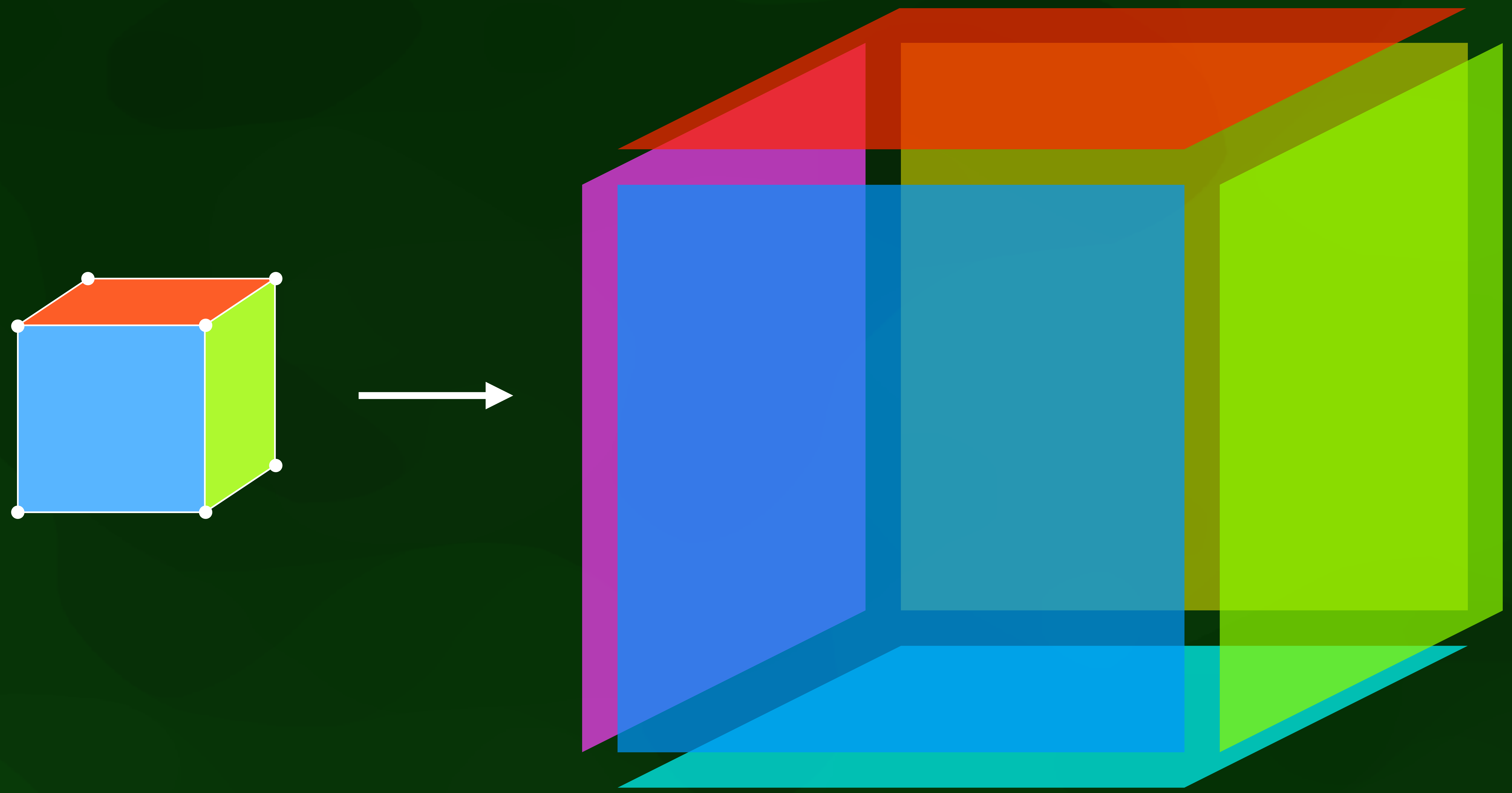
Generalised maps and combinatorial maps

GEO1004:
3D modelling of the built environment

https://3d.bk.tudelft.nl/courses/geo1004

3D geoinformation
Department of Urbanism
Faculty of Architecture and the Built Environment
Delft University of Technology

So far... (3D through b-rep)



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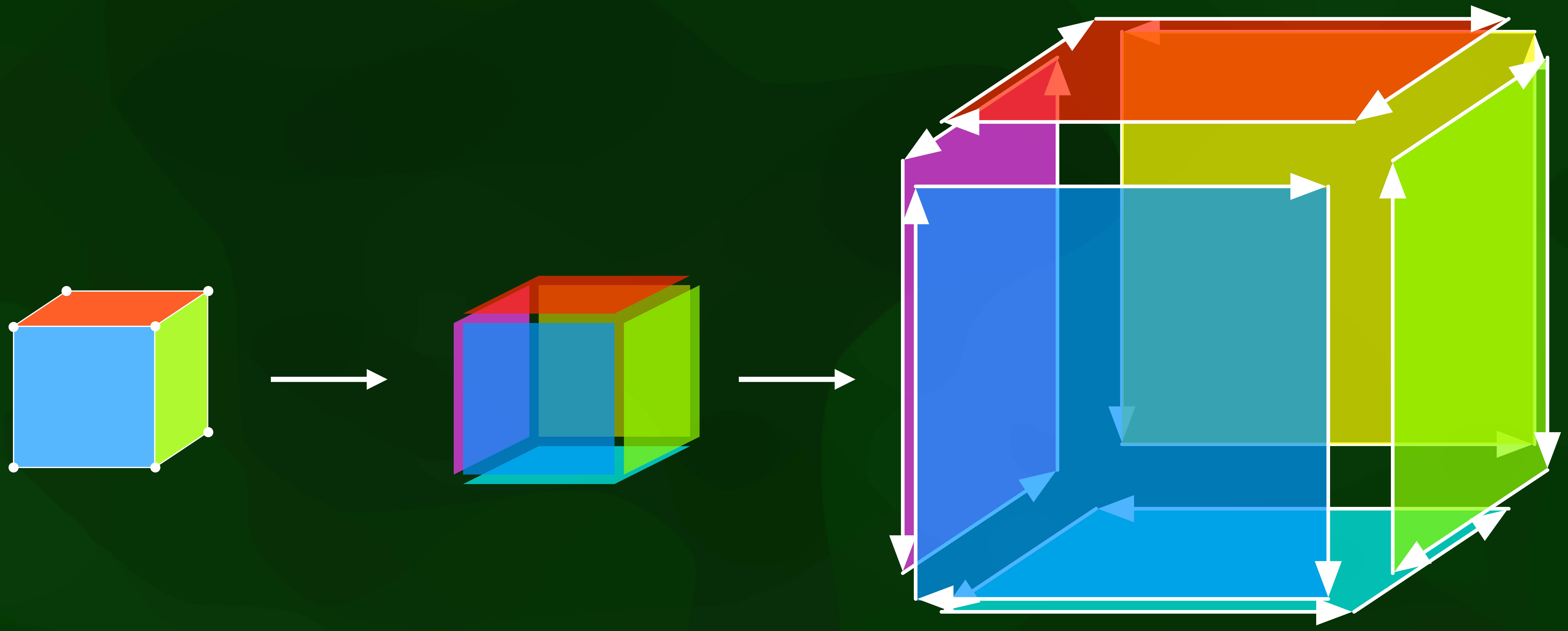
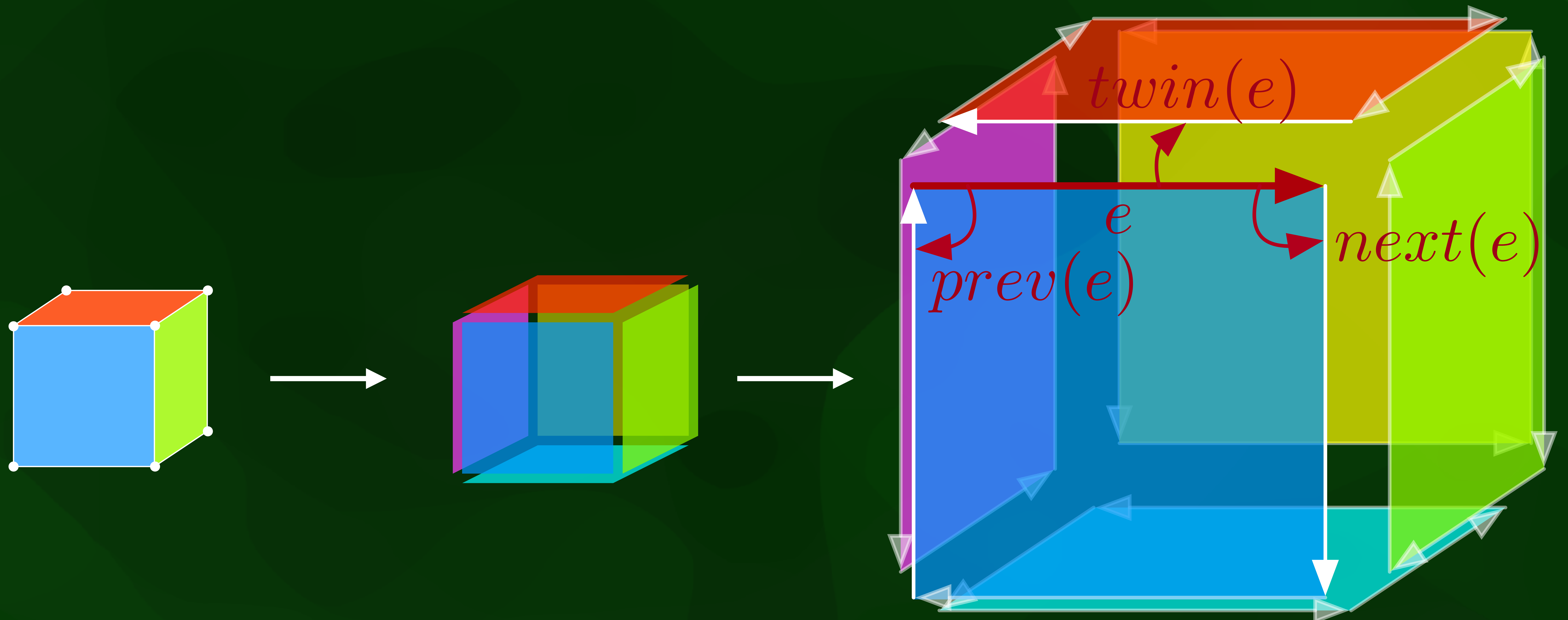
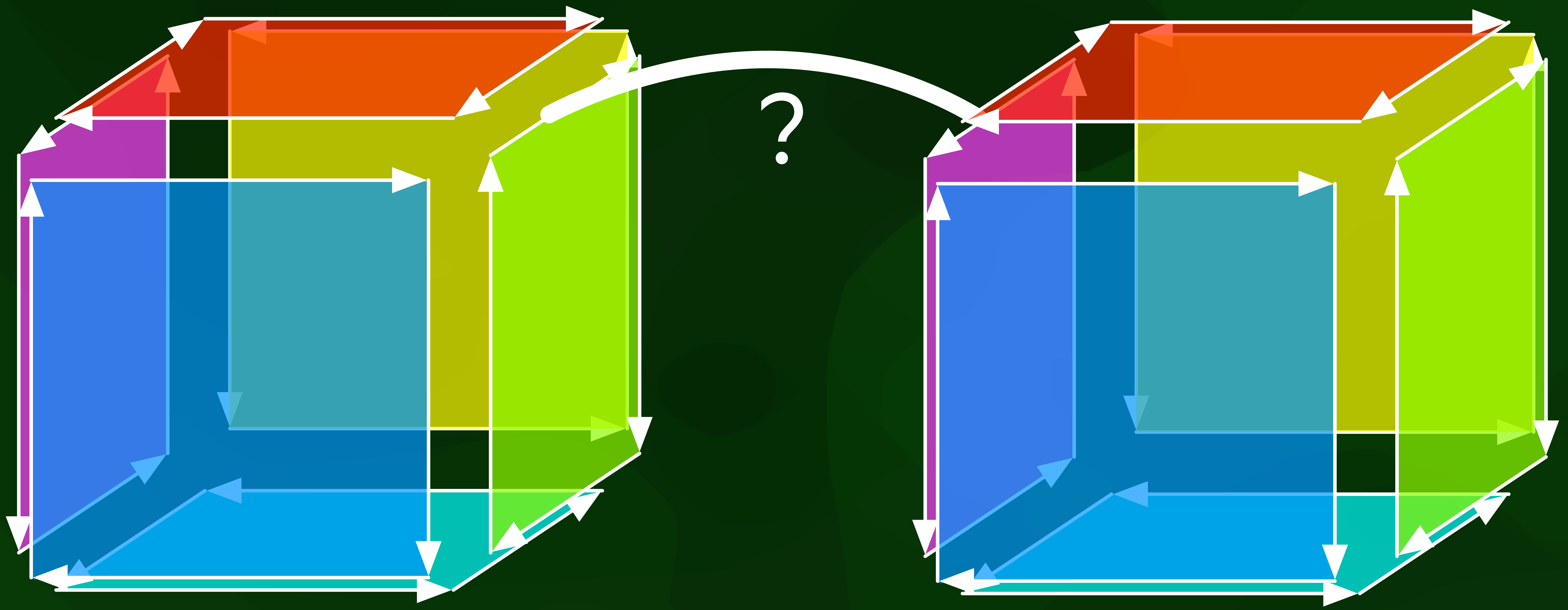


So far... (3D through b-rep)



links between 0D-2D elements

Links between 3D elements?



Drawbacks of b-rep

- Difficult to store:
  - Holes (2D and 3D)
  - Non-manifolds
  - **Multiple volumes**

Back to Jordan-Brouwer theorem

- In 2D, the Jordan curve theorem says: a closed curve separates the plane into two parts: an interior surface and an exterior surface
- In nD, the Jordan-Brouwer theorem, which in 3D says: a closed surface separates 3D space into two parts: an interior volume and an exterior volume.

Back to Jordan-Brouwer theorem

- Holes: one more exterior per hole (both 2D and 3D)
- Non-2-manifold around point: one more interior per point
- Multiple volumes: one more interior per extra volume

What are g-maps / c-maps?

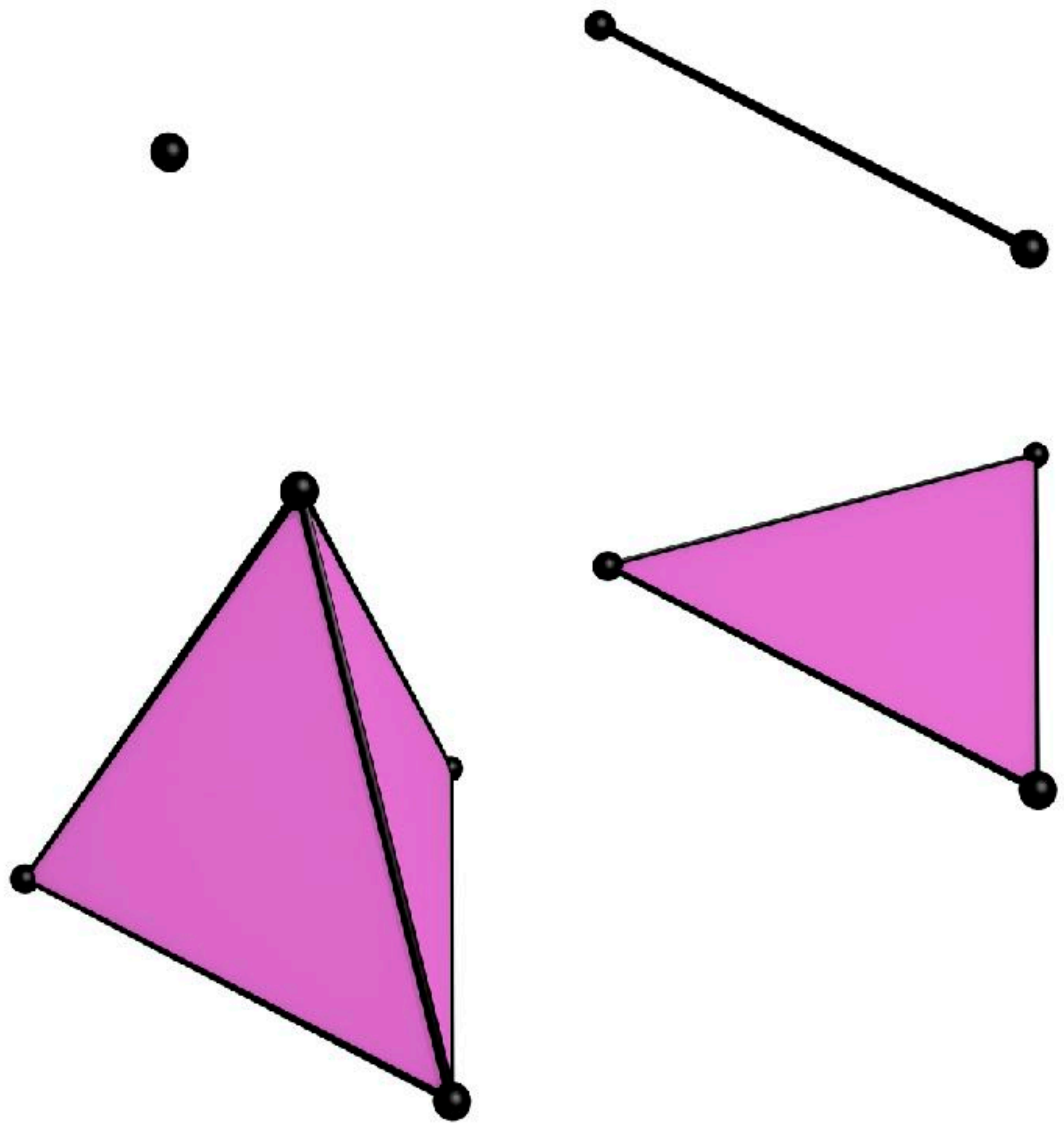
- In short:
  - c-maps: generalisation of half-edge to nD
  - g-maps: c-maps split into two to avoid oriented edges

Why g-maps / c-maps?

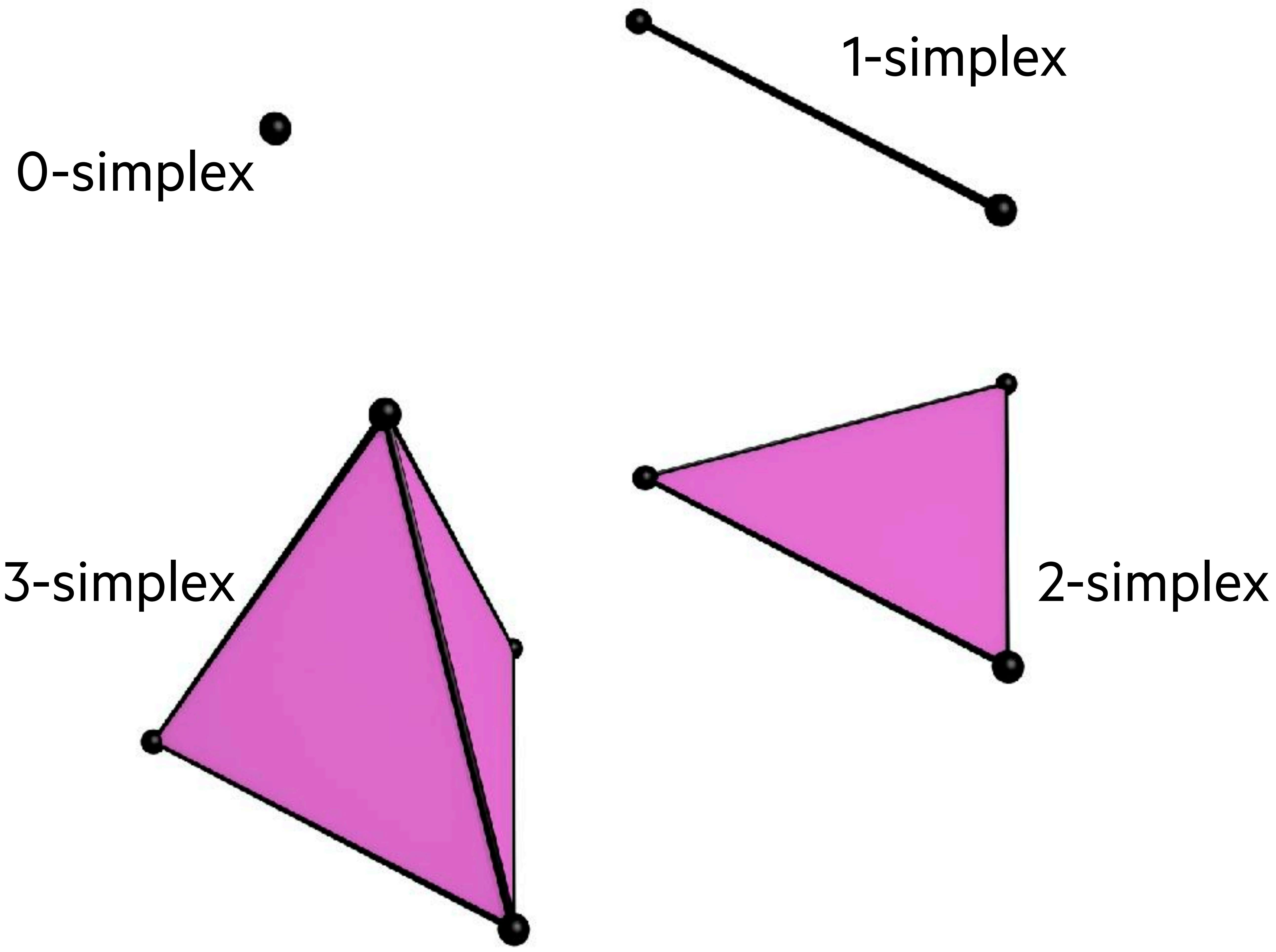
- In short:
  - Possibility to store links between nD elements (including 3D)
  - With g-maps: no orientation issues during construction

Small background

Simplex



Simplex



Simplex properties

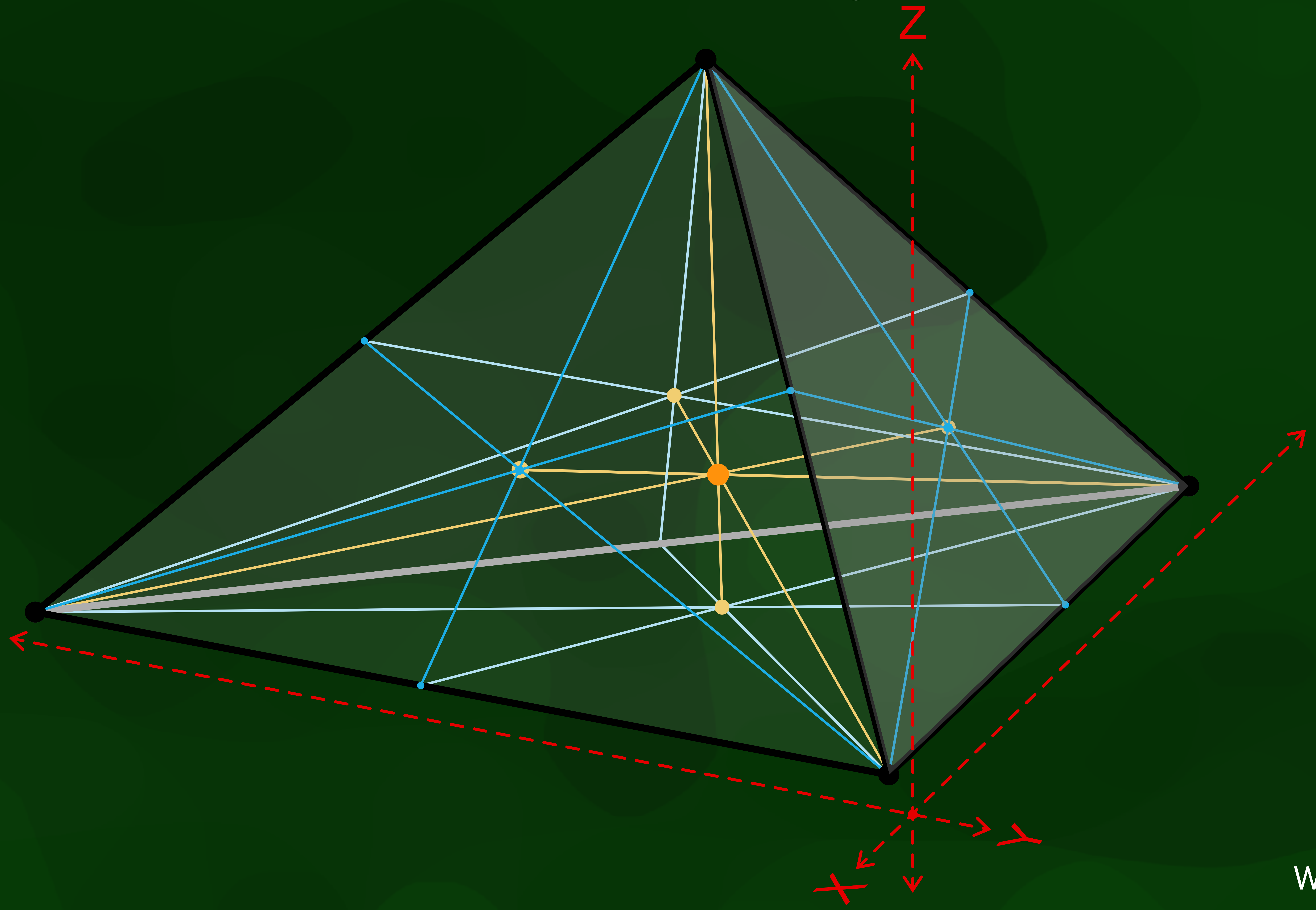
- An n-simplex in n-dimensional space:
  - is bounded by n+1 (n-1)-simplices
  - can have n+1 adjacent n-simplices, each of which shares a (n-1)-simplex on their common boundary

➔ Two adjacent n-simplices share all their vertices except for one

Cells

- 0-cell: vertex
- 1-cell: edge
- 2-cell: polygon
- 3-cell: polyhedron
- …

Barycentric triangulation

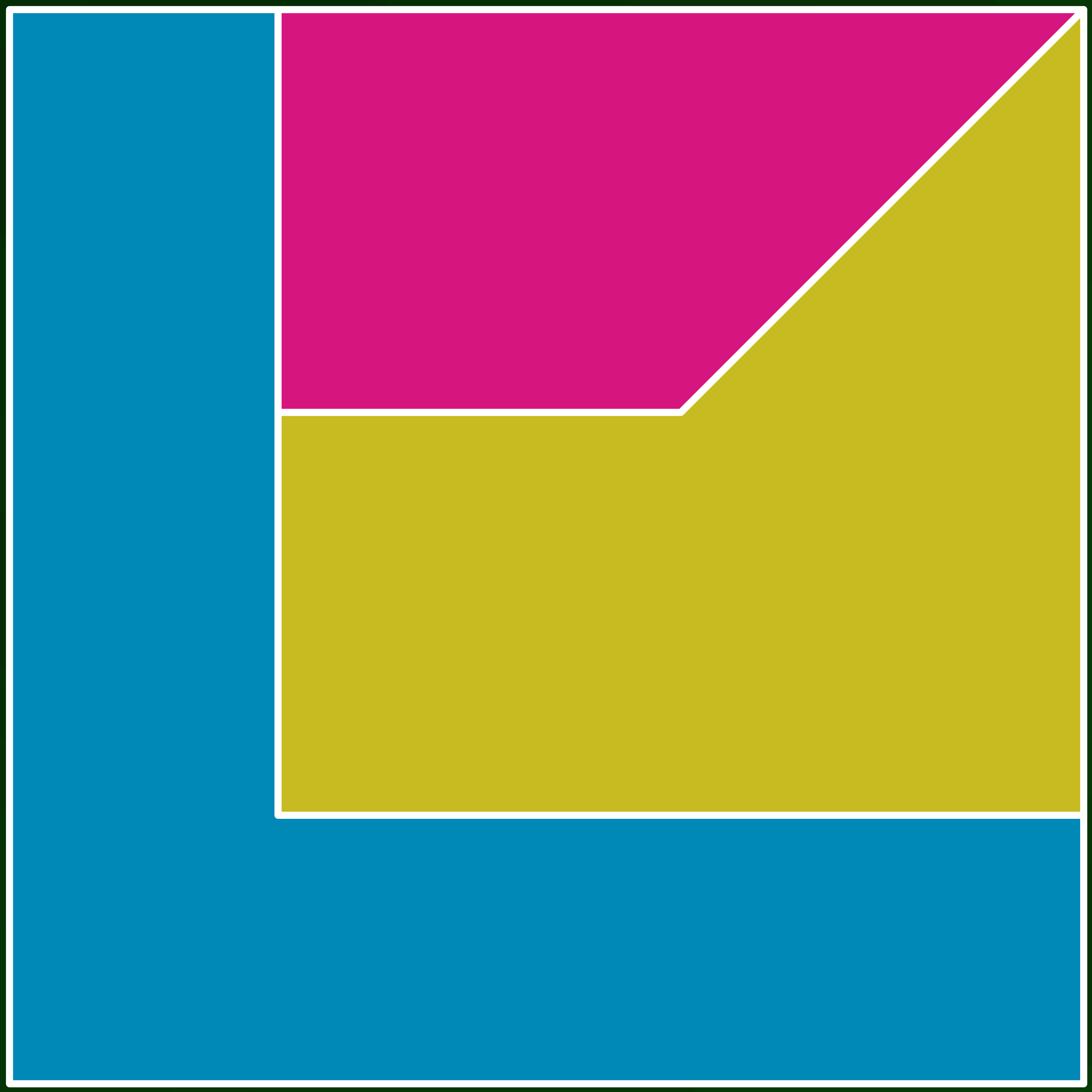


Wikimedia Commons

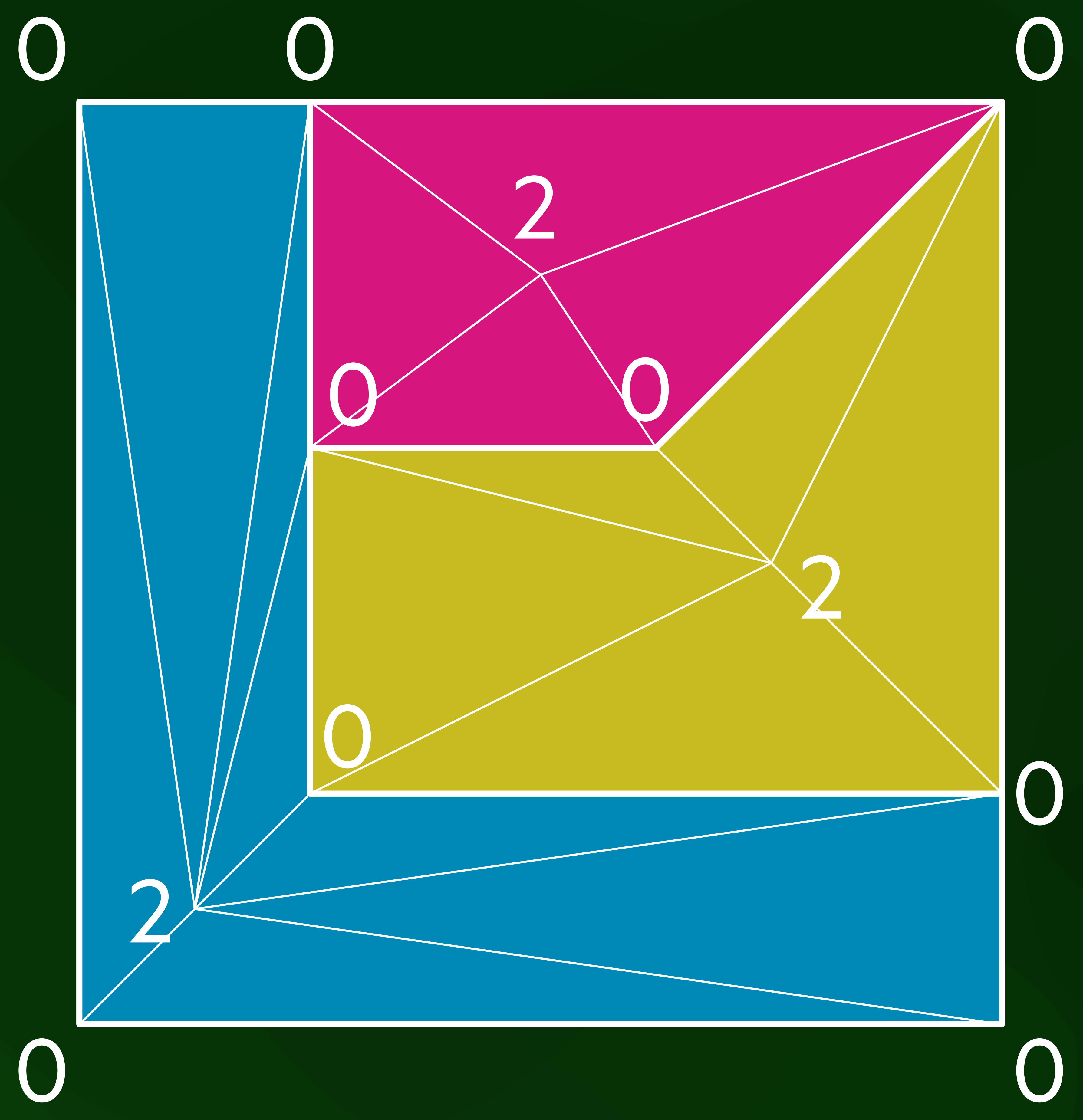
Building g-maps / c-maps

- nD c-maps (n-c-maps): barycentric triangulation n-cells from 2-cells
- nD g-maps (n-g-maps): barycentric triangulation of n-cells from 1-cells
- ... where
  - n-simplices are called **darts** and have vertices that are linked to elements of a certain dimension, and
  - only 0-cells have a location in space.

Building g-maps / c-maps

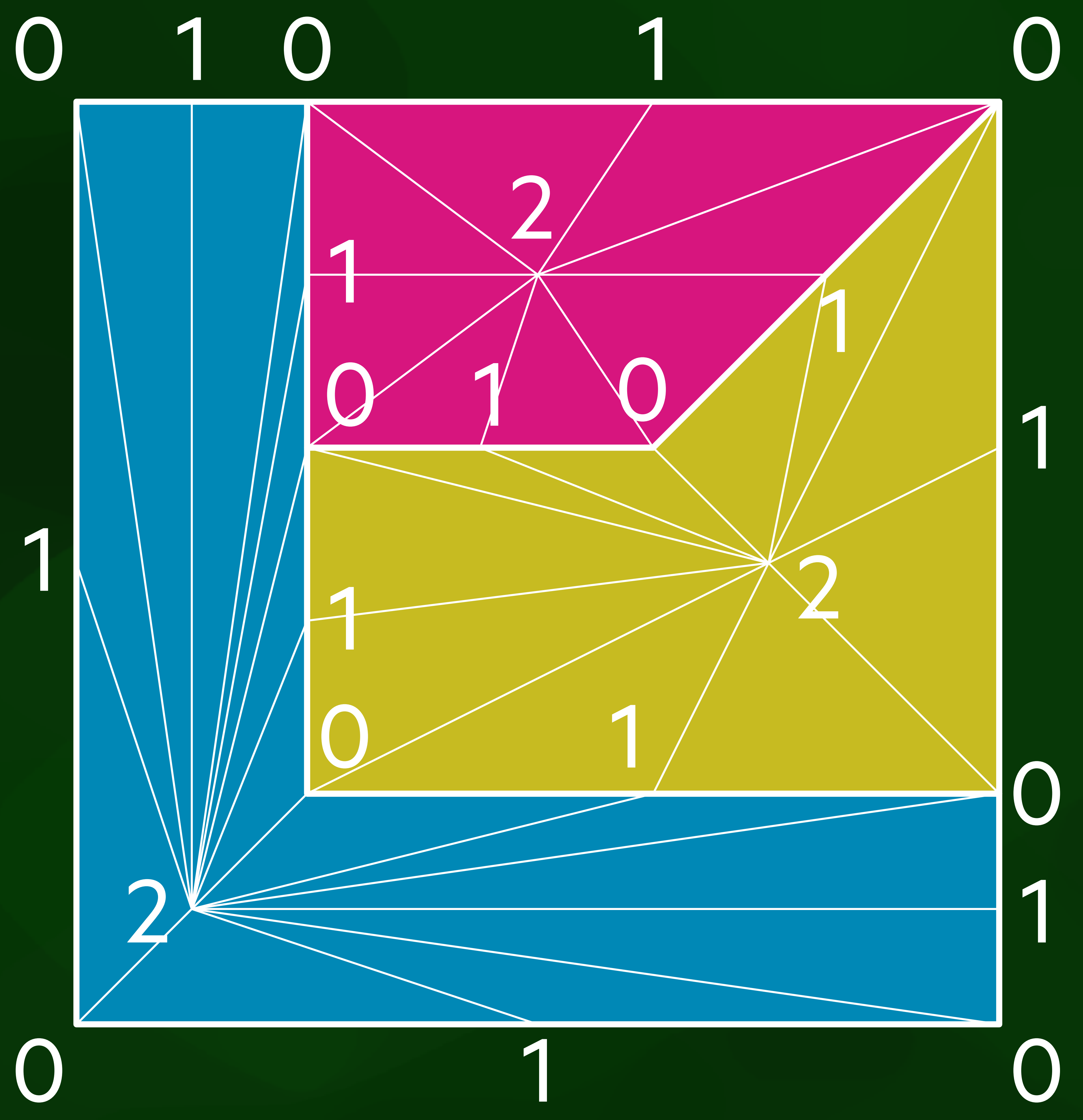


Building g-maps / c-maps



combinatorial map

(c-map)



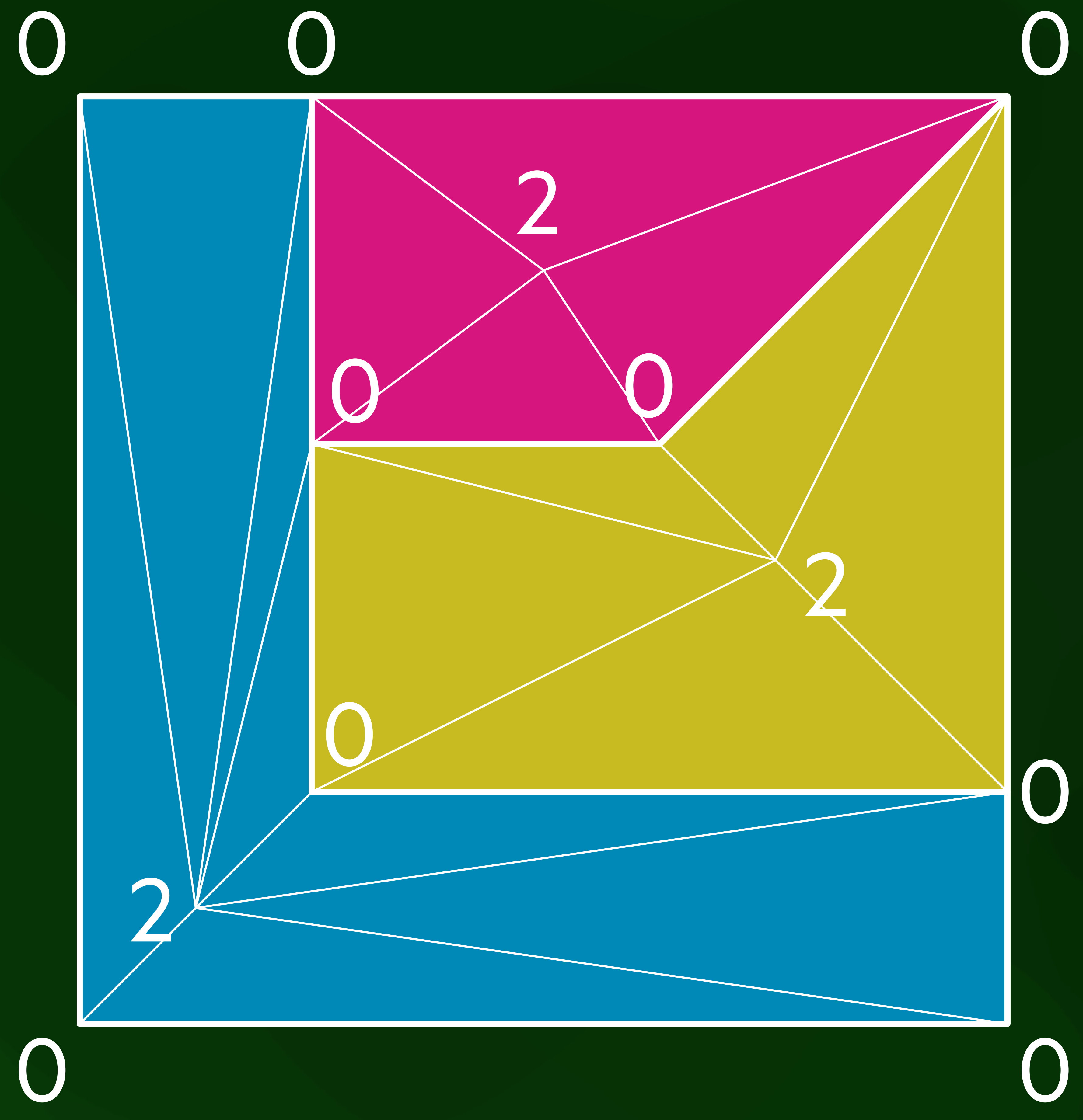
generalised map

(g-map)

Intuitive meaning of a dart

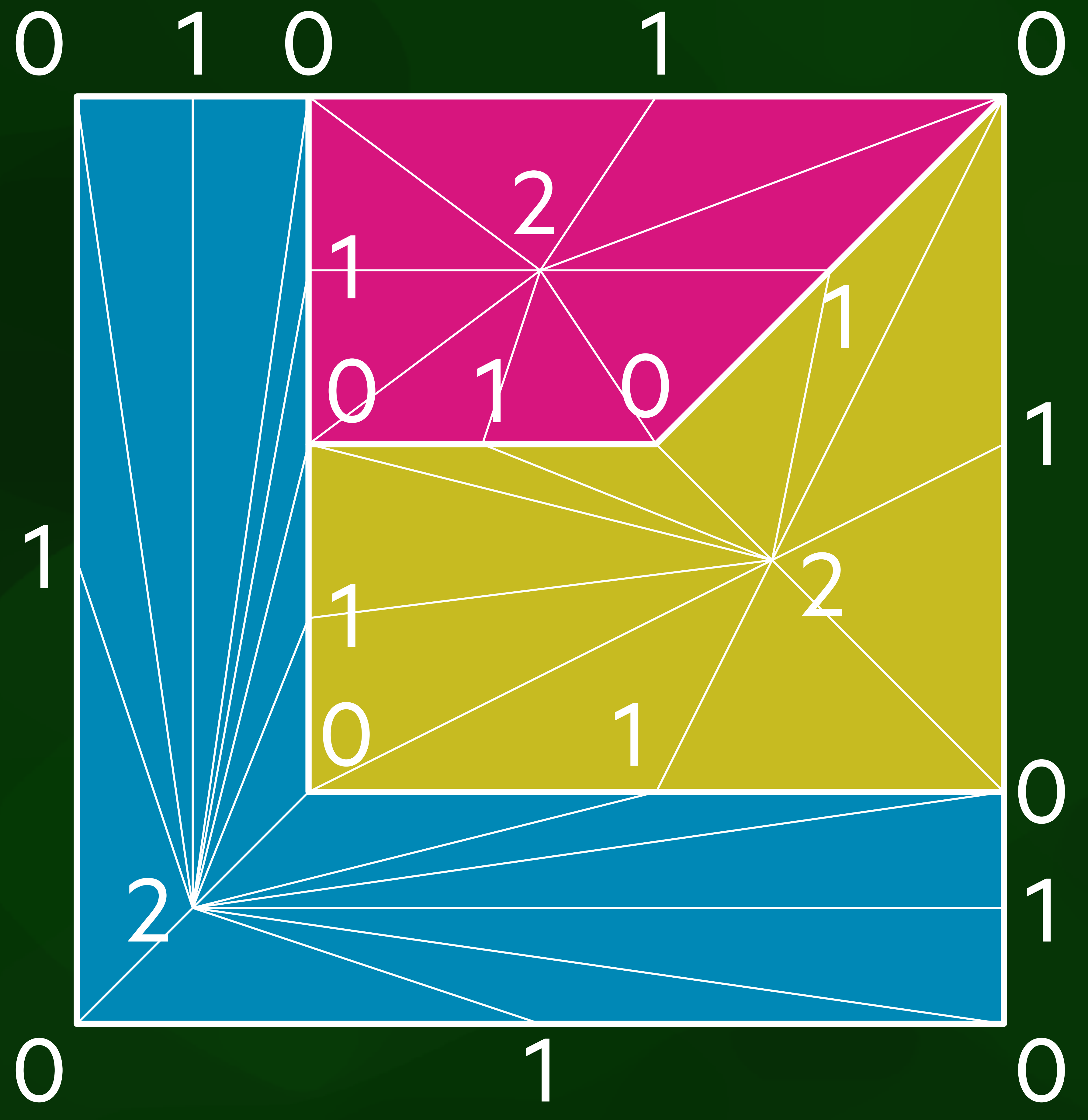
- Informally:
  - a generalised map dart is a unique combination of a cell of every dimension: vertex, edge, face, volume, …
  - a combinatorial map dart is a unique combination of a cell of every dimension from one upwards: edge, face, volume, …
- Why informal? only true for a specific linear embedding

Building g-maps / c-maps



combinatorial map

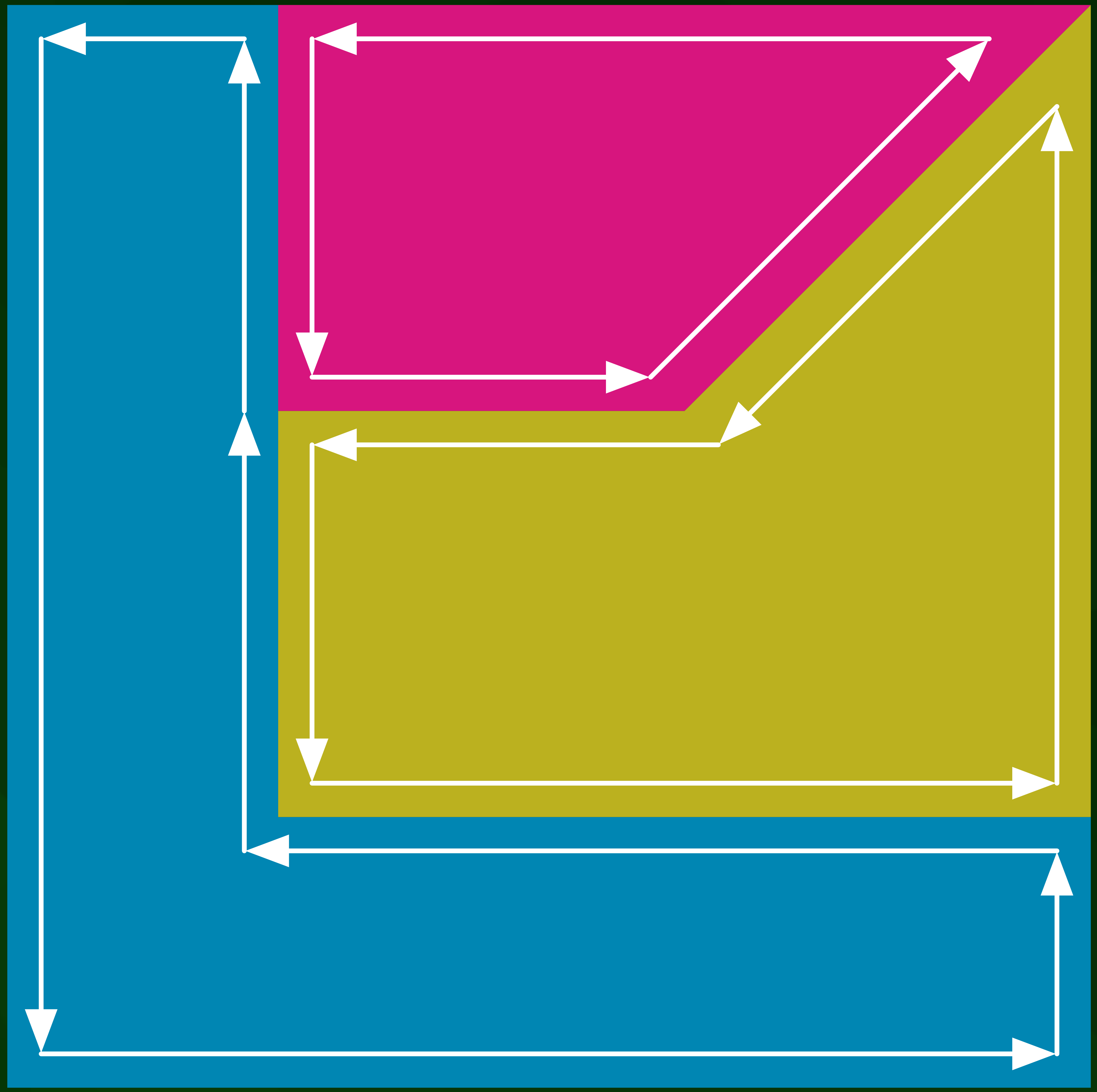
(c-map)

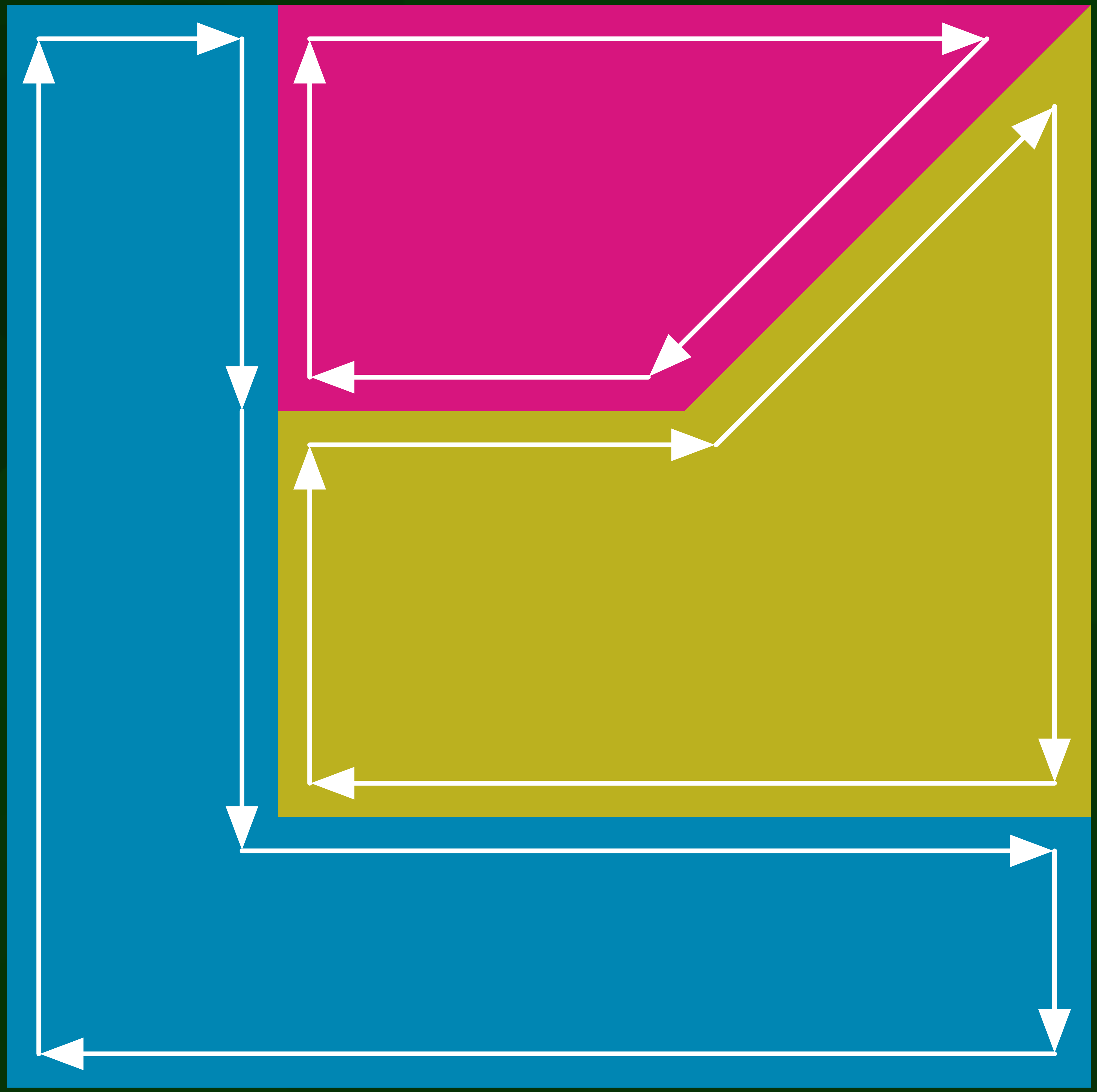


generalised map

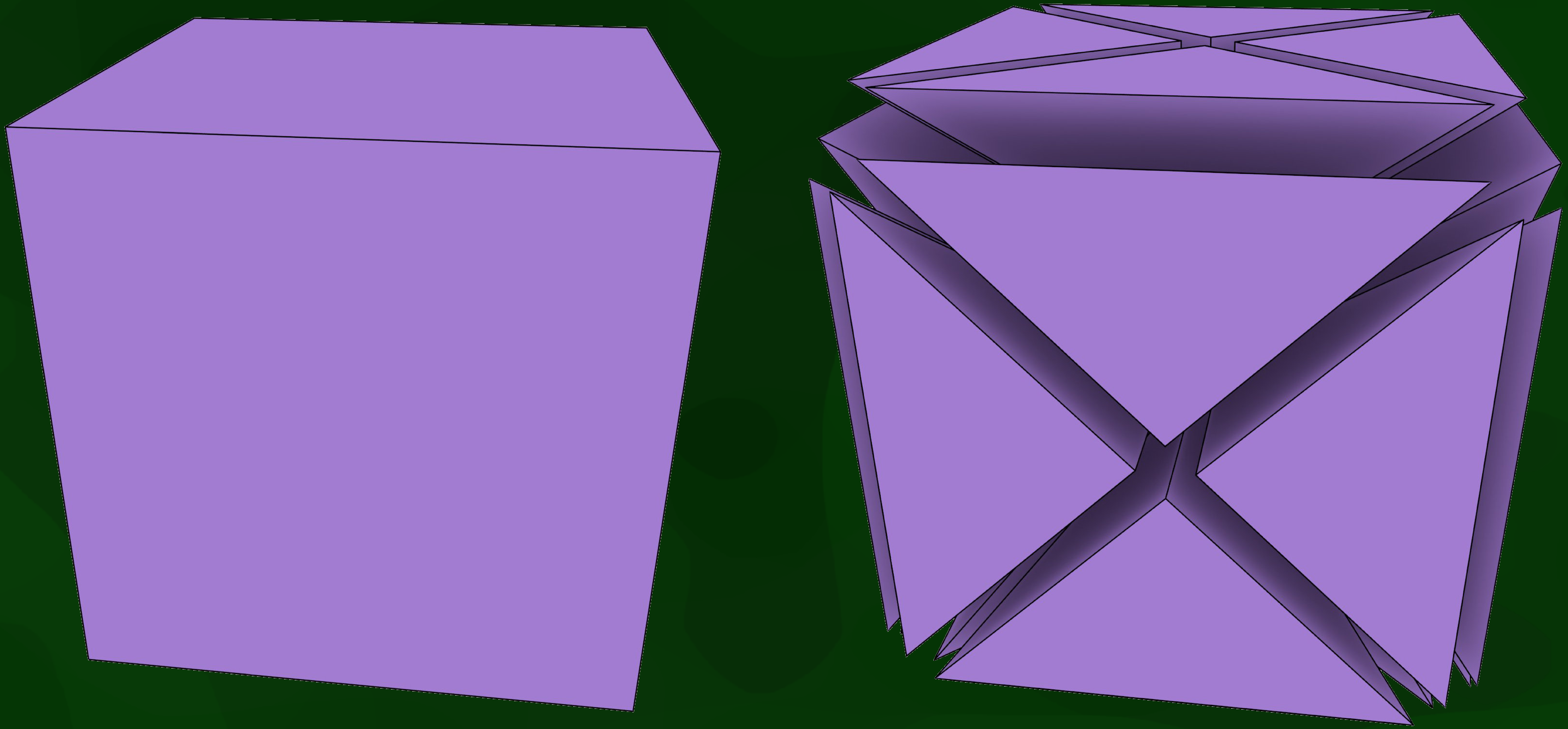
(g-map)

C-maps: orientation

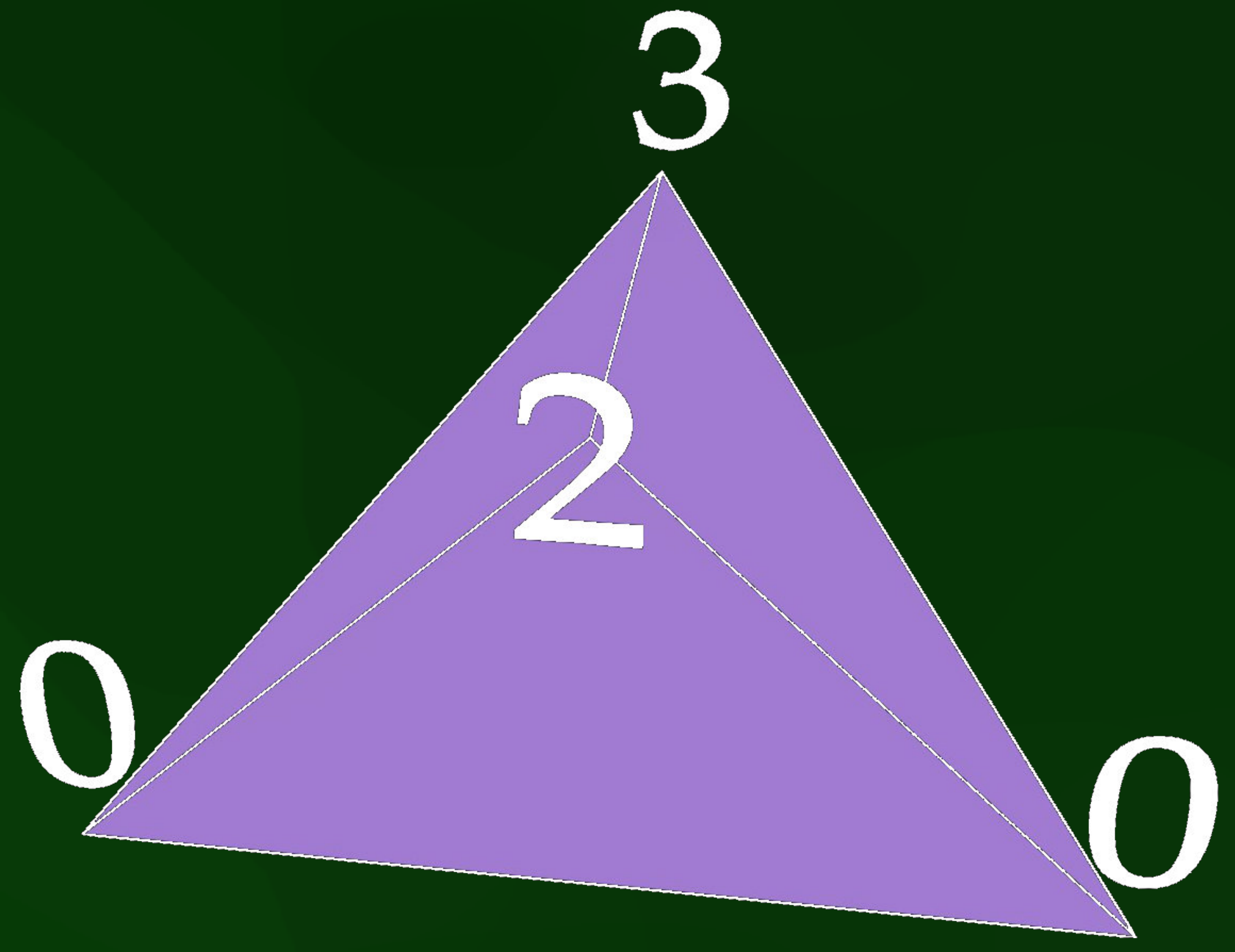


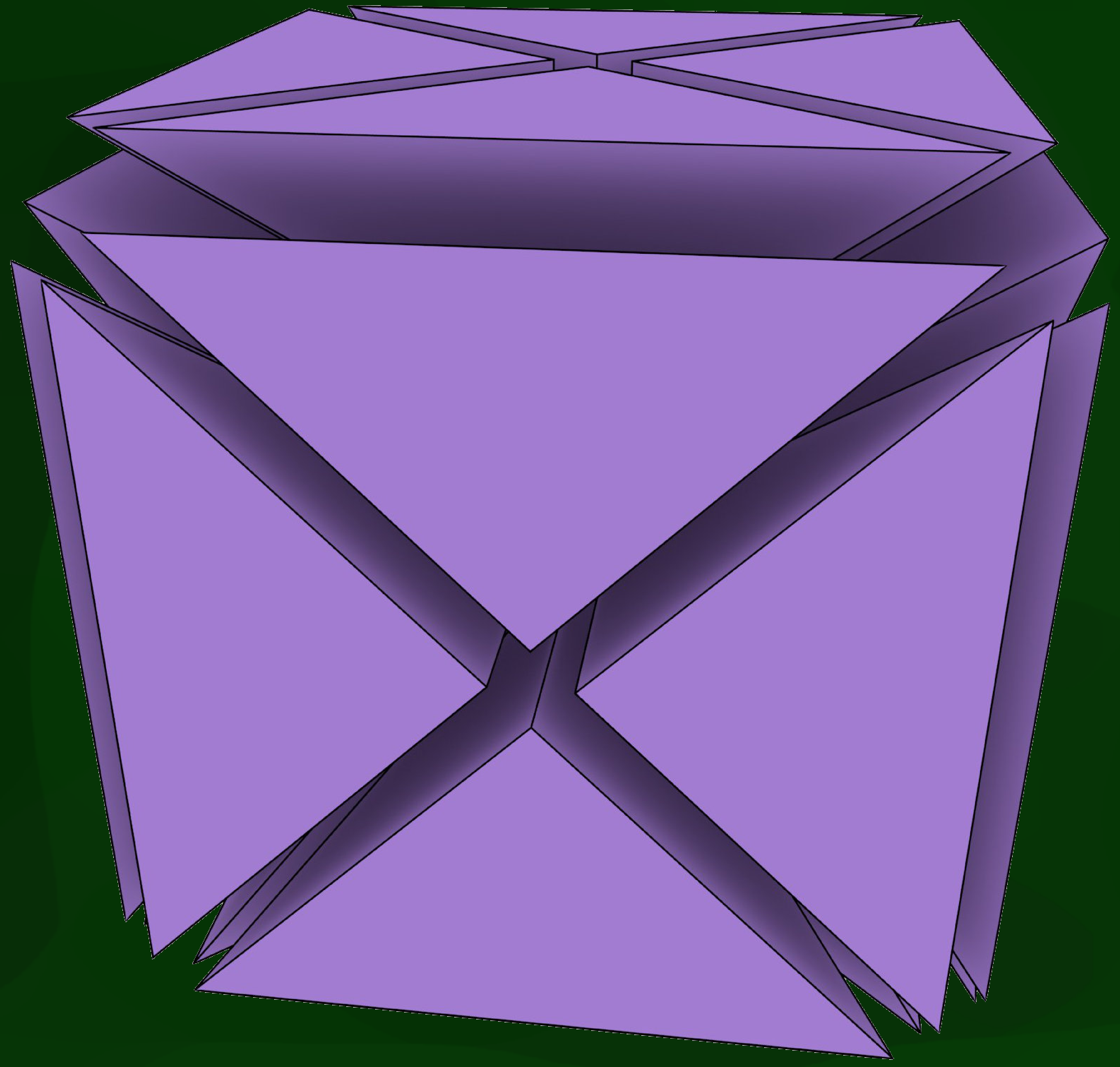


Building g-maps / c-maps



Building g-maps / c-maps





Traversing darts

- From the properties of simplices: a dart d in an nD combinatorial/generalised map has n+1 adjacent darts, each of which shares all but one of the vertices of d.

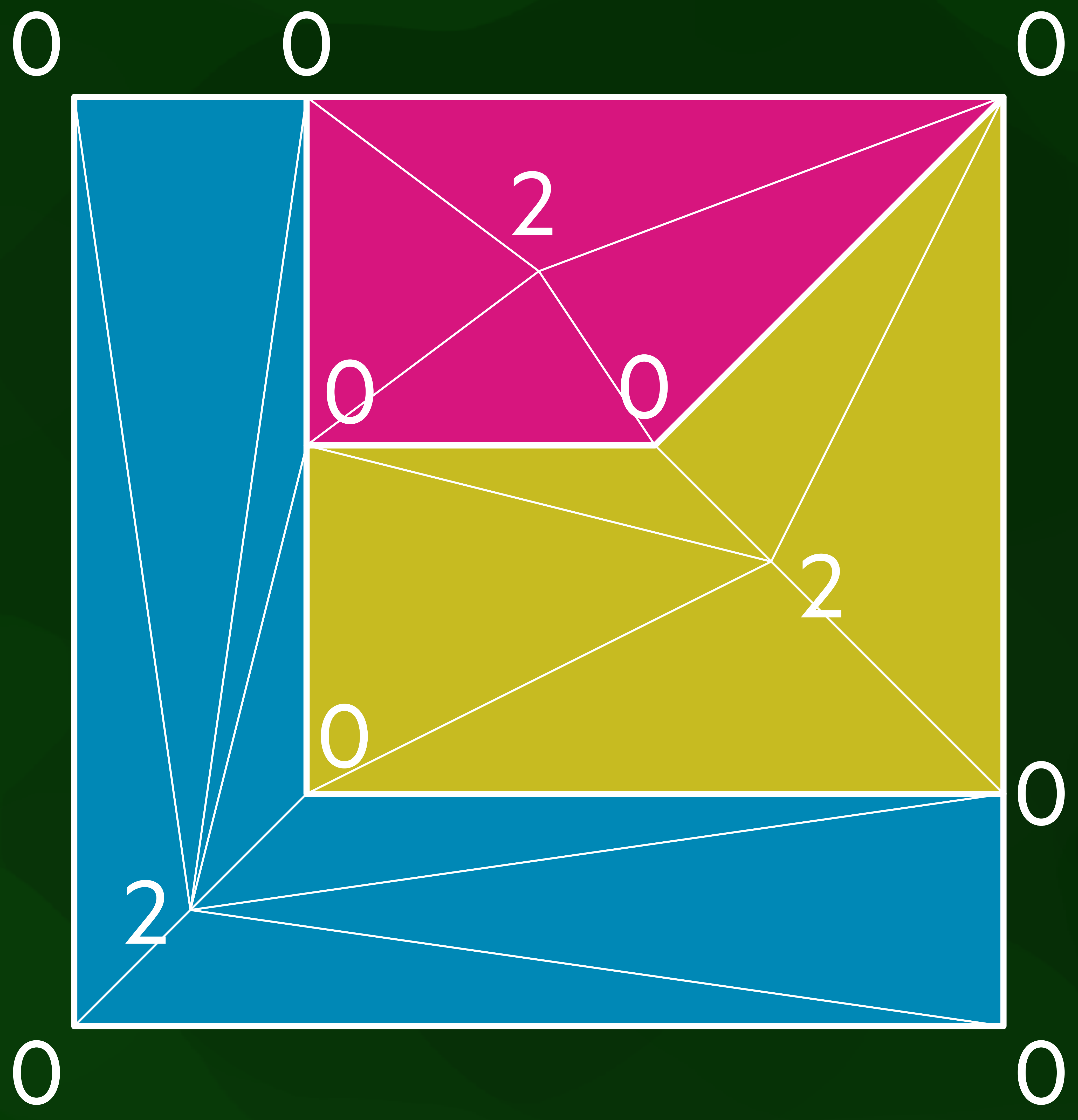
➔ Informally: Two adjacent darts share all of their cells except for one. e.g. if they differ in their edge, they share their vertex, face, volume, etc.

Traversing darts

- The link from an i-dimensional vertex of a dart d to the (other) i-dimensional vertex of its adjacent neighbour is called:
  - α_i in a generalised map
  - β_i in a combinatorial map
- Informally, it means switching the i-cell of d for the i-cell of its neighbour

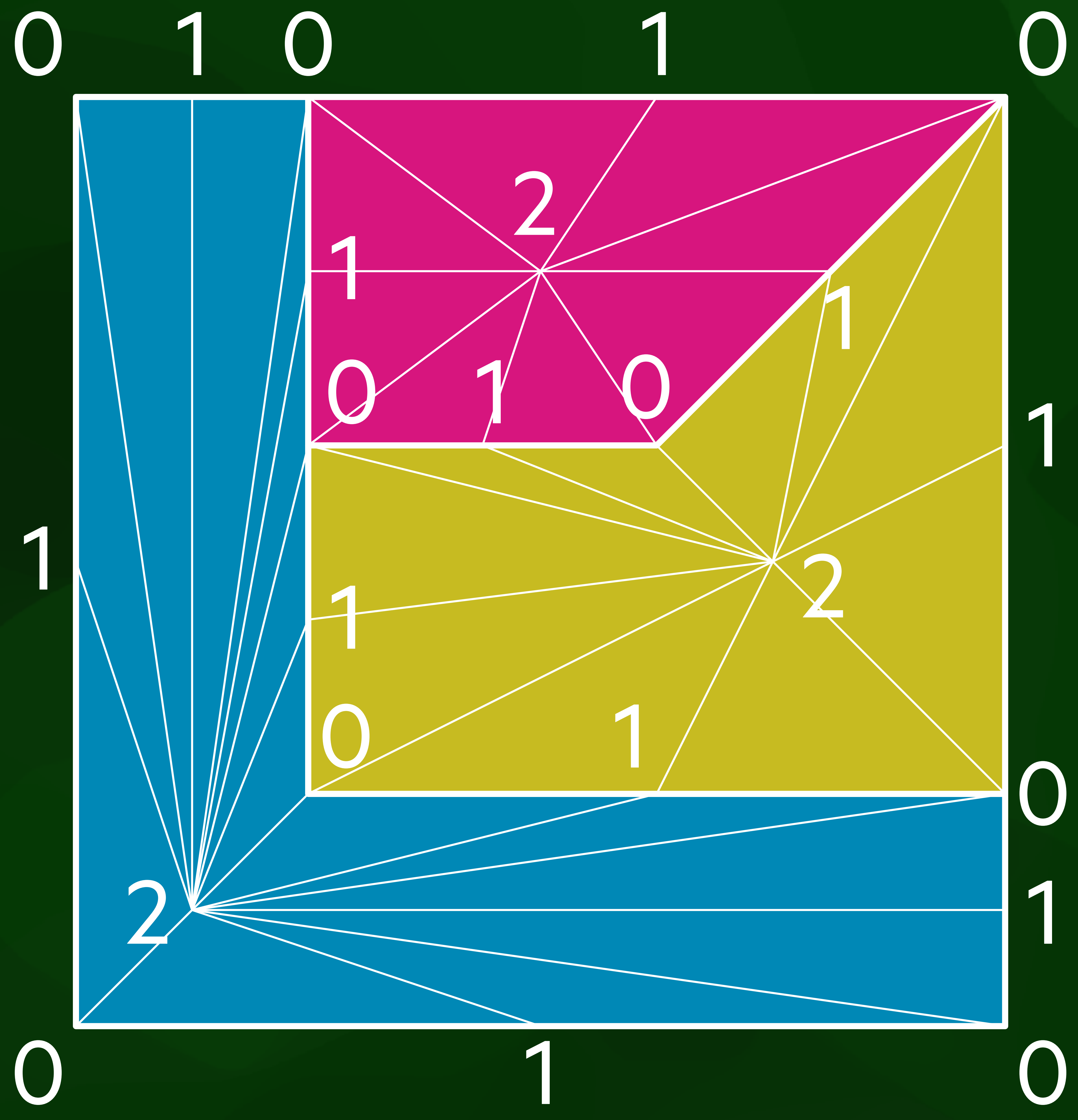
Traversing darts

- for all i, α_i is an involution
- for β > 1, β_i is also an involution
- but for for β = 1, β_i is a permutation



combinatorial map

(c-map)



generalised map

(g-map)

Storage

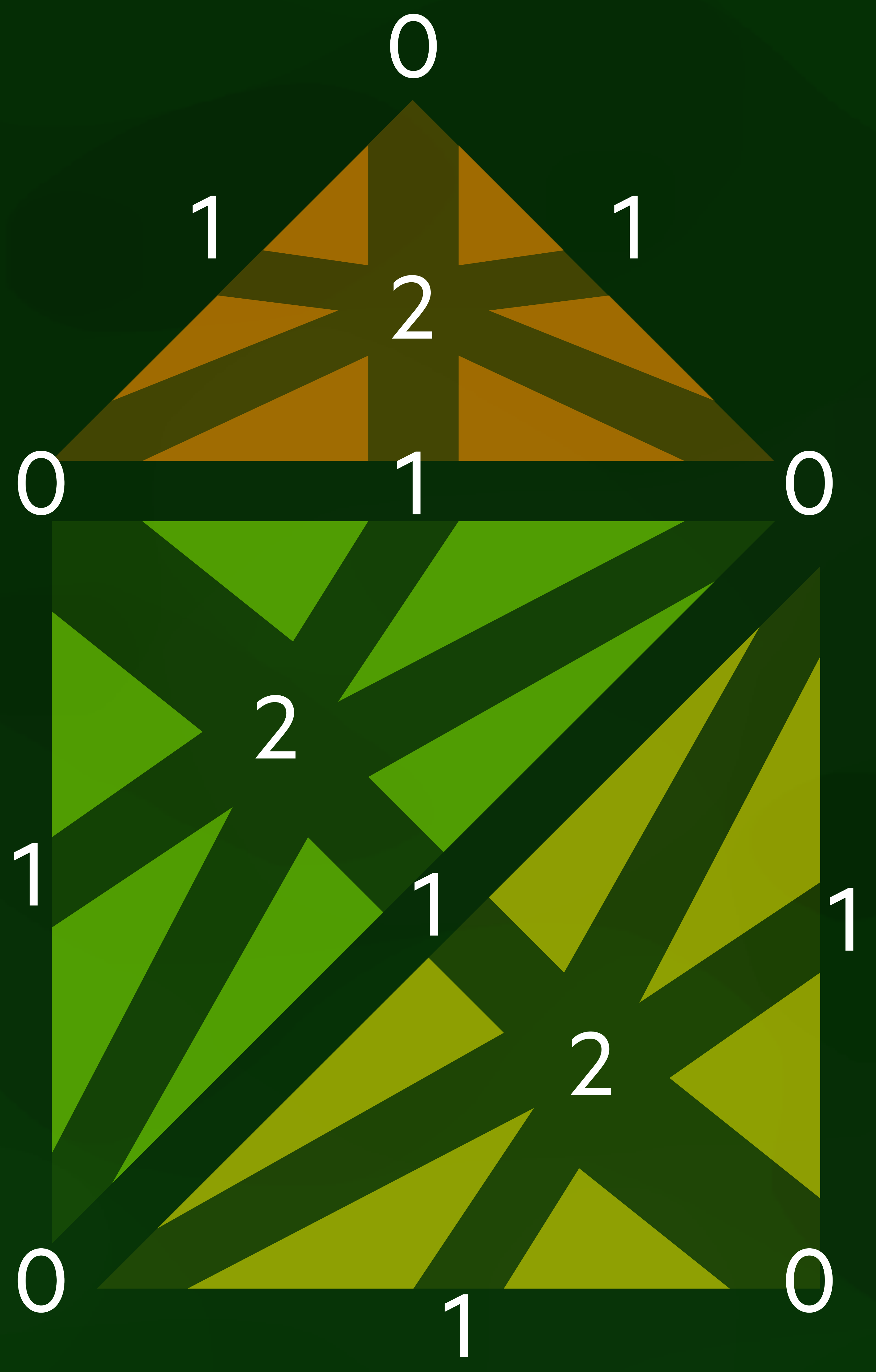
- In a generalised map, a list of darts of the form:
  - d = [[$\alpha_0(d)$, $\alpha_1(d)$, $\alpha_2(d)$, ...],

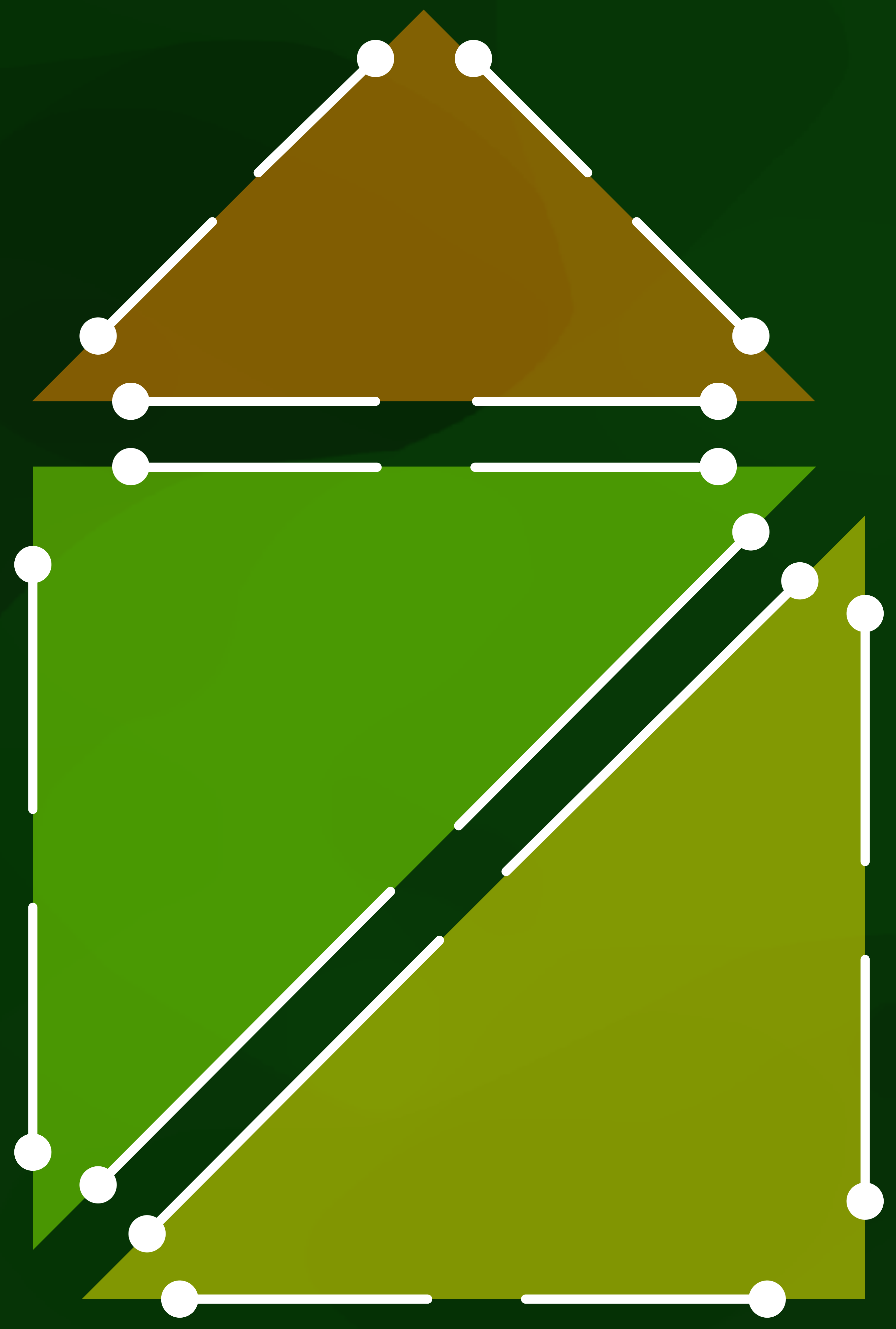
    [a_0, a_1, a_2, ...]]
  - where each α is a link (ID, pointer) to another dart, and
- In a combinatorial map, a list of darts of the form:
  - d = [[β_1, β_2, ...],

    [a_0, a_1, a_2, ...]]
  - where each β is a link (ID, pointer) to another dart, and
  - each a is an optional link to a data structure with the attributes for the i-cell of d, including the coordinates in a_0.

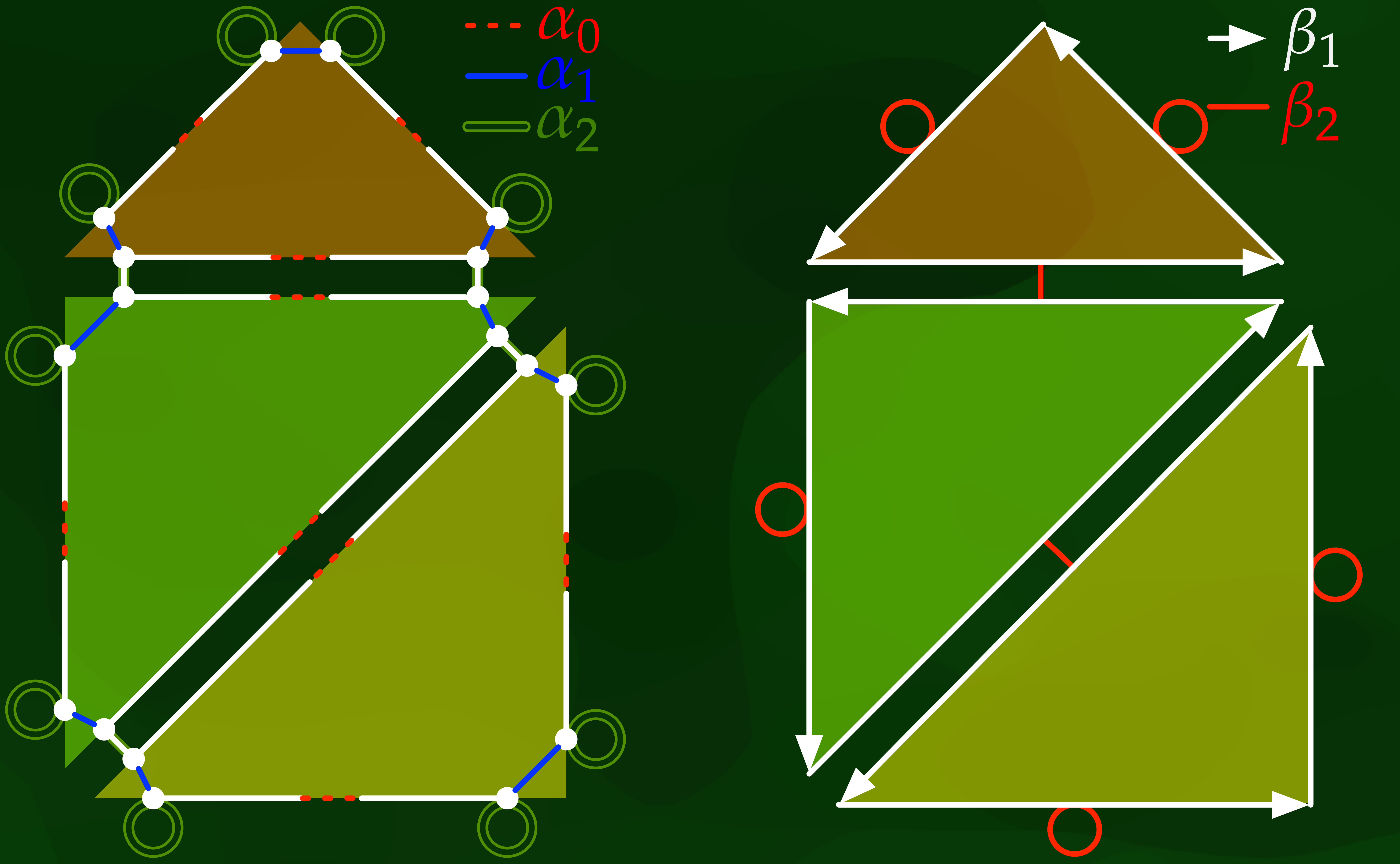
In practice? CGAL

Simpler representation

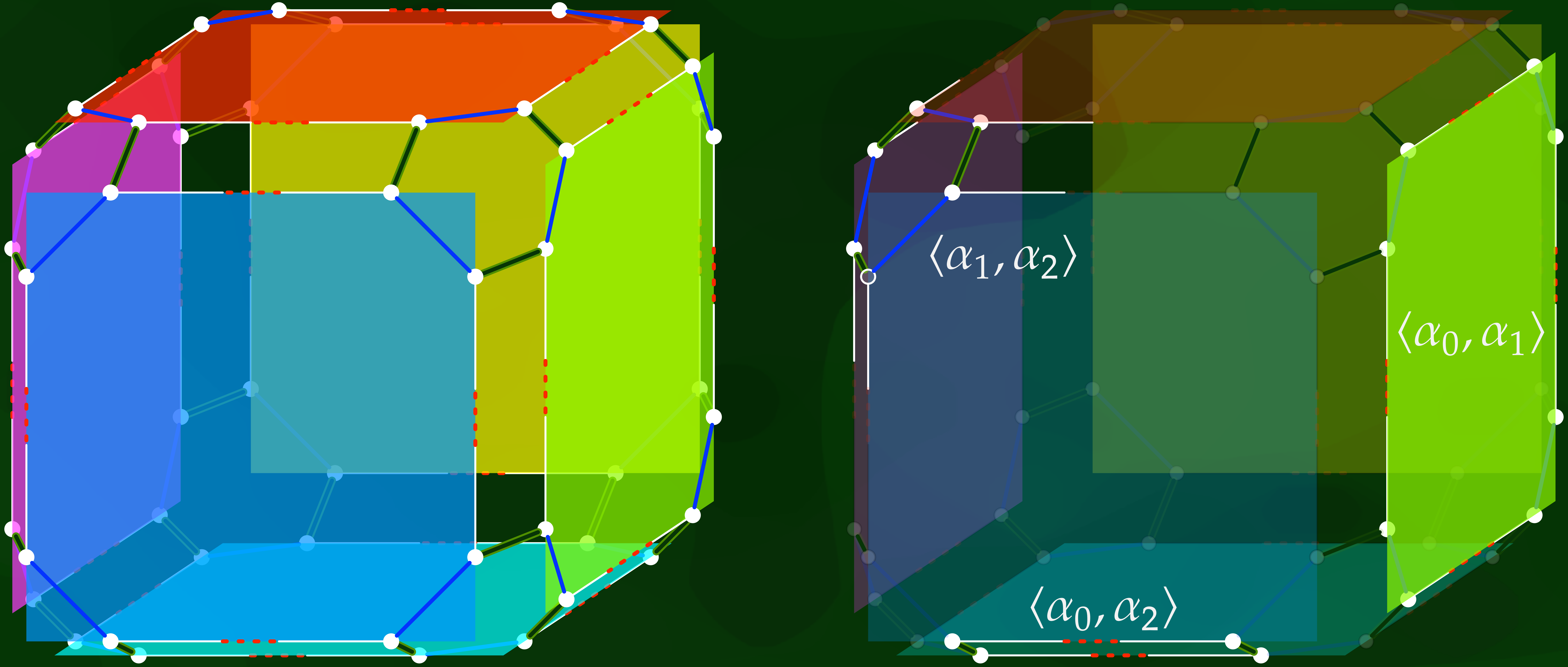




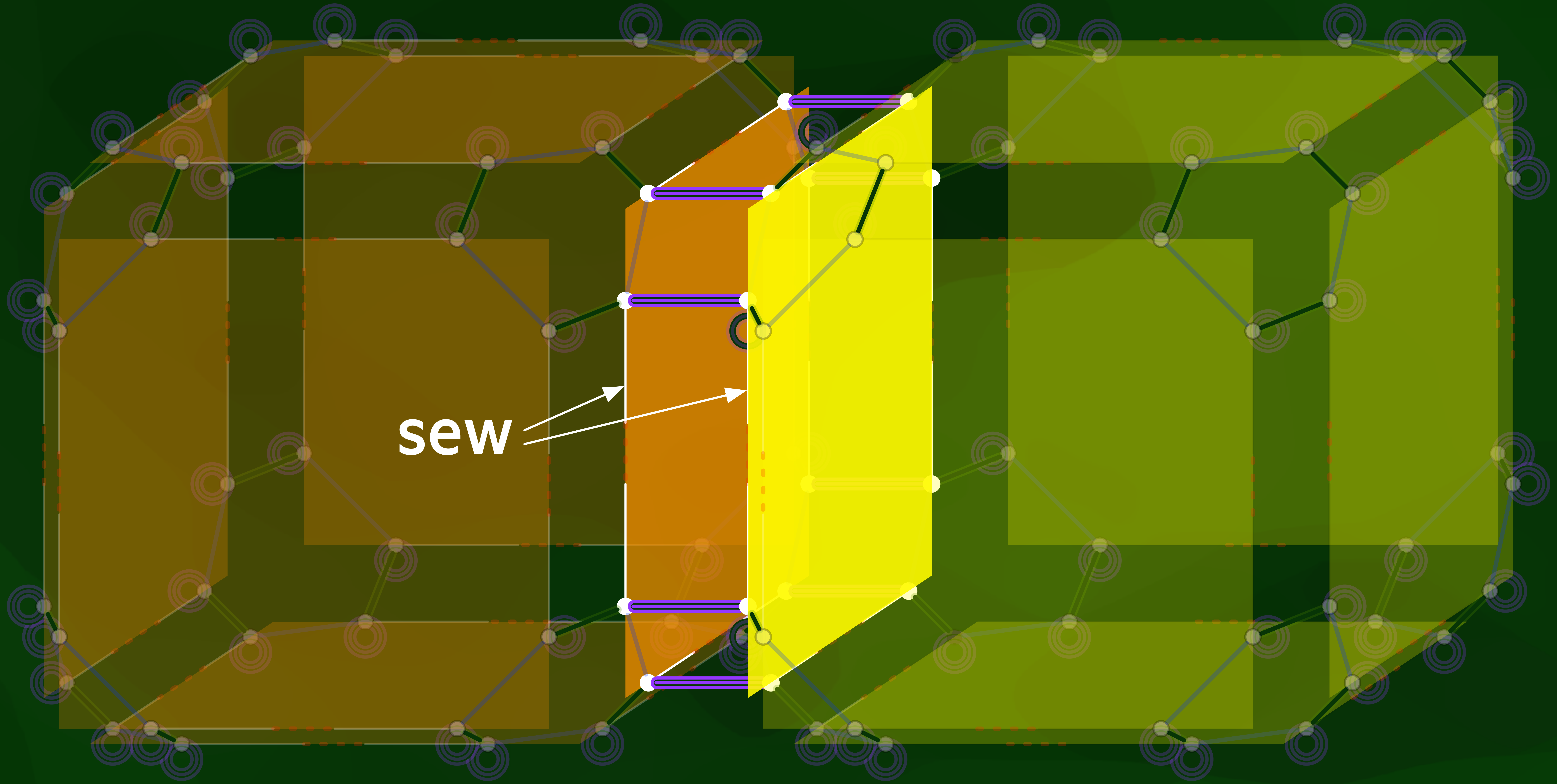
Involutions and permutations



Orbits


$\langle \alpha_1, \alpha_2 \rangle$
$\langle \alpha_0, \alpha_1 \rangle$
$\langle \alpha_0, \alpha_2 \rangle$

Sewing



What to do next?

1. Today:
    - Continue with Homework 2 (generalisation of a 3D city model)
    - Study for midterm exam (Lessons 1.1-4.1)
    - Go to geo1004 website and study today's lesson (3D book Chapter 8)
2. Wednesday: midterm exam (1st hour) and help with lessons or Hw 2 (2nd hour)
3. Thursday: help session with Dimitris

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