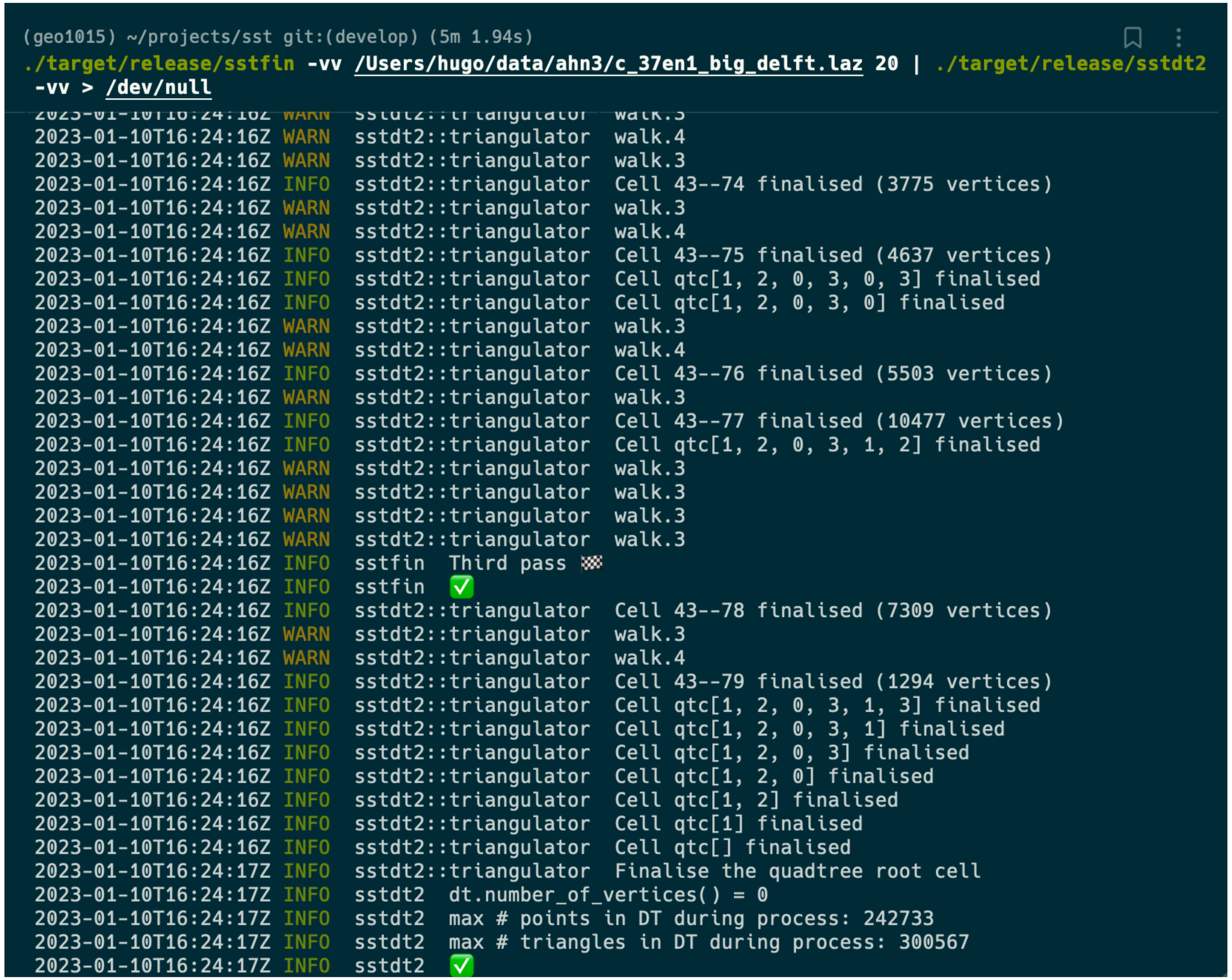

occupied cells of grid at depth six
color codes position of point in stream
center color indicates first point
boundary color indicates last point

(a) 6 million point Baisman Run dataset in Broadmoor, Maryland


sample points of the actual terrain
all points from beginning of stream
all points from end of stream
points from beginning and end of stream

(b) 500 million point Neuse river basin dataset in North Carolina

Figure 2.4: Spatial coherence as inherent property of real-world datasets. Adapted from Isenburg et al. [2006b, p. 3].



```
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./target/release/sstfin -vv /Users/hugo/data/ahn3/c_37en1_big_delft.laz 20 | ./target/release/sstdt2
 -vv > /dev/null
 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.3
 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.4
 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.3
 2023-01-10T16:24:16Z INFO  sstdt2::triangulator  Cell 43--74 finalised (3775 vertices)
 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.3
 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.4
 2023-01-10T16:24:16Z INFO  sstdt2::triangulator  Cell 43--75 finalised (4637 vertices)
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 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.3
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 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.3
 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.3
 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.3
 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.3
 2023-01-10T16:24:16Z INFO  sstfin  Third pass 🏁
 2023-01-10T16:24:16Z INFO  sstfin  ✅
 2023-01-10T16:24:16Z INFO  sstdt2::triangulator  Cell 43--78 finalised (7309 vertices)
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 2023-01-10T16:24:16Z WARN  sstdt2::triangulator  walk.4
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 2023-01-10T16:24:16Z INFO  sstdt2::triangulator  Cell qtc[1, 2, 0, 3] finalised
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 2023-01-10T16:24:16Z INFO  sstdt2::triangulator  Cell qtc[1, 2] finalised
 2023-01-10T16:24:16Z INFO  sstdt2::triangulator  Cell qtc[1] finalised
 2023-01-10T16:24:16Z INFO  sstdt2::triangulator  Cell qtc[] finalised
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 2023-01-10T16:24:17Z INFO  sstdt2  ✅
```

Some questions

- The raw point clouds are seldom used, instead gridded terrains (at for instance 0.5mX0.5m) are used. How are these created?
- The ideas behind streaming are very useful for certain local problems, but unfortunately they cannot be used directly for global problems such as visibility or flow modelling". Explain why that is with a concrete example.