Lesson 4.1
Semantic 3D city models

GEO1004:
3D modelling of the built environment

https://3d.bk.tudelft.nl/courses/geo1004

3D geoinformation
Department of Urbanism
Faculty of Architecture and the Built Environment
Delft University of Technology

JSON

JSON

- **J**ava**S**cript **O**bject **N**otation
- Lightweight data-interchange format
- It is easy for humans to read and write.
- It is easy for machines to parse and generate.
- It is originally based on a subset of JavaScript (but is now in most languages)
- It is text-based


```
{
  "firstName": "John",
  "lastName": "Smith",
  "isAlive": true,
  "age": 27,
  "address": {
    "streetAddress": "21 2nd Street",
    "city": "New York",
    "state": "NY",
    "postalCode": "10021-3100"
  },
  "phoneNumbers": [
    {
      "type": "home",
      "number": "212 555-1234"
    },
    {
      "type": "office",
      "number": "646 555-4567"
    },
    {
      "type": "mobile",
      "number": "123 456-7890"
    }
  ],
  "children": [],
  "spouse": null
}
```

Virtually all languages have native support for it

It is built on two (very common) **data structures**:

1. collection of name-value pairs
   - Python dictionaries
   - C++ `std::map`
   - `also called a “hash”`
2. arrays
   - Python lists
   - C++ `std::vector` / `std::array`

Data types:

- number (float and double)
- integer
- string
- Boolean
- null
- *(no support for datetime type)*

To represent a person

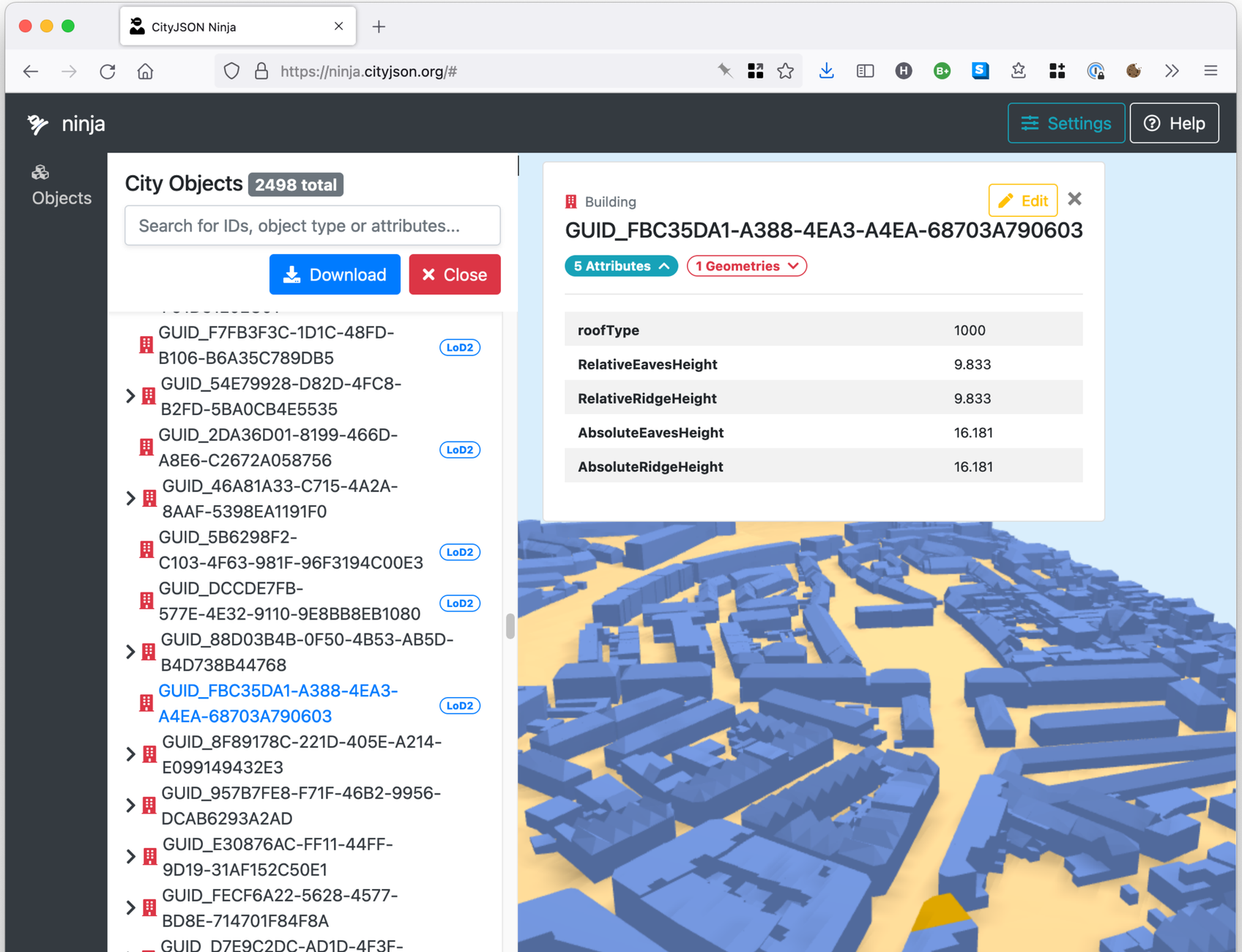
```
{
  "firstName": "John",
  "lastName": "Smith",
  "isAlive": true,
  "age": 27,
  "address": {
    "streetAddress": "21 2nd Street",
    "city": "New York",
    "state": "NY",
    "postalCode": "10021-3100"
  },
  "phoneNumbers": [
    {
      "type": "home",
      "number": "212 555-1234"
    },
    {
      "type": "office",
      "number": "646 555-4567"
    },
    {
      "type": "mobile",
      "number": "123 456-7890"
    }
  ],
  "children": [],
  "spouse": null
}
```

A CityJSON file


```
{
  "type": "CityJSON",
  "version": "1.1",
  "metadata": {
    "referenceSystem": "https://www.opengis.net/def/crs/EPSG/0/7415"
  },
  "transform": {…}
  "CityObjects": {
    "id-1": {
      "type": "Building",
      "attributes": {
        "measuredHeight": 22.3,
        "roofType": "gable",
        "owner": "Elvis Presley"
      },
      "geometry": [
        {
          "type": "MultiSurface",
          "boundaries": [
            [[0, 3, 2, 1]], [[4, 5, 6, 7]], [[0, 1, 5, 4]]
          ]
        }
      ]
    }
  },
  "vertices": [
    [231, 23212, 110],
    [1111, 3211, 120],
    ...
  ],
  "appearance": {
    "materials": [],
    "textures":[],
    "vertices-texture": []
  }
}
```

CityJSON software

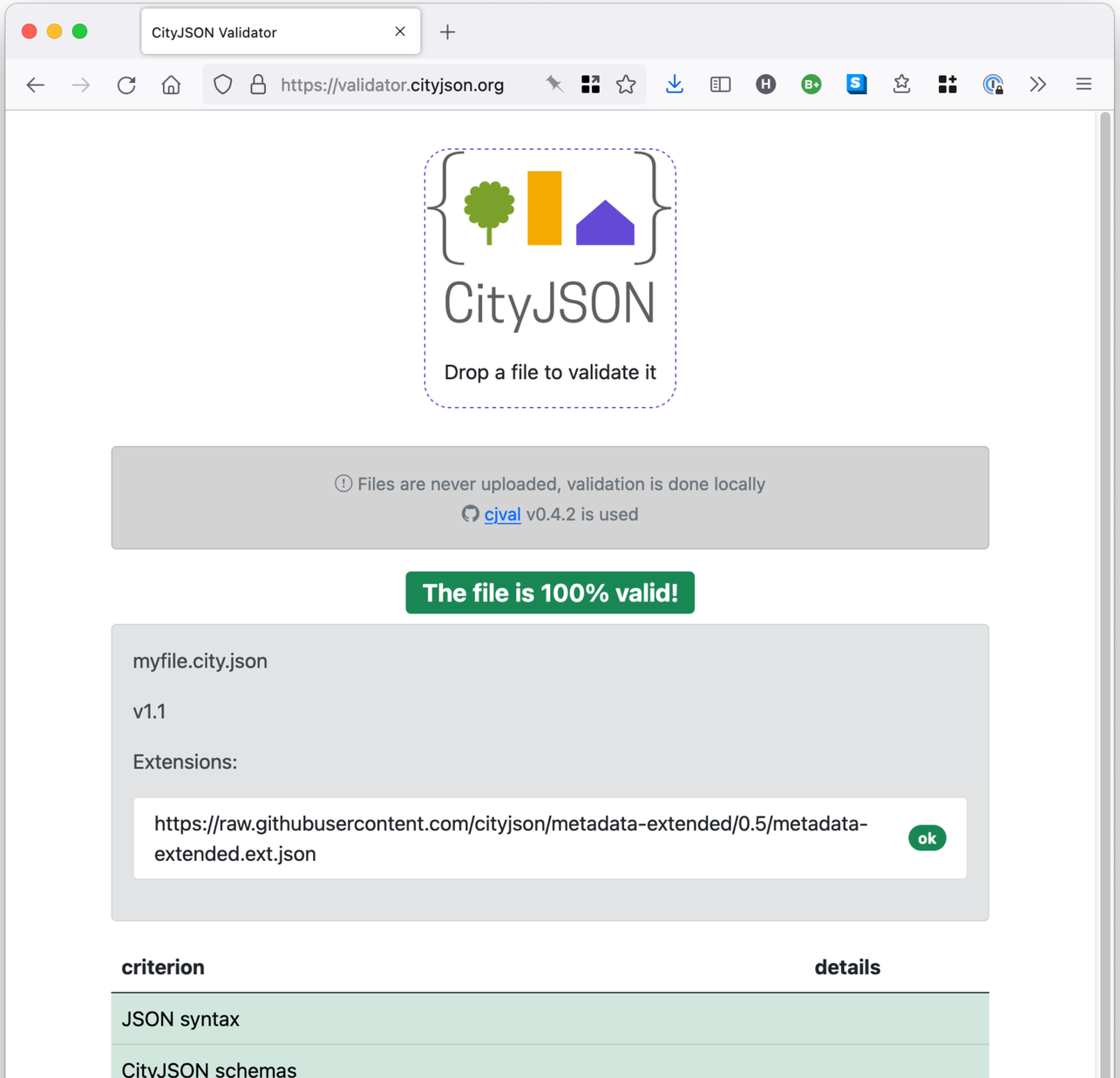
web-viewer: ninja.cityjson.org



Cool fact: programmed mostly by 2 Geomatics students, as projects for GE05010

You can do the same (or similar!)

Validation of the syntax of a file: cjval




```
{
  "type": "CityJSON",
  "version": "1.1",
  "metadata": {
    "referenceSystem": "https://www.opengis.net/def/crs/EPSG/0/7415"
  },
  "transform": {…}
  "CityObjects": {
    "id-1": {
      "type": "Buildnig",
      "attributes": {
        "measuredHeight": 22.3,
        "roofType": "gable",
        "owner": "Elvis Presley"
      },
      "geometry": [
        {
          "type": "MultiSurface",
          "boundaries": [
            [[0, 3, 2, 1]], [[4, 5, 6, 7], [[0, 1, 5, 4]]
          ]
        }
      ]
    }
  },
  "vertices": [
    [231, 23212, 110.223],
    [1111, 3211, 120],
    ...
  ],
  "appearance": {
    "materials": [],
    "textures":[],
    "vertices-texture": []
  }
}
```

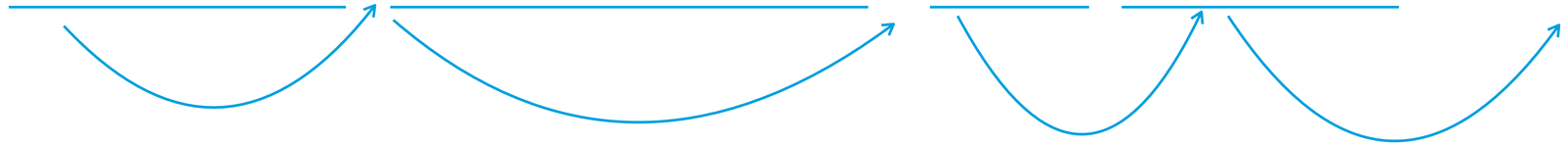
cjio (CityJSON/io) => pip install cjio

2. bash

```
Hugos-MacBook-Pro:rotterdam hugo$ cjio
Usage: cjio [OPTIONS] INPUT COMMAND1 [ARGS]... [COMMAND2 [ARGS]...]...

  Process and manipulate a CityJSON file, and allow different outputs. The
  different operators can be chained to perform several processing in one
  step, the CityJSON model goes through the different operators.

  To get help on specific command, eg for 'validate':

      cjio validate --help

  Usage examples:

      cjio example.json validate
      cjio example.json remove_textures info
      cjio example.json subset --id house12 remove_materials save out.json

Options:
  --version                Show the version and exit.
  --off                    Load an OFF file and convert it to one CityJSON
                           GenericCityObject.
  --ignore_duplicate_keys  Load a CityJSON file even if some City Objects have
                           the same IDs (technically invalid file)
  --help                   Show this message and exit.

Commands:
  compress                   Compress a CityJSON file, ie stores its...
  decompress                 Decompress a CityJSON file, ie remove the...
  info                       Output info in simple JSON.
  merge                      Merge the current CityJSON with others.
  remove_duplicate_vertices  Remove duplicate vertices a CityJSON file.
  remove_materials           Remove all materials from a CityJSON file.
  remove_orphan_vertices     Remove orphan vertices a CityJSON file.
  remove_textures            Remove all textures from a CityJSON file.
  save                       Save the CityJSON to a file.
  subset                     Create a subset of a CityJSON file.
  update_bbox                Update the bbox of a CityJSON file.
  update_crs                 Update the CRS with a new value.
  validate                   Validate the CityJSON file: (1) against its...
```

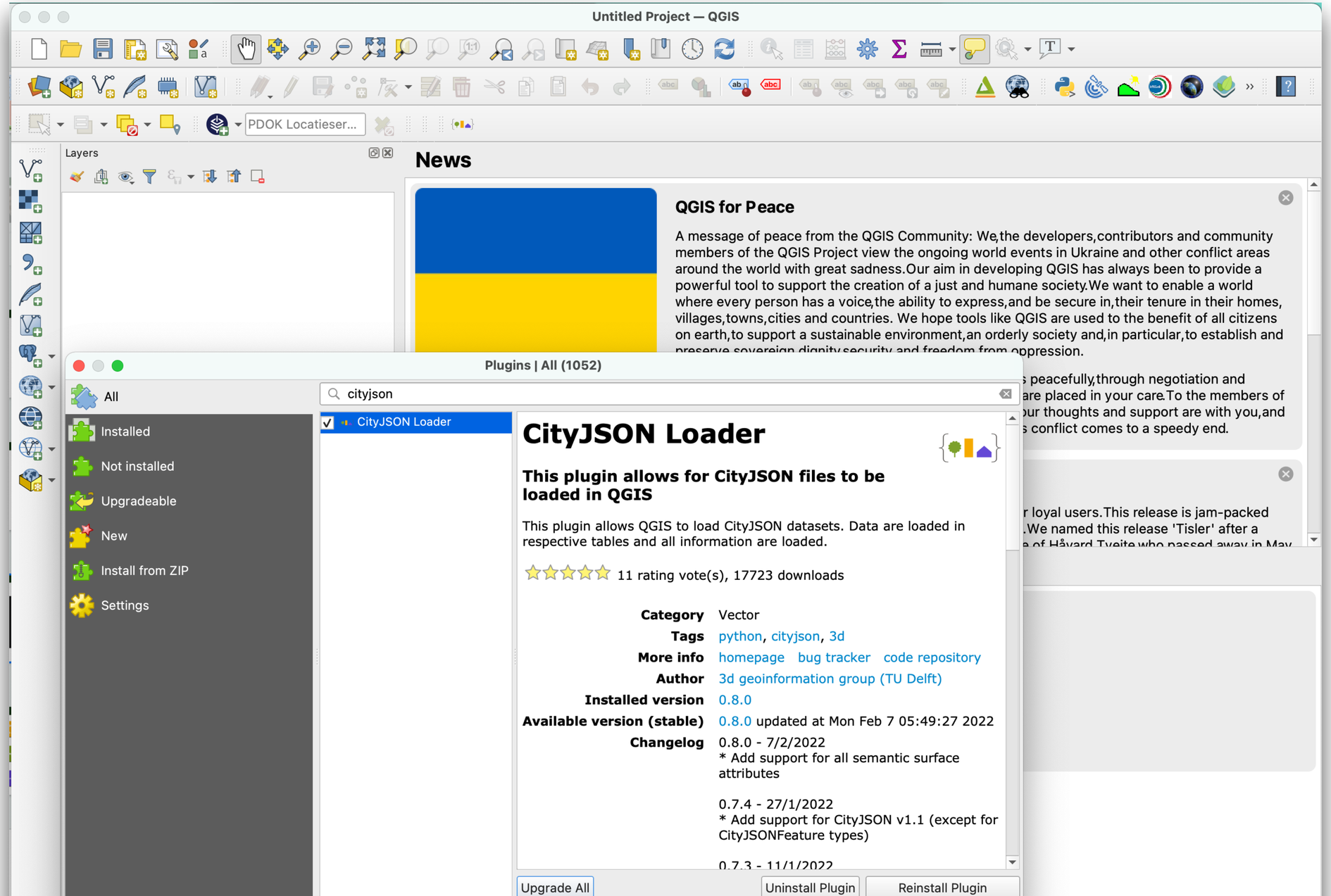
cjio (CityJSON/io)

```
$ cjio myfile.json crs_assign 7415 subset --cotype Building lod_filter 2.2 save out.json
```

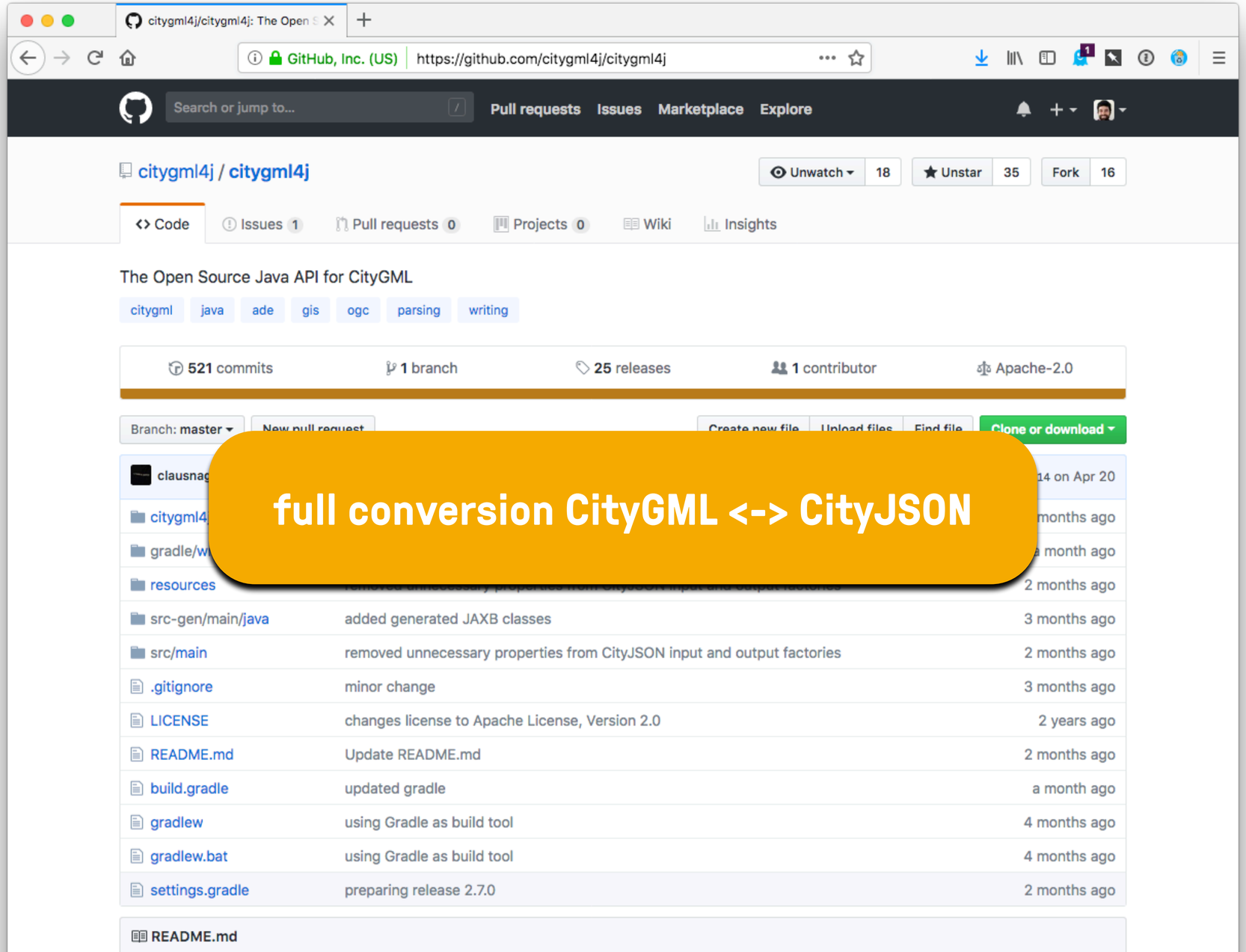


No need to save temp files between operators: pipeline is used

QGIS plugin


Untitled Project — QGIS
PDOK Locatieser...
Layers
News
QGIS for Peace
A message of peace from the QGIS Community: We,the developers,contributors and community members of the QGIS Project view the ongoing world events in Ukraine and other conflict areas around the world with great sadness.Our aim in developing QGIS has always been to provide a powerful tool to support the creation of a just and humane society.We want to enable a world where every person has a voice,the ability to express,and be secure in,their tenure in their homes, villages,towns,cities and countries. We hope tools like QGIS are used to the benefit of all citizens on earth,to support a sustainable environment,an orderly society and,in particular,to establish and
peacefully,through negotiation and
are placed in your care.To the members of
our thoughts and support are with you,and
conflict comes to a speedy end.
loyal users.This release is jam-packed
.We named this release 'Tisler' after a
Plugins | All (1052)
All
Installed
Not installed
Upgradeable
New
Install from ZIP
Settings
cityjson
CityJSON Loader
CityJSON Loader
This plugin allows for CityJSON files to be loaded in QGIS
This plugin allows QGIS to load CityJSON datasets. Data are loaded in respective tables and all information are loaded.
11 rating vote(s), 17723 downloads
Category Vector
Tags python, cityjson, 3d
More info homepage bug tracker code repository
Author 3d geoinformation group (TU Delft)
Installed version 0.8.0
Available version (stable) 0.8.0 updated at Mon Feb 7 05:49:27 2022
Changelog 0.8.0 - 7/2/2022
* Add support for all semantic surface attributes
0.7.4 - 27/1/2022
* Add support for CityJSON v1.1 (except for CityJSONFeature types)
0.7.3 - 11/1/2022
Upgrade All
Uninstall Plugin
Reinstall Plugin

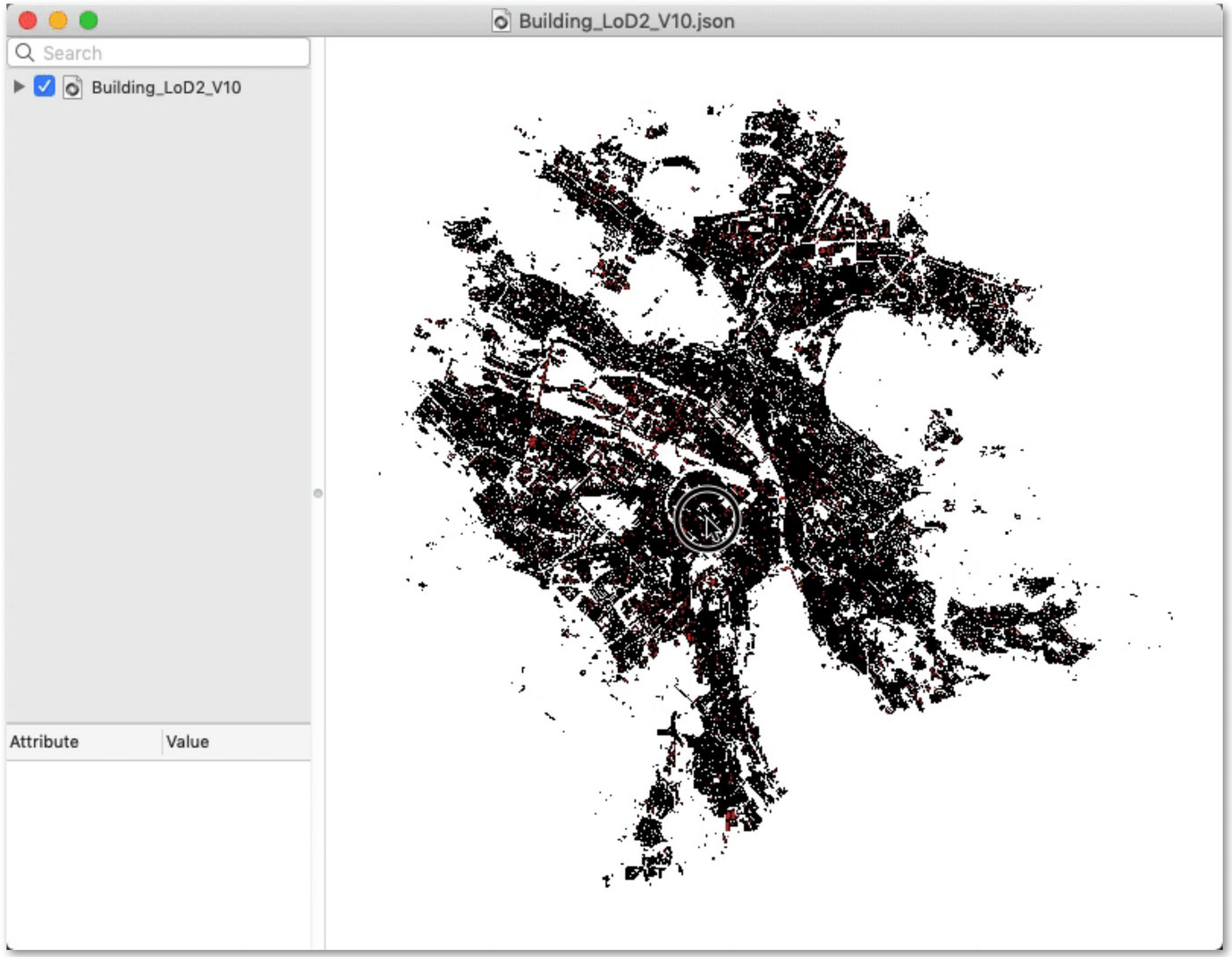
citygml4j


citygml4j/citygml4j: The Open
GitHub, Inc. (US)
https://github.com/citygml4j/citygml4j
Search or jump to...
Pull requests
Issues
Marketplace
Explore
citygml4j / citygml4j
Unwatch 18
Unstar 35
Fork 16
Code
Issues 1
Pull requests 0
Projects 0
Wiki
Insights
The Open Source Java API for CityGML
citygml
java
ade
gis
ogc
parsing
writing
521 commits
1 branch
25 releases
1 contributor
Apache-2.0
Branch: master
Clone or download
full conversion CityGML <-> CityJSON
resources
src-gen/main/java
added generated JAXB classes
3 months ago
src/main
removed unnecessary properties from CityJSON input and output factories
2 months ago
.gitignore
minor change
3 months ago
LICENSE
changes license to Apache License, Version 2.0
2 years ago
README.md
Update README.md
2 months ago
build.gradle
updated gradle
a month ago
gradlew
using Gradle as build tool
4 months ago
gradlew.bat
using Gradle as build tool
4 months ago
settings.gradle
preparing release 2.7.0
2 months ago
README.md

Compression factor == ~6X

file	CityGML size (original)	CityGML size (w/o spaces)	textures?	CityJSON	CityJSON compressed	compression factor
CityGML demo "GeoRes"	4.3MB	4.1MB	yes	582KB	524KB	8.0
CityGML v2 demo "Railway"	45MB	34MB	yes	4.5MB	4.3MB	8.1
Den Haag "tile 01"	23MB	18MB	no, material	3.1MB	2.9MB	6.2
Montréal VM05	56MB	42MB	yes	5.7MB	5.4MB	7.8
New York LoD2 (DA13)	590MB	574MB	no	110MB	105MB	5.5
Rotterdam Delfshaven	16MB	15MB	yes	2.8MB	2.6MB	5.4
Vienna	37MB	36MB	no	5.6MB	5.3MB	6.8

One example: Zürich LoD2 buildings



CityGML = 3.0GB

(but 1GB of spaces/CRs/tabs!)

CityJSON = 292MB

Compression == 7.1X

Getting started?

Getting started with CityJSON | City

https://www.cityjson.org/tutorials/getting-started/

CityJSON

Search CityJSON

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  - Converting to/from CityGML files
  - Validation of a CityJSON file
  - Mapping the Noise ADE to a CityJSON Extension
  - Upgrading to v1.1

Need help? Want to contribute?
Spotted an error?

Tutorials / Getting started with CityJSON

Getting started with CityJSON

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1 Download a simple file with 2 buildings
2 Visualise it
3 Manipulate and edit it with cjio
4 What else?
5 Questions and need help?

Download a simple file with 2 buildings

Download twobuildings.city.json, a simple file with 2 buildings.

You can open that file in any text editor to see its structure, and notice that you can manually edit it to change values and/or add new buildings, new metadata, or delete some attributes.

twobuildings.city.json

```
{
    "type": "CityJSON",
    "version": "1.1",
    "metadata":
    {
        "geographicalExtent":
        [
            300578.235,
            5041258.061,
            13.688,
            300618.138,
            5041289.394,
            29.45
        ]
    },
    "CityObjects":
    {
        "Building_1":
        {
            "geometry":
            [
                {
                    "boundaries":
                    [
                        [
                            [
                                0,
                                1
```

Validation of a CityJSON file | CityJS

https://www.cityjson.org/tutorials/validation/

CityJSON

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Tutorials / Validation of a CityJSON file

Validation of a CityJSON file

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1 Schema validation (is the syntax of the file OK?)
2 Geometry (are the geometric primitives valid?)

Validation of a CityJSON dataset means that one must ensure that it respects the standardised specifications and definitions as given in the specifications.

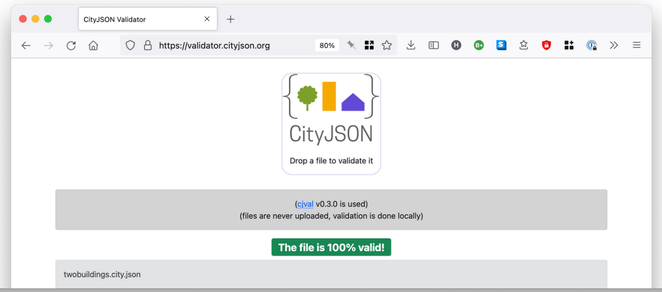
Schema validation (is the syntax of the file OK?)

The JSON schemas of CityJSON can be downloaded, for each version, at https://www.cityjson.org/schemas/. These are based on the JSON Schema project.

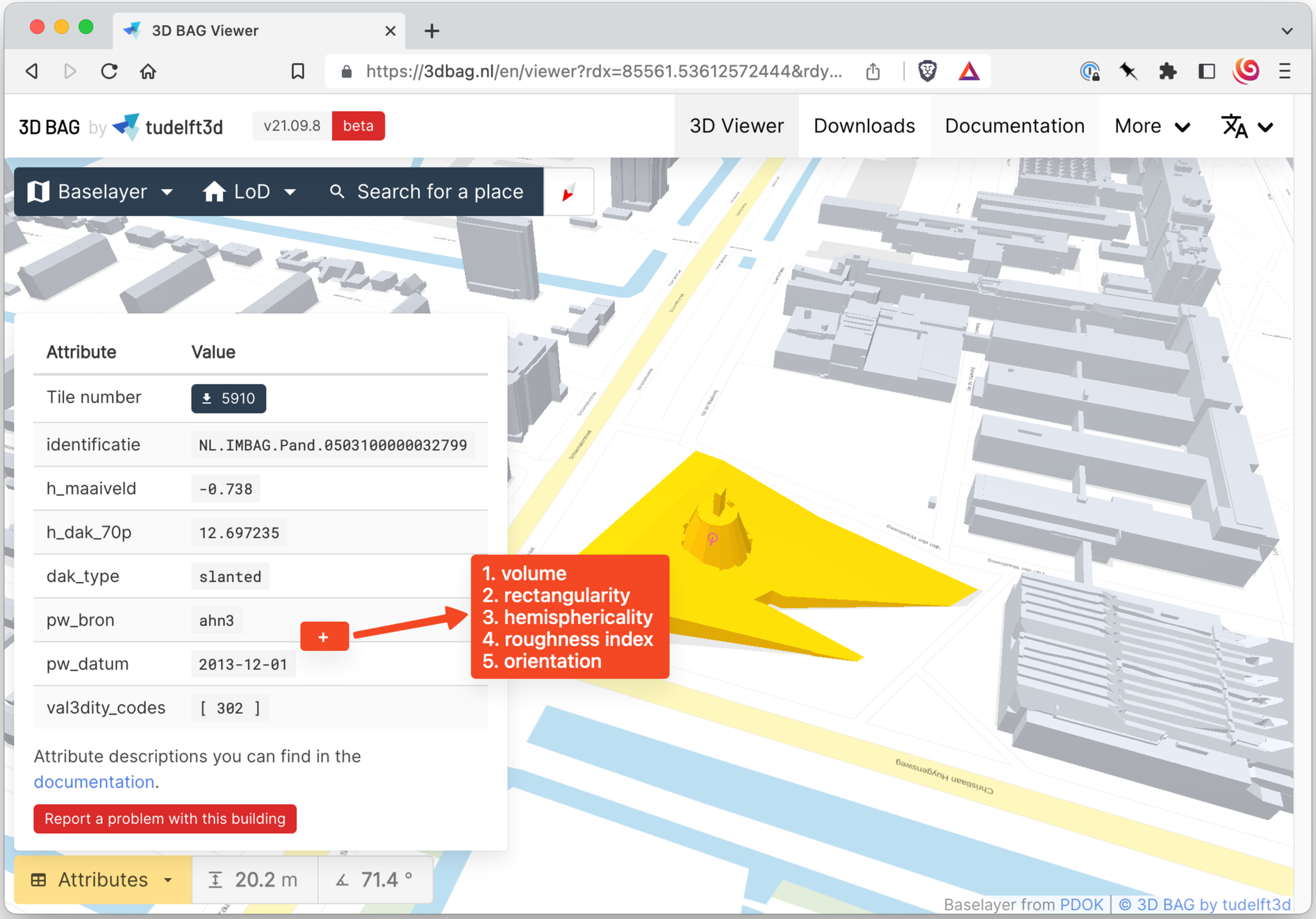
To validate a given file you can use any software listed here. However, it is rather tricky to stitch all the schemas together, and the handling of Extensions will not work.

The "official validator" for CityJSON is cjval, which is available as a web-app and with cjio.

To validate the file twobuildings.city.json, simply drag it to https://validator.cityjson.org:



hw02


3D BAG Viewer
https://3dbag.nl/en/viewer?rdx=85561.53612572444&rdy...
3D BAG by tudelft3d
v21.09.8
beta
3D Viewer
Downloads
Documentation
More
Baselayer
LoD
Search for a place
Attribute
Value
Tile number
5910
identificatie
NL.IMBAG.Pand.0503100000032799
h_maaiveld
-0.738
h_dak_70p
12.697235
dak_type
slanted
pw_bron
ahn3
pw_datum
2013-12-01
val3dity_codes
[302]
Attribute descriptions you can find in the documentation.
Report a problem with this building
+
1. volume
2. rectangularity
3. hemisphericality
4. roughness index
5. orientation
Attributes
20.2 m
71.4 °
Baselayer from PDOK | © 3D BAG by tudelft3d

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RESEARCH ARTICLE

OPEN ACCESS Check for updates

3D building metrics for urban morphology

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[a]3D Geoinformation Group, Delft University of Technology, Delft, The Netherlands; [b]Department of Architecture, National University of Singapore, Singapore, Singapore; [c]Department of Real Estate, National University of Singapore, Singapore, Singapore

ABSTRACT

Urban morphology is important in a broad range of investigations across the fields of city planning, transportation, climate, energy, and urban data science. Characterising buildings with a set of numerical metrics is fundamental to studying the urban form. Despite the rapid developments in 3D geoinformation science, and the growing 3D data availability, most studies simplify buildings to their 2D footprint, and when taking their height into account, they at most assume one height value per building, i.e. simple 3D. We take the first step in elevating building metrics into full/true 3D, uncovering the use of higher levels of detail, and taking into account the detailed shape of a building. We set the foundation of the new research line on 3D urban morphology by providing a comprehensive set of 3D metrics, implementing them in openly released software, generating an open dataset containing 2D and 3D metrics for 823,000 buildings in the Netherlands, and demonstrating a use case where clusters and architectural patterns are analysed through time. Our experiments suggest the added value of 3D metrics to complement existing counterparts, reducing ambiguity, and providing advanced insights. Furthermore, we provide a comparative analysis using different levels of detail of 3D building models.

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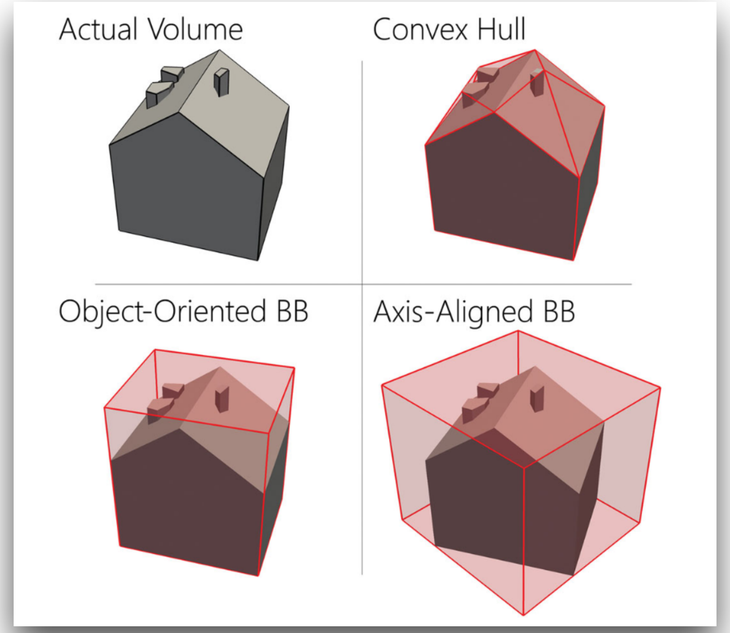
KEYWORDS
3D GIS; urban fabric; urban modelling; morphometrics; urban indicators



Appendix Table 1. Calculating shape indices in 2D and 3D.

Index	2D	3D
Circularity/Hemisphericality	$CI = \frac{A_{PN}}{A_{PC}} = \frac{4\pi A_{PN}}{P_{PN}^2}$	$HEM = \frac{3\sqrt{2\pi V}}{\sqrt{A_{VS}^3}}$
Convexity	$CNV_2 = \frac{A_{PN}}{A_{CH}}$	$CNV_3 = \frac{V_{VS}}{V_{CH}}$
Fractality	$FR_2 = 1 - \frac{\log(A_{PN})}{2 \times \log(P_{PN})}$	$FR_3 = 1 - \frac{\log(V_{VS})}{\frac{3}{2} \times \log(A_{VS})}$
Rectangularity/Cuboidness	$REC = \frac{A_{PN}}{A_{MABR}}$	$CBD = \frac{V_{VS}}{V_{OOBB}}$
Squareness/Cubeness	$SQN = \frac{P_{EAC}}{P_{PN}} = \frac{4\sqrt{A_{PN}}}{P_{PN}}$	$CBN = \frac{A_{DS}}{A_{VS}} = \frac{6\sqrt[3]{V_{VS}^2}}{A_{VS}}$
Cohesion	$nCI_2 = \frac{0.9054 \times \sqrt{\frac{A_{PN}}{\pi}}}{\frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{dgp_{ij}}}$	$nCI_3 = \frac{\frac{16}{15} \times \sqrt[3]{\frac{3V_{VS}}{4\pi}}}{\frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{dgp_{ij}}}$
Proximity	$nPxI_2 = \frac{\frac{2}{3} \times r_{EAC}}{\mu_{r_{dgp}}} = \frac{\frac{1}{3} \times \sqrt{\frac{A_{PN}}{\pi}}}{\mu_{r_{dgp}}}$	$nPxI_3 = \frac{\frac{3}{4} \times r_{EVS}}{\mu_{r_{dgp}}} = \frac{\frac{3}{4} \times \sqrt[3]{\frac{3V_{VS}}{4\pi}}}{\mu_{r_{dgp}}}$
Exchange	$nEI_2 = \frac{A_{(PN \cap EAC)}}{A_{PN}}$	$nEI_3 = \frac{V_{(VS \cap EVS)}}{V_{VS}}$
Spin	$nSI_2 = \frac{0.5 \times \Delta_{PN}}{\mu_{r_{dgp}}^2}$	$nSI_3 = \frac{\frac{1}{2} \times \sqrt{\frac{5 \times V_{VS}}{4\pi}}^2}{\mu_{r_{dgp}}^2}$
Perimeter/Circumference	$nPmI_2 = \frac{P_{EAC}}{P_{PN}} = \frac{2\sqrt{\pi A_{PN}}}{P_{PN}}$	$nPmI_3 = \frac{4\pi \times \sqrt{\frac{5 \times V_{VS}}{4\pi}}^2}{A_{VS}}$
Depth	$nDpI_2 = \frac{\mu_{PN_{d_{dgp,b}}}}{\mu_{EAC_{d_{dgp,b}}}} = \frac{3\mu_{PN_{d_{dgp,b}}}}{\sqrt{\frac{A_{PN}}{\pi}}}$	$nDpI_3 = \frac{4 \times \mu_{VS_{d_{dgp,b}}}}{\sqrt{\frac{5 \times V_{VS}}{4\pi}}}$
Girth	$nGI_2 = \frac{r_{LIC}}{r_{EAC}} = \frac{r_{LIC}}{\sqrt{\frac{A_{PN}}{\pi}}}$	$nGI_3 = \frac{r_{LIS}}{\sqrt[3]{\frac{3 \times V_{VS}}{4\pi}}}$
Dispersion	$nDsI_2 = 1 - \frac{\mu_{r_{dev.}}}{r_{ADC}} = 1 - \frac{\mu_{r_{dev.}}}{\mu_{r_{dgp}}}$	$nDsI_3 = 1 - \frac{\mu_{r_{dev.}}}{r_{ADS}} = 1 - \frac{\mu_{r_{dev.}}}{\mu_{r_{dgp}}}$
Range	$nRI_2 = \frac{r_{LAC}}{r_{SCC}} = \frac{\sqrt{\frac{A_{PN}}{\pi}}}{r_{SCC}}$	$nRI_3 = \frac{\sqrt[3]{\frac{3 \times V_{VS}}{4\pi}}}{r_{SCS}}$
Equivalent rectangular/ cuboid index	(1) $k = \sqrt{\frac{A_{PN}}{A_{MABR}}}$ $(k \leq 1)$ (2) $P_{EAR} = k \times P_{MABR}$ (3) $ERI = \frac{P_{EAR}}{P_{PN}} = \sqrt{\frac{A_{PN}}{A_{MABR}}} \times \frac{P_{MABR}}{P_{PN}}$	$ECI = \sqrt[3]{\frac{V_{VS}}{V_{OOBB}}}^2 \times \frac{A_{OOBB}}{A_{VS}}$
Roughness index*	$RI_2 = \frac{\mu_{r_{bp}}^2}{A_{PN} \div P_{PN}} \times 42.62$	$RI_3 = \frac{\mu_{r_{bp}}^3}{V_{VS} \div \sqrt{A_{VS}}} \times 48.735$
Elongation	$1 - \frac{s}{l}$ (Computed for the three axes respectively)	
Form factor	–	$\frac{A}{V^{\frac{2}{3}}}$

*For *roughness index* the constants 42.62 and 48.735 are used because without them the resulting values for a circle or a sphere would be $RI_2 = 1/42.62$ and $RI_3 = 1/48.735$, respectively.

How to calculate the volume of a (invalid) solid

TU Delft 3Dgeoinfo

Vector primer
Main geometric algorithms

Area of polygon

Area of a simple polygon 1/2

- Let P be a simple polygon (no boundary self-intersections) with vertex vectors

$$P = ((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n))$$

where $(x_1, y_1) = (x_n, y_n)$

- Then the area of the polygon P is

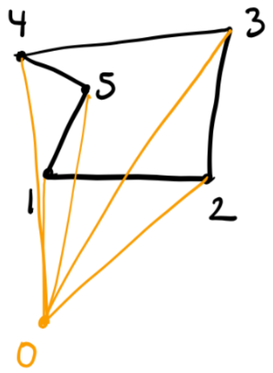
$$\mathbf{area}(P) = \frac{1}{2}\sum_{i=1}^{n-1} x_i y_{i+1} - x_{i+1} y_i$$

area of 1 triangle

- In the case of a triangle *pqr*

$$\mathbf{area}(pqr) = \frac{x_p y_q - x_q y_p + x_q y_r - x_r y_q + x_r y_p - x_p y_r}{2}$$

14



Area → signed area

Δ 012 – negative
Δ 023 – positive
+ Δ 034 – positive
Δ 045 – negative
Δ 051 – positive

Inverse distance weighting (IDW). The generalisation of this method to three dimensions is straightforward: a searching *sphere* with a given radius is used. The same problems with the one-dimensionality of the method (the value for the search radius) will be even worse because the search must be performed in one more dimension. The method has too many problems to be considered has a viable solution for fields as found in geosciences: the interpolant is not guaranteed to be continuous, especially when the dataset has an anisotropic distribution, and the criterion has to be selected carefully by the user. Note that the implementation problems are also similar to the ones encountered with the previous method, and an auxiliary data structure must be used to avoid testing all the points in a dataset.

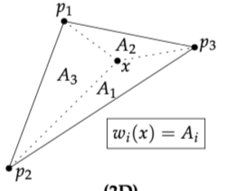
anisotropic distribution

Linear interpolation in tetrahedra. This is the generalisation of the popular linear interpolation in TINs where the tetrahedra of the Delaun[illegible] tetrahedralisation (DT) are used. The barycentric coordinates can be used to linearly interpolate inside a tetrahedron, as shown in Figure 13.3 the volumes of 4 tetrahedra are used (instead of the area for the 2[illegible] case.)

The volume of a d-simplex σ is easily computed:

$$vol(\sigma) = \frac{1}{d!}\left|\det\begin{pmatrix} v^0 & \cdots & v^d \\ 1 & \cdots & 1 \end{pmatrix}\right| \quad (13.1)$$

where v^i is a d-dimensional vector representing the coordinates of a vertex and det() is the determinant of the matrix.



(2D)

TU Delft 3Dgeoinfo

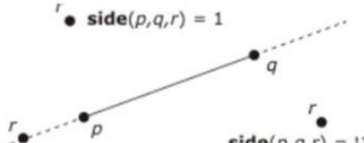
Vector primer
Main geometric algorithms

Area of polygon

Area of a simple polygon 2/2

- Note that the area may be positive or negative, in fact **area**(pqr) = -**area**(qpr)
- If *p* is to the left of *qr* then the area is positive, if *p* is to the right of *qr* then the area is negative

$$\mathbf{side}(p,q,r) = \begin{cases} 1 & \text{if } \mathbf{area}(pqr) > 0 \quad (p \text{ is left of } qr) \\ 0 & \text{if } \mathbf{area}(pqr) = 0 \quad (pqr \text{ are collinear}) \\ -1 & \text{if } \mathbf{area}(pqr) < 0 \quad (p \text{ is right of } qr) \end{cases}$$



How to calculate the volume of a (invalid) solid

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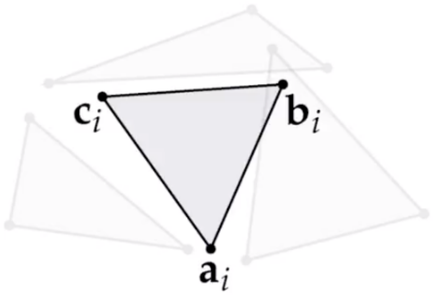
Need the volume of a broken triangle mesh?

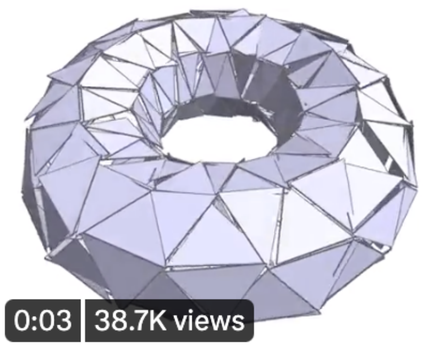
Don't bother fixing it.

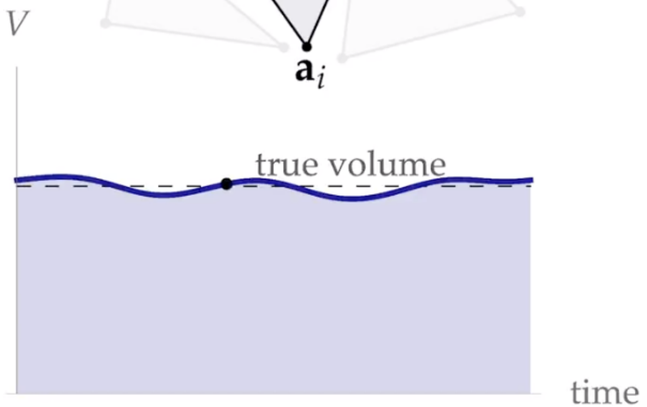
Just sum over all triangles ($\mathbf{a}_i$, $\mathbf{b}_i$, $\mathbf{c}_i$) the dot product of one vertex with the cross product of the other two, and divide by 6.

[Caveat: all normals must point out. Works even for nonconvex shapes!]

$$V = \frac{1}{6}\sum_i \mathbf{a}_i \cdot (\mathbf{b}_i \times \mathbf{c}_i)$$







Keenan Crane @keenanisalive · 22 Nov 2022

Need the area of a broken polygon?

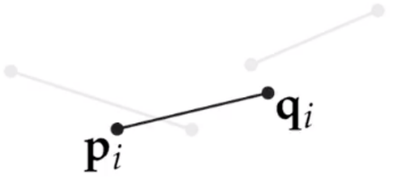
Don't bother fixing it.

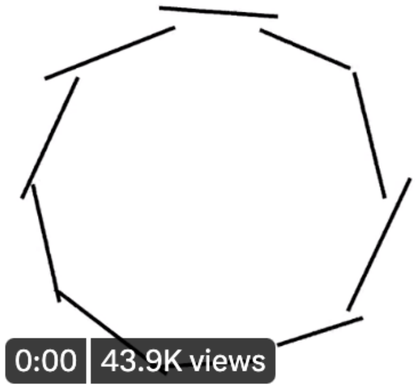
Just sum up the cross product of the two endpoints, and divide by two.

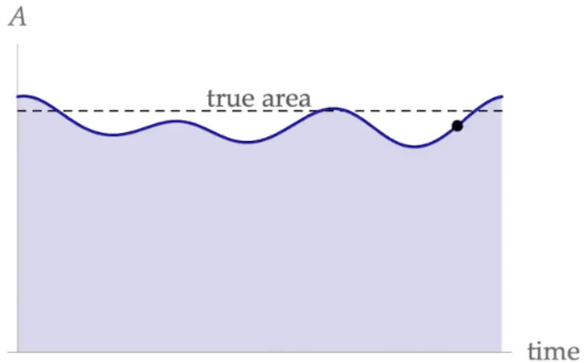
[Here $u \times v := u_1v_2 - u_2v_1$. Works even if the polygon is nonconvex. Caveat: all segments must point counter-clockwise, since $u \times v = -v \times u$!]

Show this thread

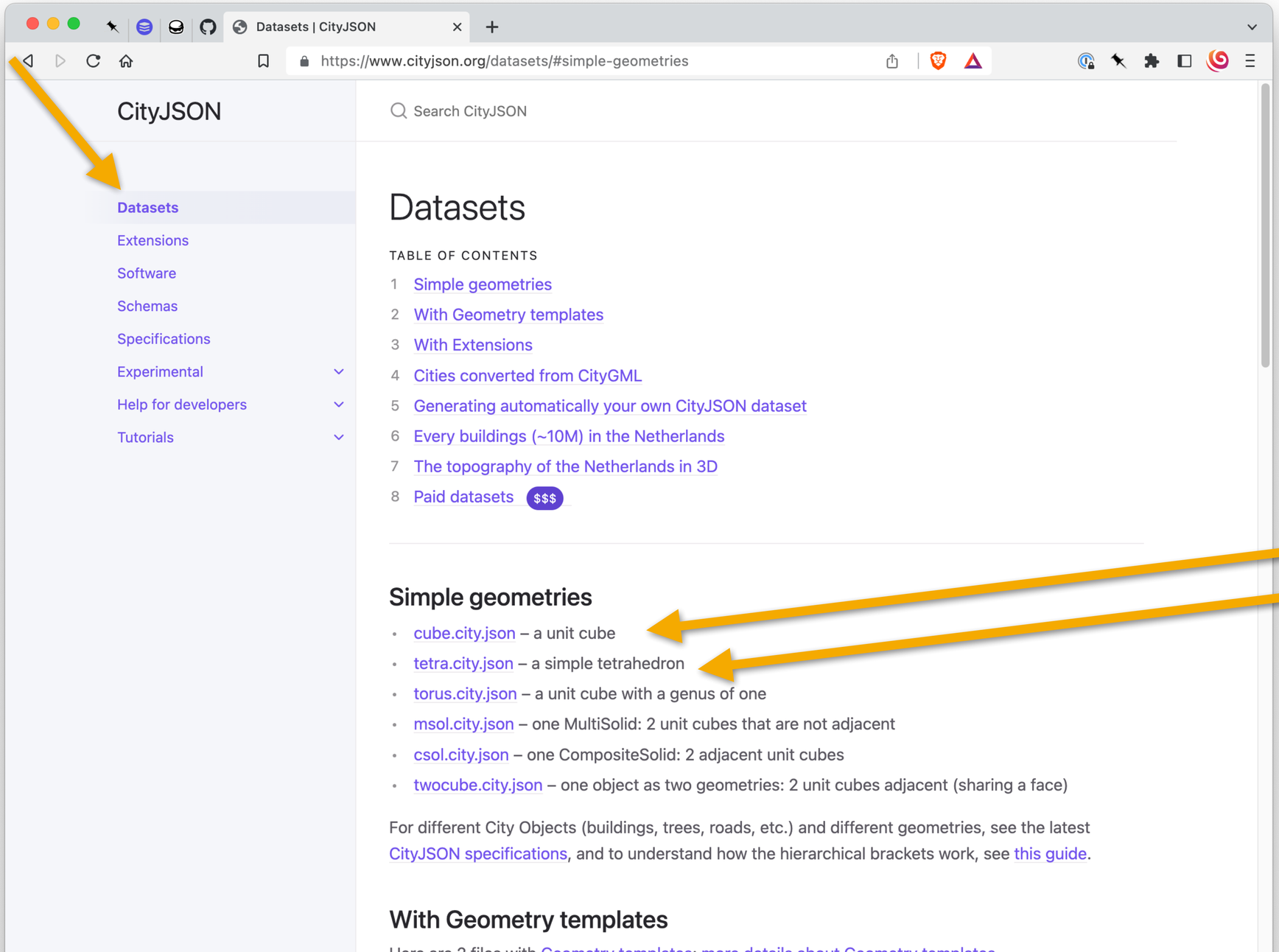
$$A = \frac{1}{2}\sum_{i=1}^{n} \mathbf{p}_i \times \mathbf{q}_i$$







Tip: start with simple shapes, not BK-City...



Download and modify those